City University of Hong Kong
Institute for Advanced Study

Stress Field
Photoelastic Visualisation

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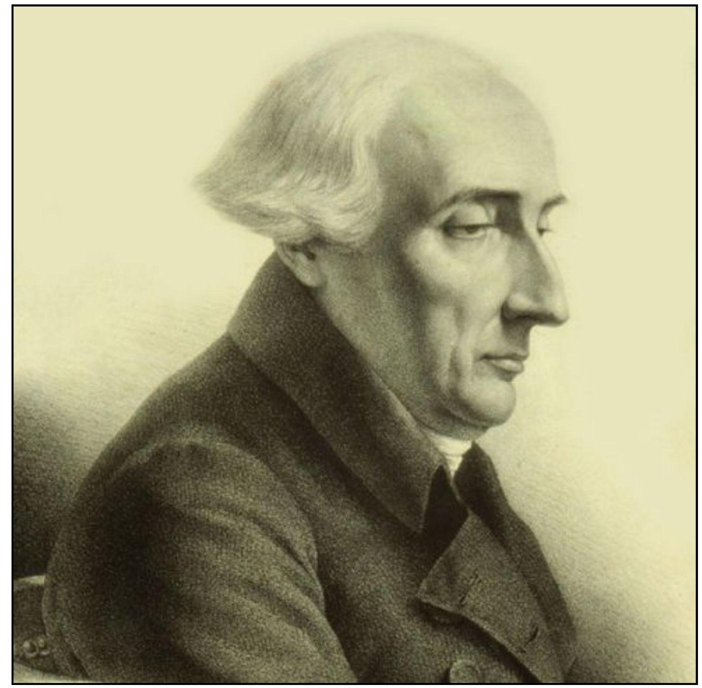
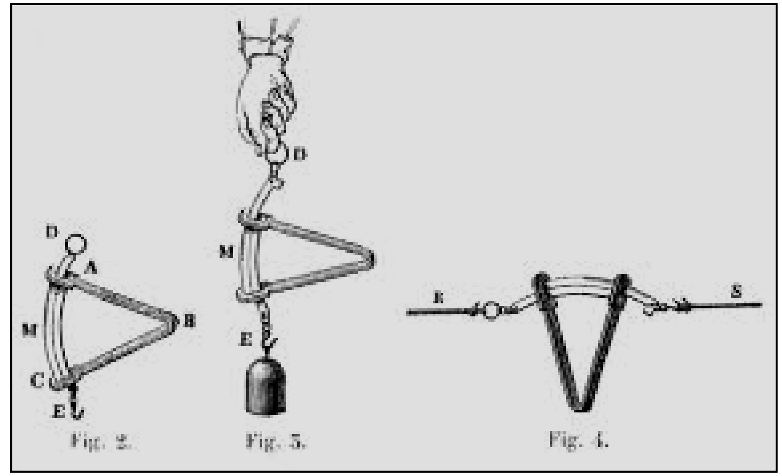
Stress Field Photoelastic Visualisation

One of the many reasons why the emergence of the concept of force was so difficult and controversial lies in the fact that forces, either external or internal, cannot be observed directly.



Stress Field Photoelastic Visualisation

Dynamometers classically used to measure the magnitude of "concentrated" forces are based on a known established relationship between the **observed effect** and the **force** that causes this effect.



The most common mechanical property used for this purpose is linear **elasticity** which states a **linear relationship** between the displacement of the end of a "spring" and the force that causes this displacement.

Stress Field Photoelastic Visualisation

Photoelastic materials are transparent polymers which exhibit **induced birefringence** upon the application of stresses.
The property is local, meaning that at each point in the material the birefringence effect is only dependent on the state of stress at that point.

A loaded photoelastic specimen observed through a polariscope displays patterns of **black-and-light or colour fringes**, depending on the specificities of the light source, which visualise the stress field and make it possible to determine its magnitude.

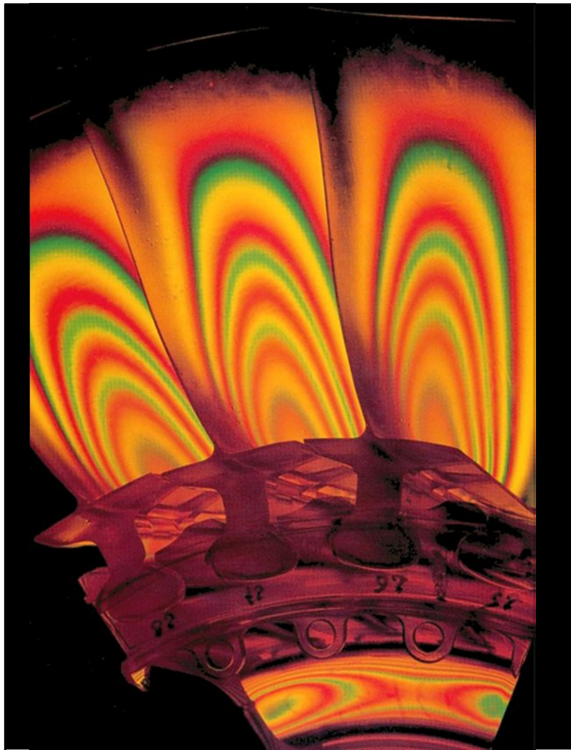
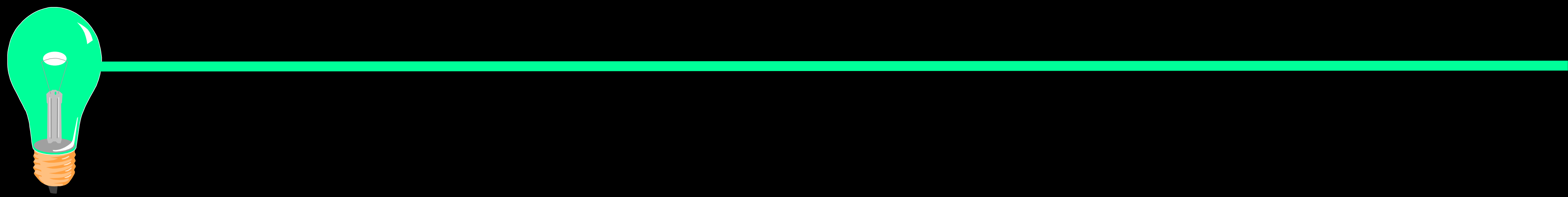


Photo : SNECMA

Basic Experimental Data

Consider an ordinary monochromatic light source (single wave length) naturally not polarised.



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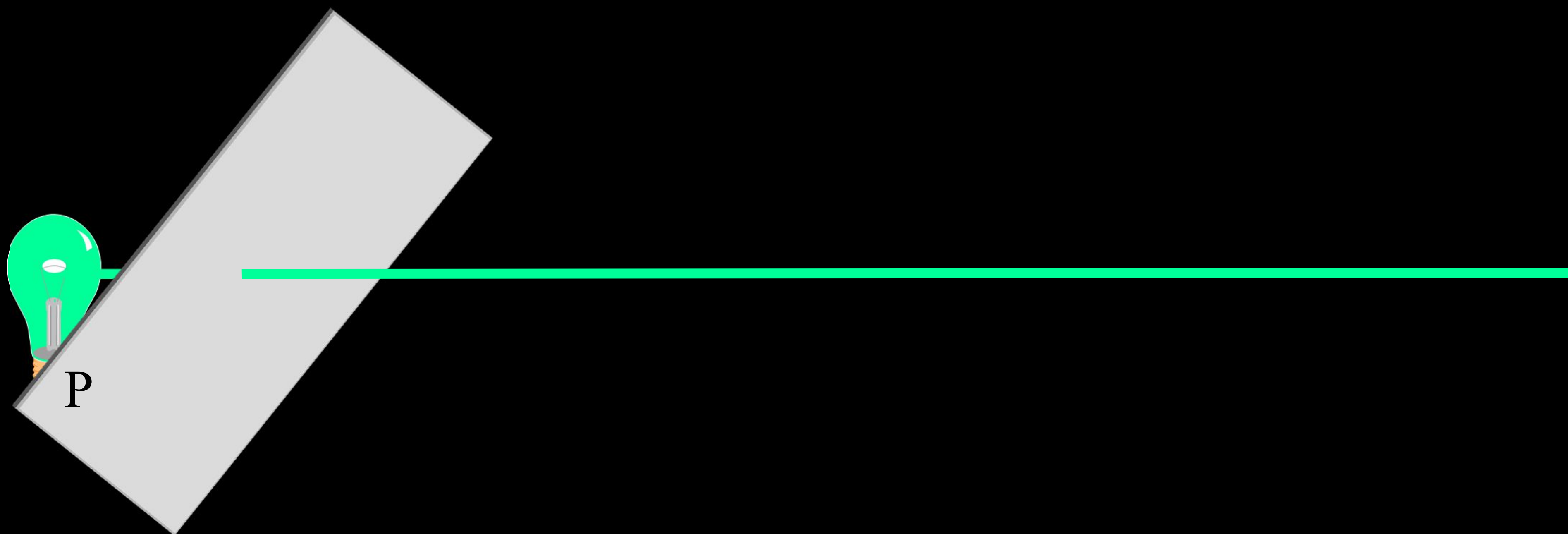
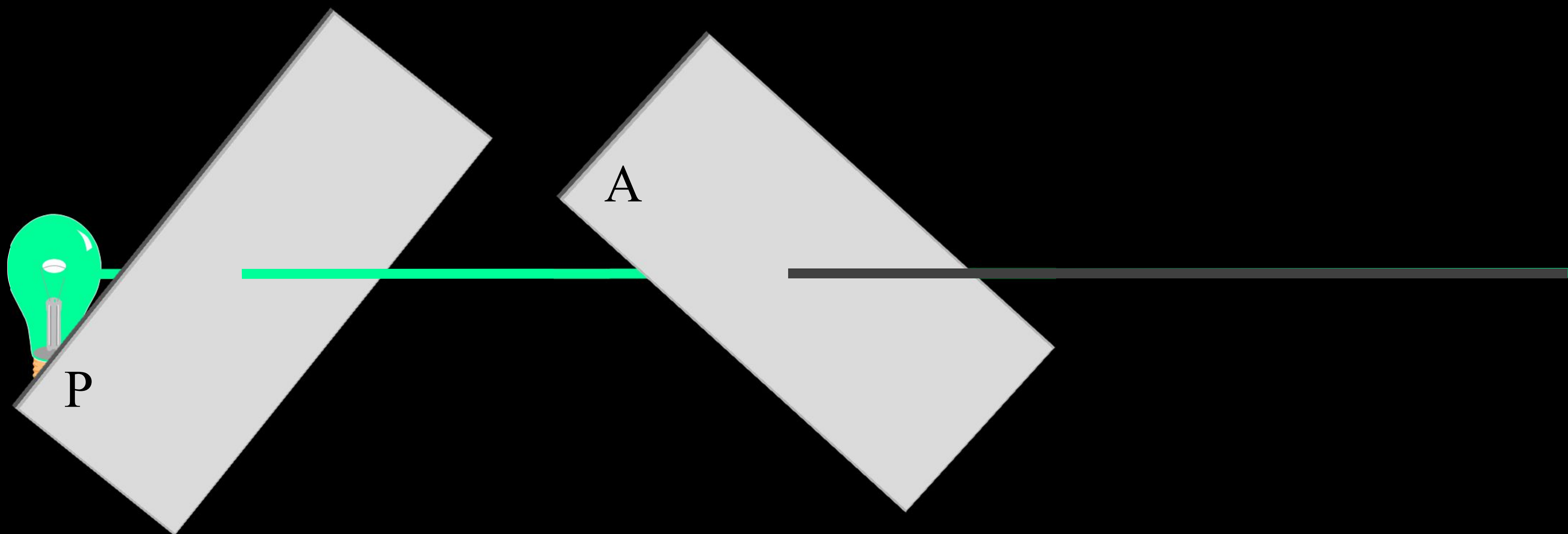


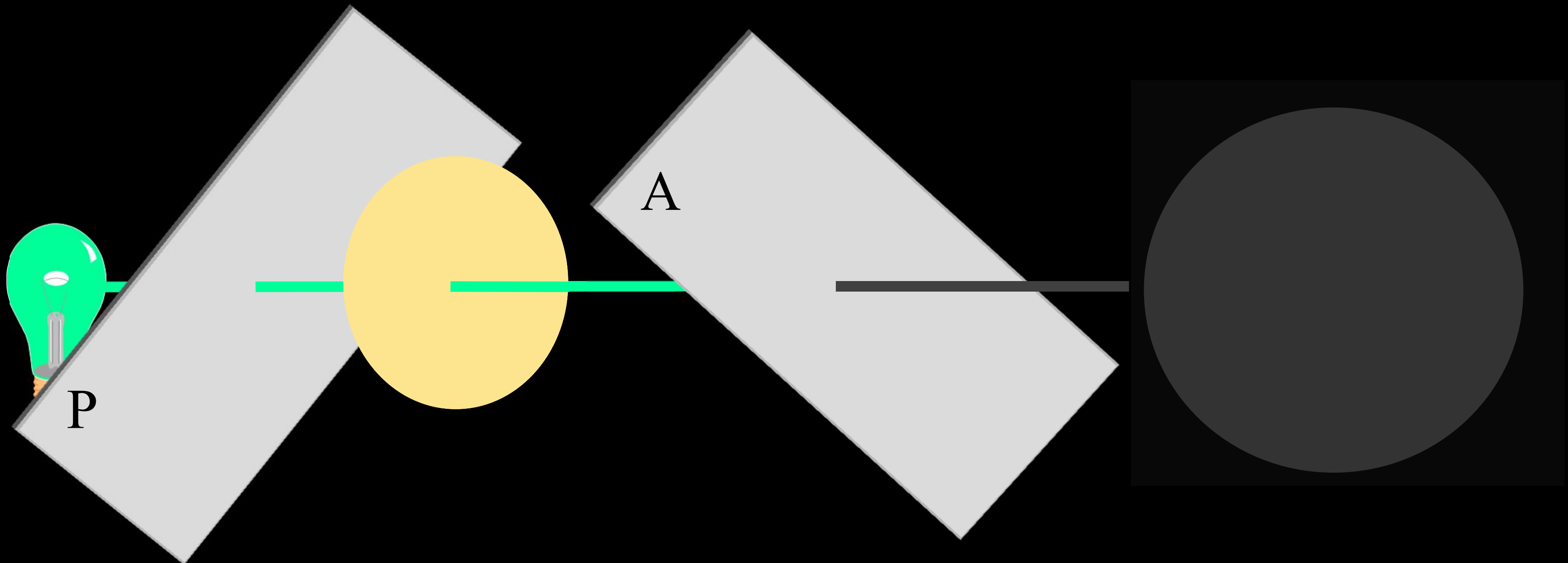
Basic Experimental Data

The light passing through a polariser and a crossed analyser is shut down



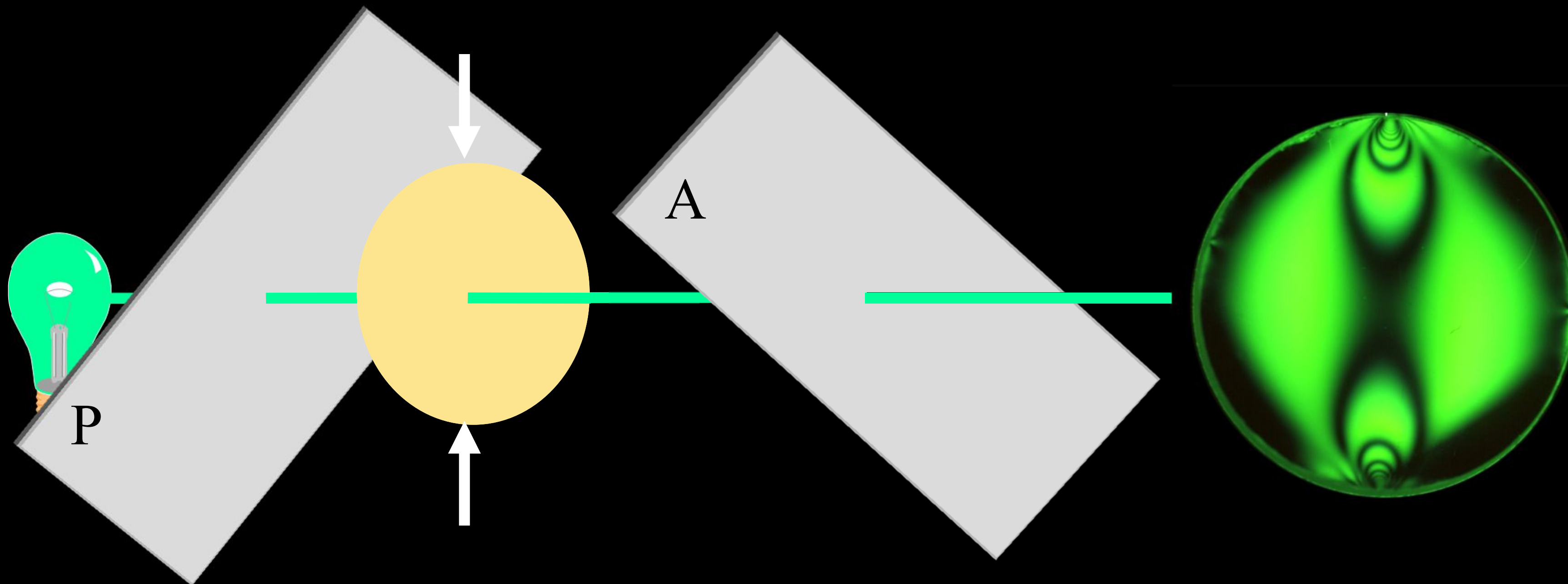
Basic Experimental Data

If a plane specimen made of a photoelastic material is introduced between the polariser and the analyser no change occurs as long as the specimen is not loaded.



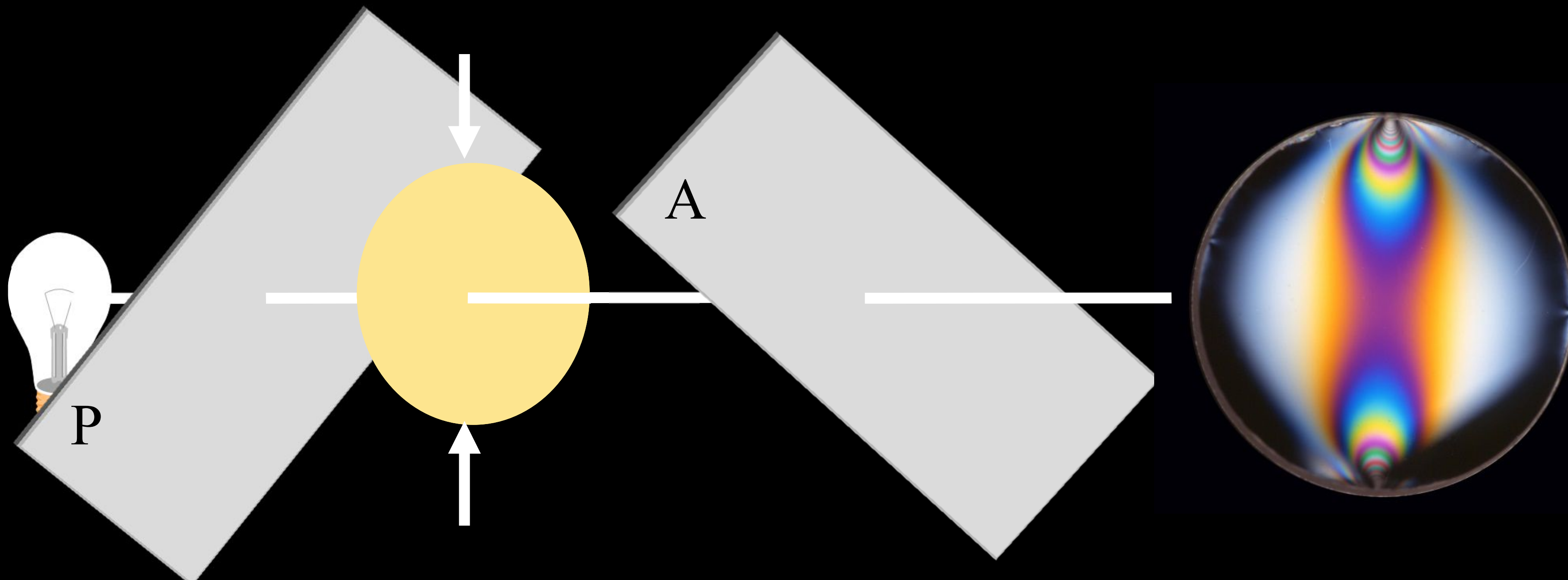
Basic Experimental Data

When the specimen is loaded the generated stress field induces a birefringence field and a pattern of black-and-light fringes is observed

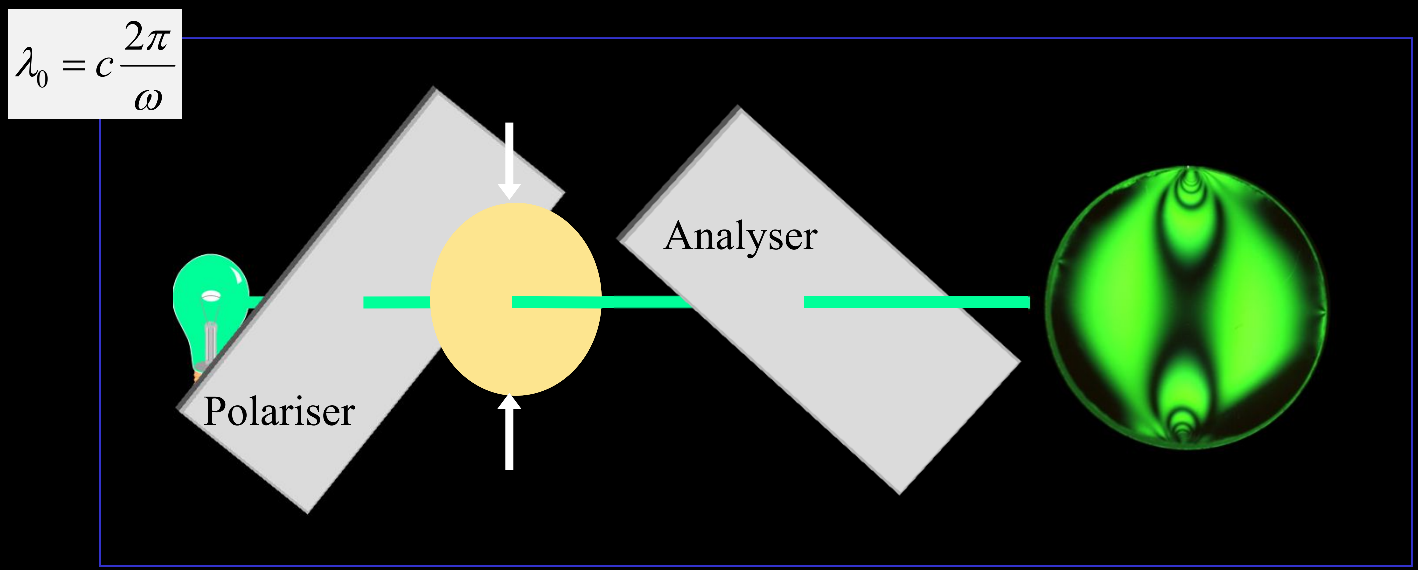


Basic Experimental Data

In the case of a white light source the observed pattern is made of black and colour fringes.

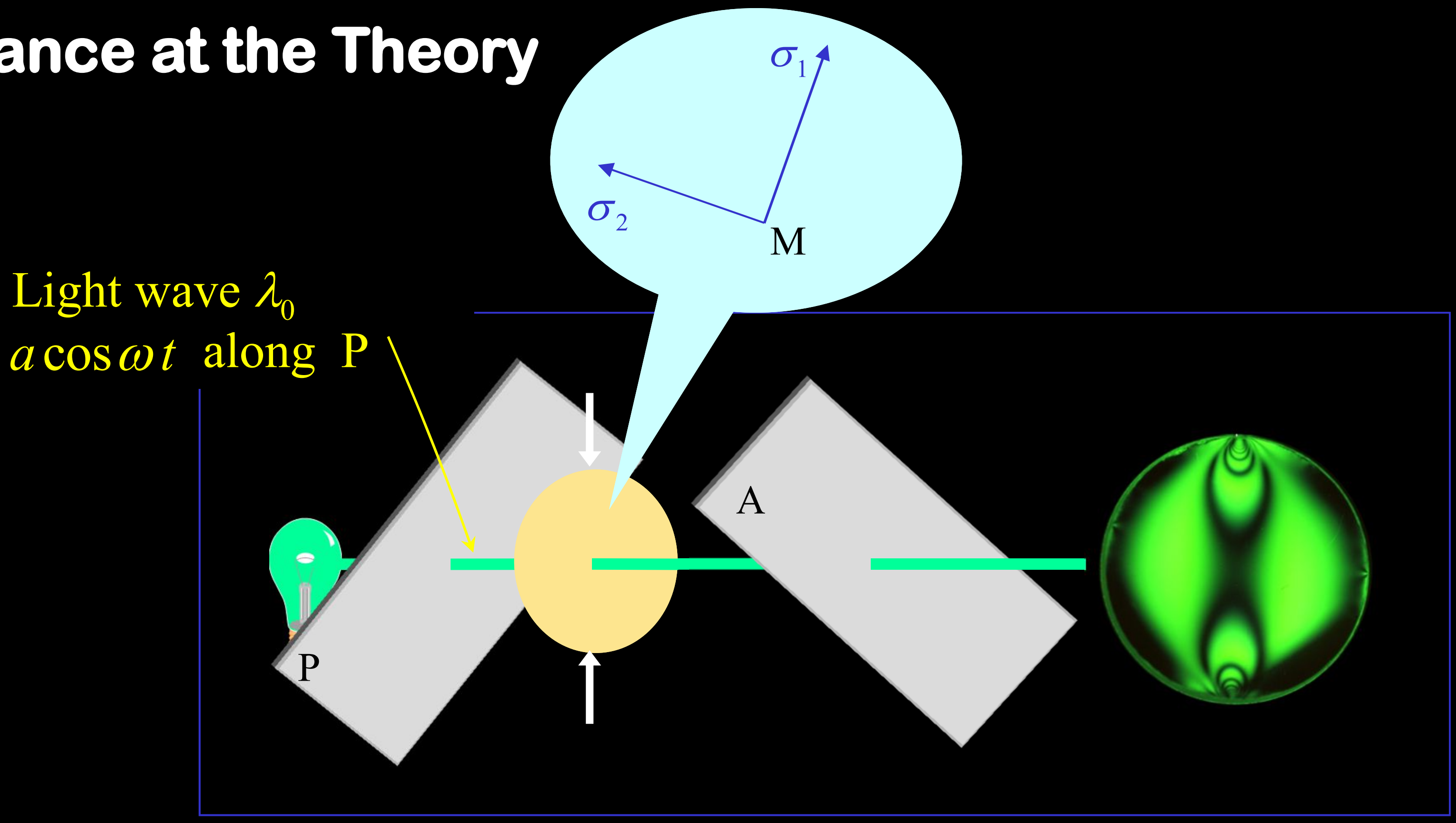


A Glance at the Theory



At each point of the loaded specimen, the stress tensor is plane: the principal stress σ_3 orthogonal to the plane of the specimen is zero.

A Glance at the Theory



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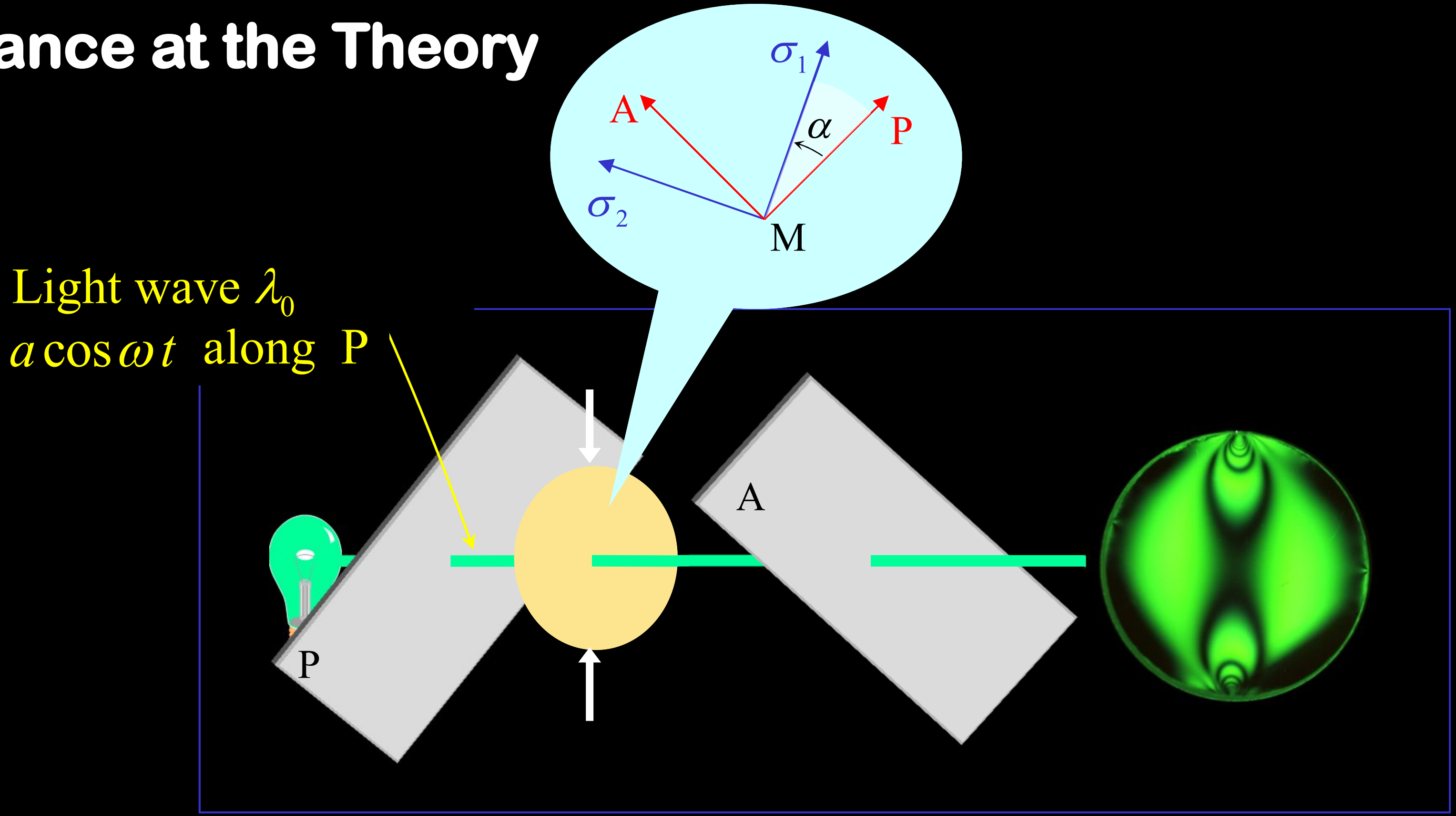
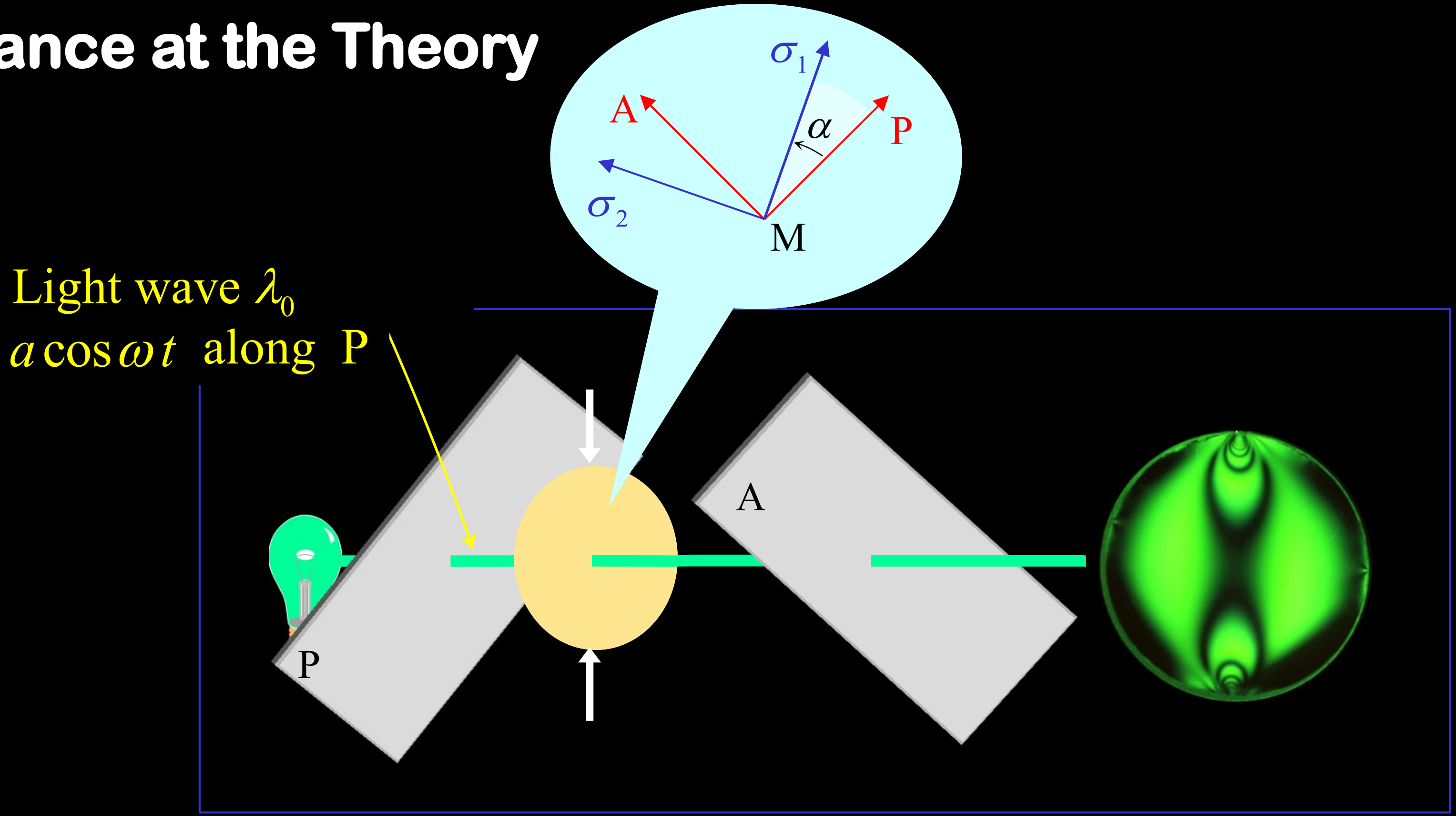


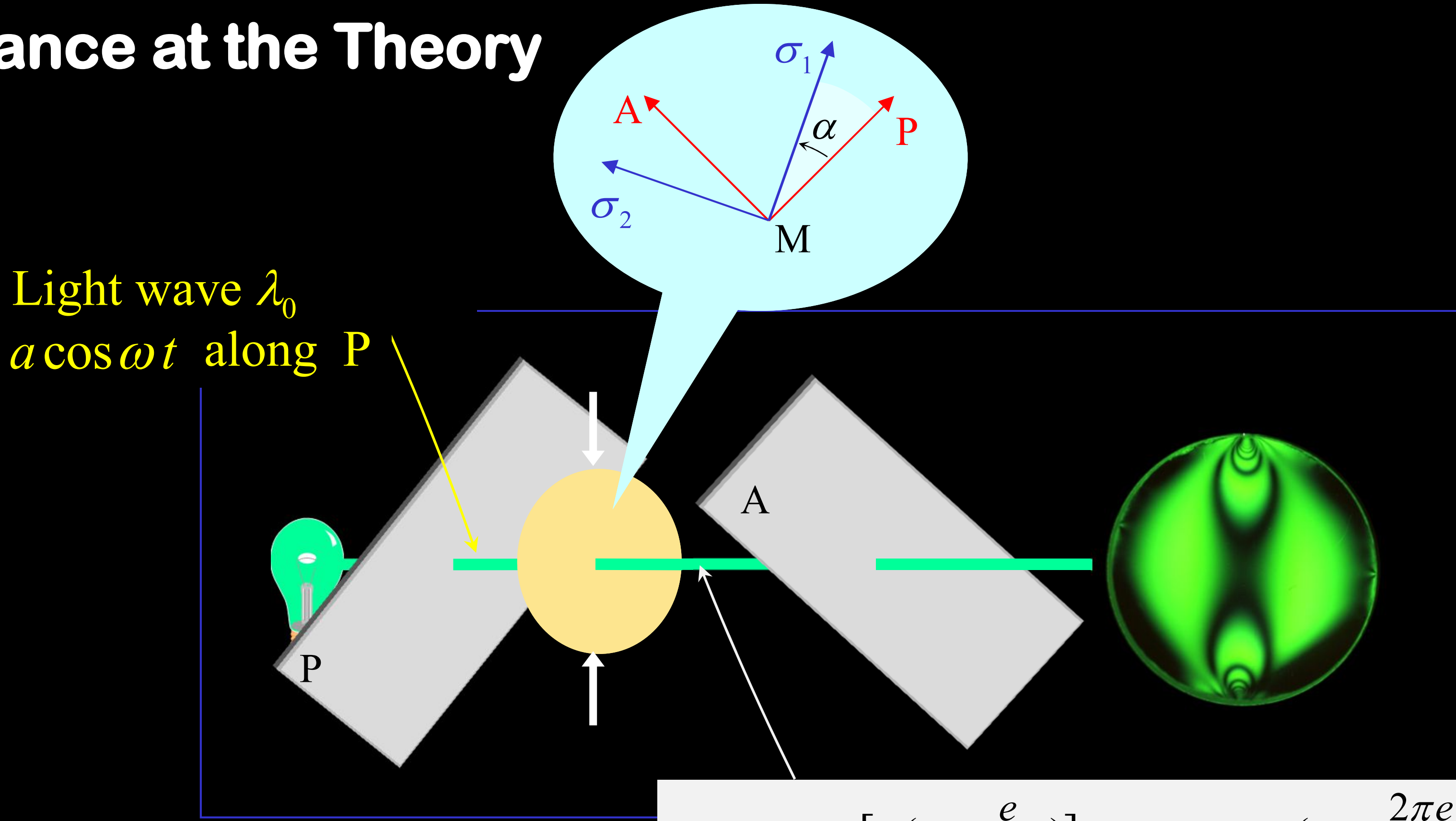
The induced birefringence is characterised by refraction indices n_1 and n_2 along the principal axes of the stress tensor in the plane of the specimen.

A Glance at the Theory



After passing through the specimen, the components of the polarised monochromatic light wave along the principal axes are out-of-phase.

A Glance at the Theory



$$a\cos\alpha\cos\left[\omega\left(t-\frac{e}{c/n_1}\right)\right]=a\cos\alpha\cos\left(\omega t-\frac{2\pi e}{\lambda_0}n_1\right)\quad \text{along}\quad \sigma_1$$

$$-a\sin\alpha\cos\left[\omega\left(t-\frac{e}{c/n_2}\right)\right]=-a\sin\alpha\cos\left(\omega t-\frac{2\pi e}{\lambda_0}n_2\right)\quad \text{along}\quad \sigma_2$$

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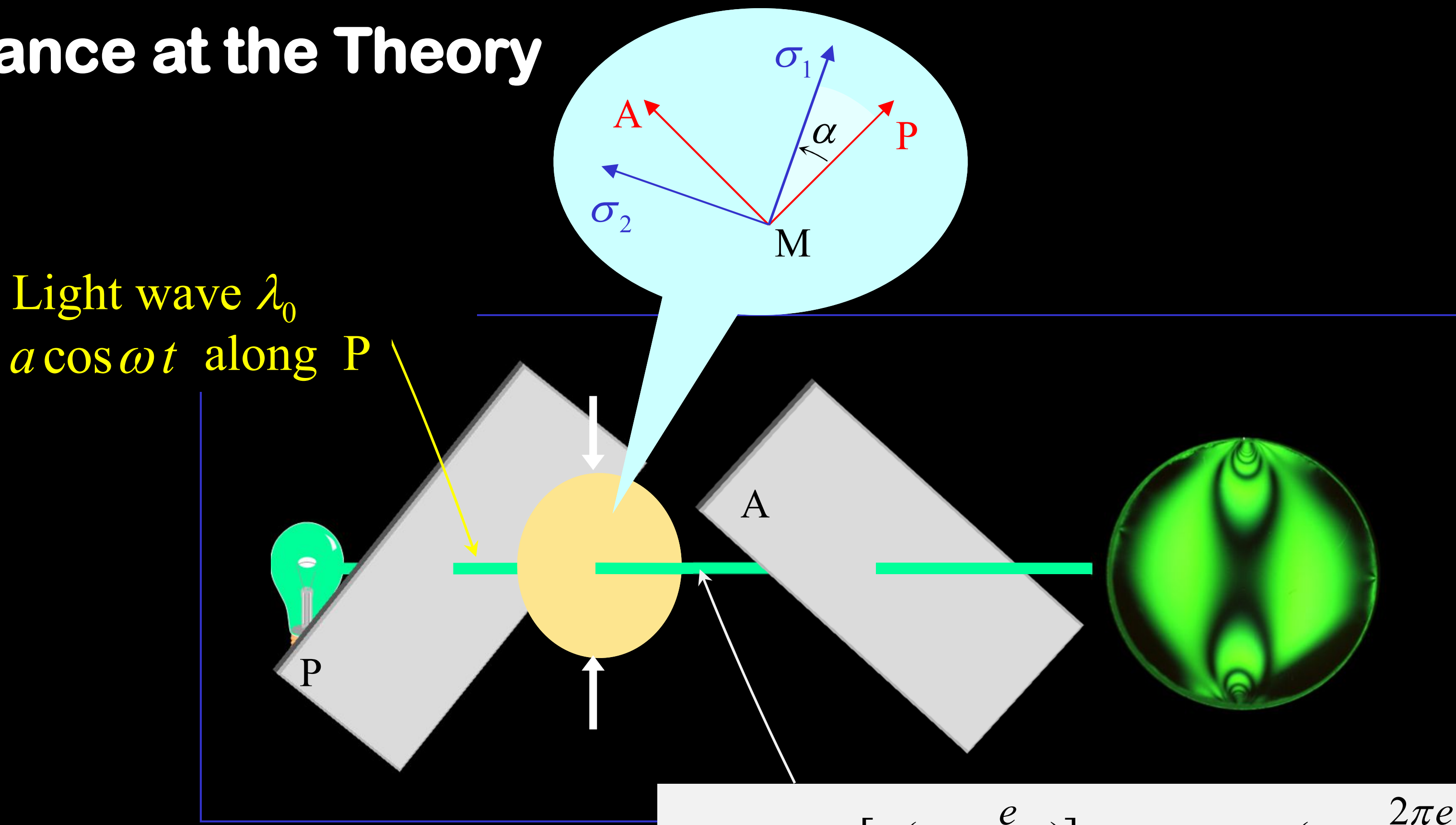
A Glance at the Theory



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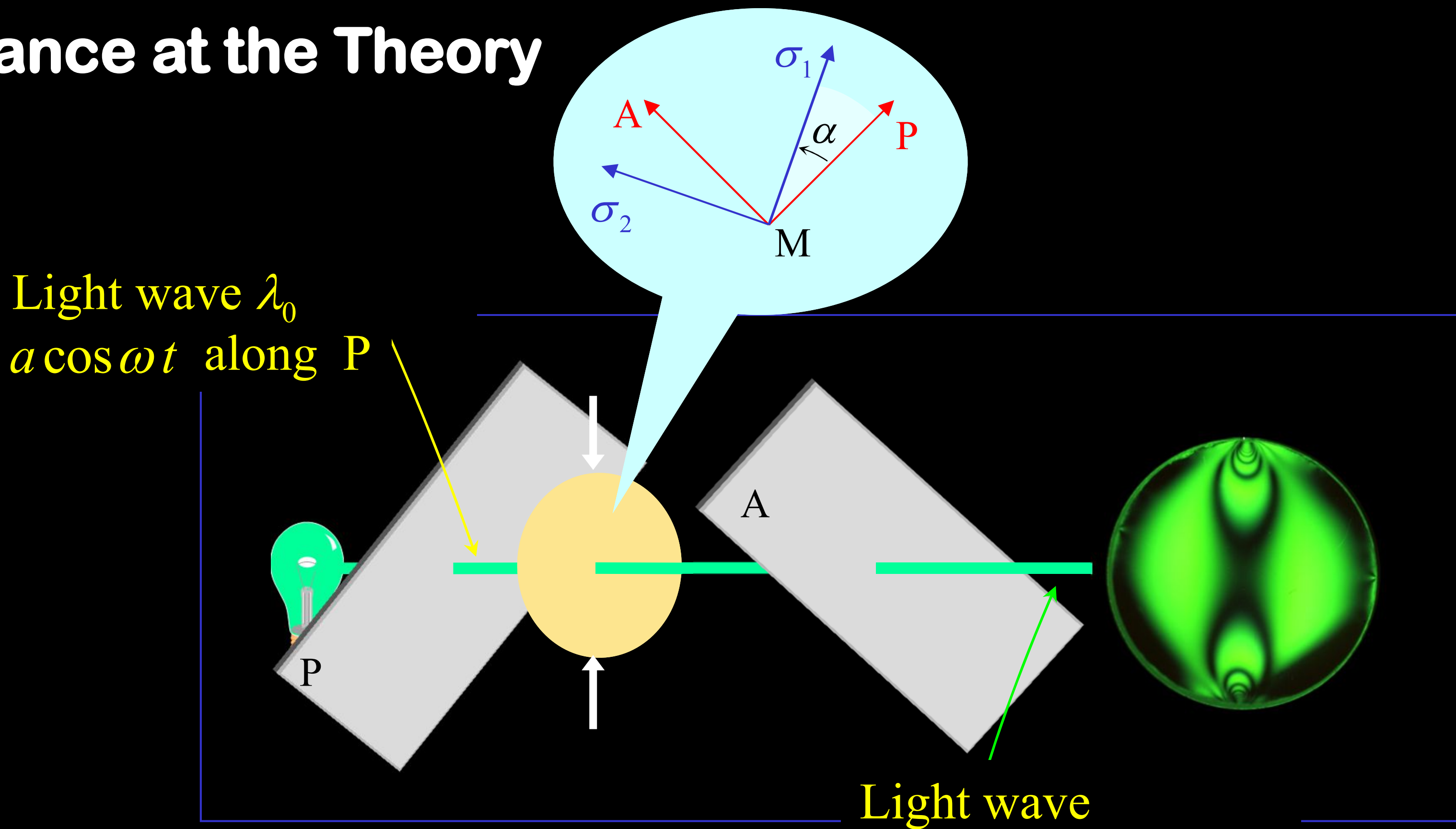
After passing through the crossed analyser the amplitude of the light wave depends on the orientation of the principal axes of the stress tensor at the considered point with respect to the orientation of the polariser and analyser and the refraction-index difference .

A Glance at the Theory



$$a \sin 2\alpha \sin \frac{\pi (n_1 - n_2)\, e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

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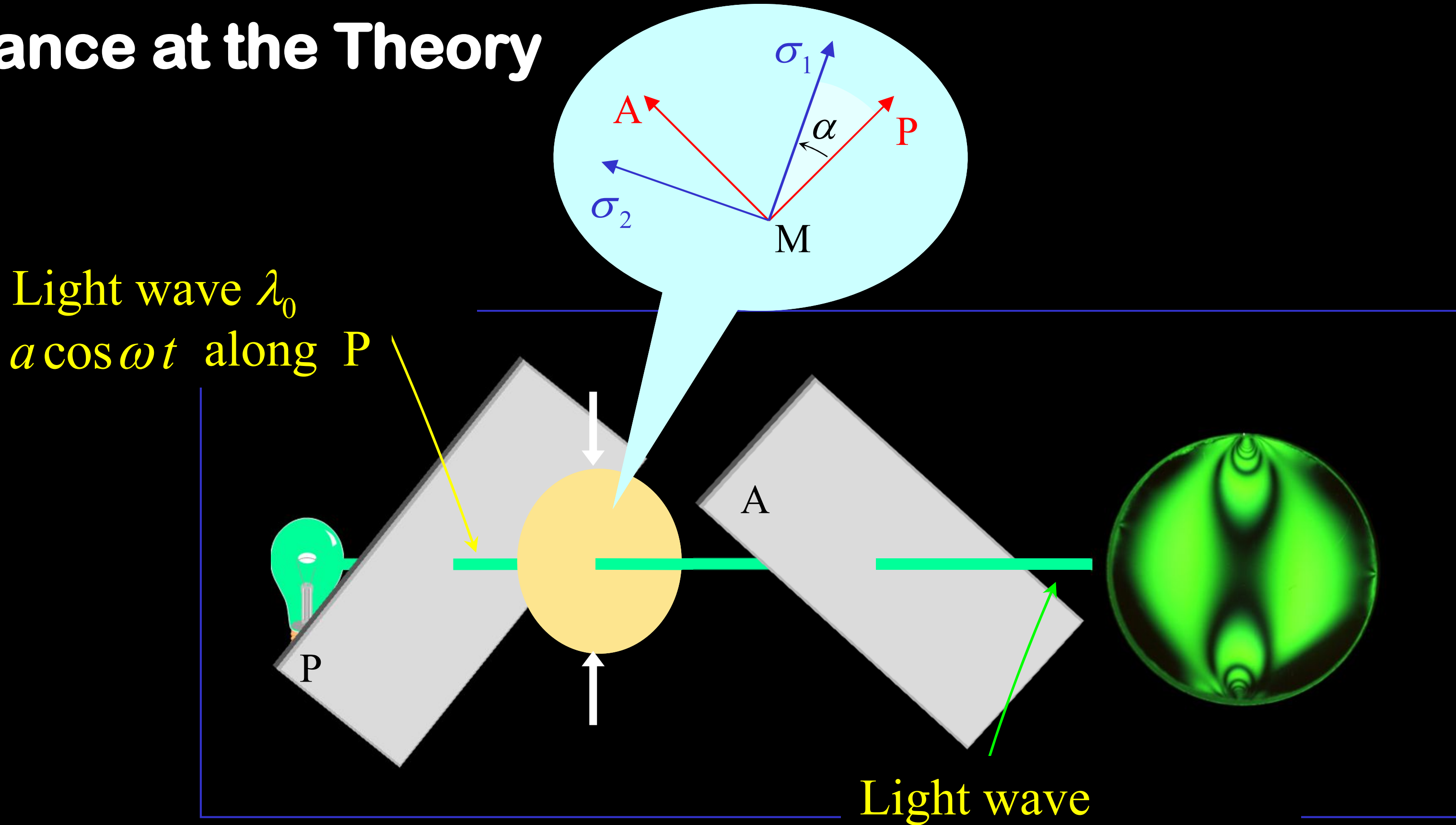
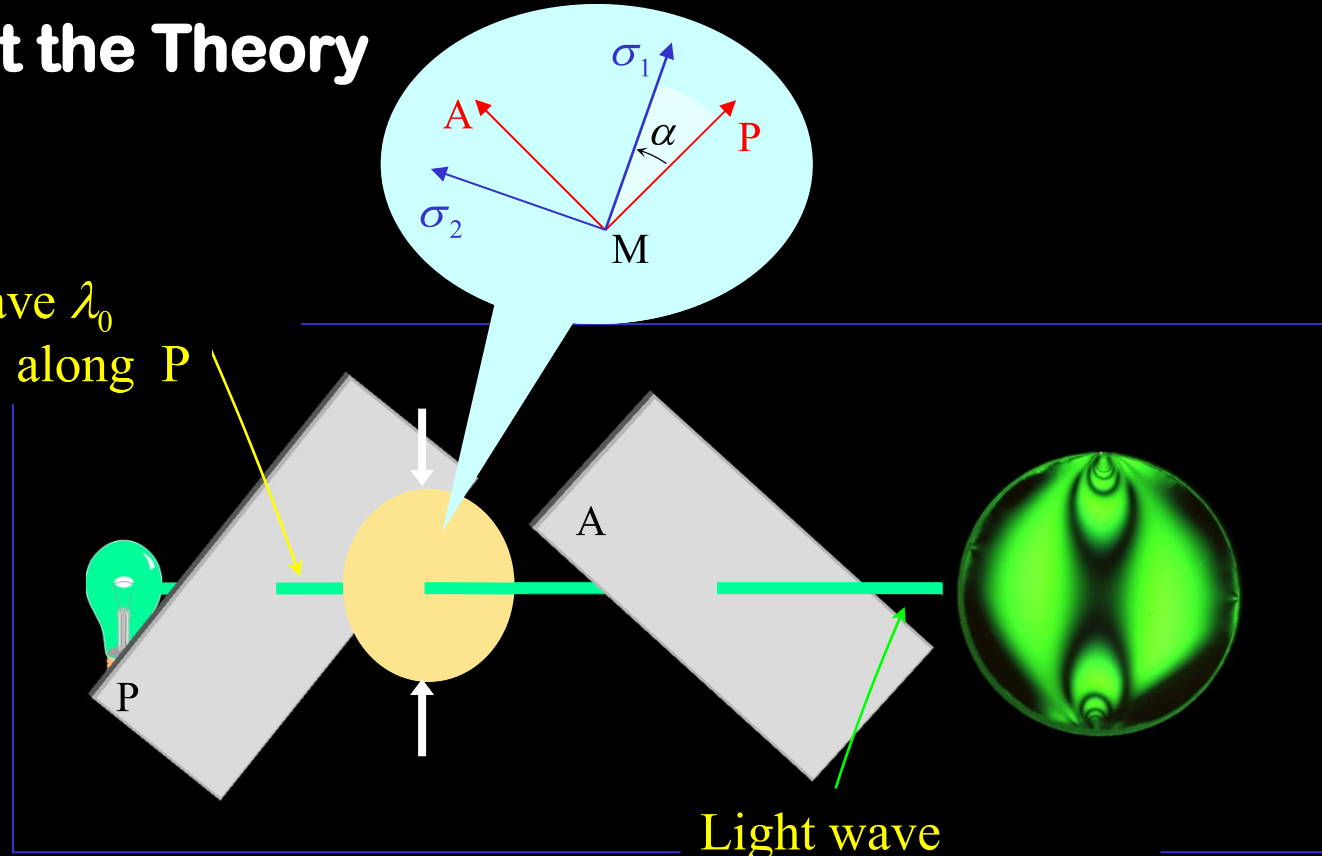
A Glance at the Theory



$$a\sin 2\alpha \sin\frac{\pi(n_1-n_2)e}{\lambda_0}\sin\left(\omega t-\frac{\pi e}{\lambda_0}(n_1+n_2)\right) \text{ along A}$$

From Brewser Law (1816), the refraction-index difference is proportional to the principal-stress difference $(n_1-n_2)=C(\sigma_1-\sigma_2)$

A Glance at the Theory



$$a \sin 2\alpha \sin \frac{\pi C(\sigma_1 - \sigma_2)\, e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

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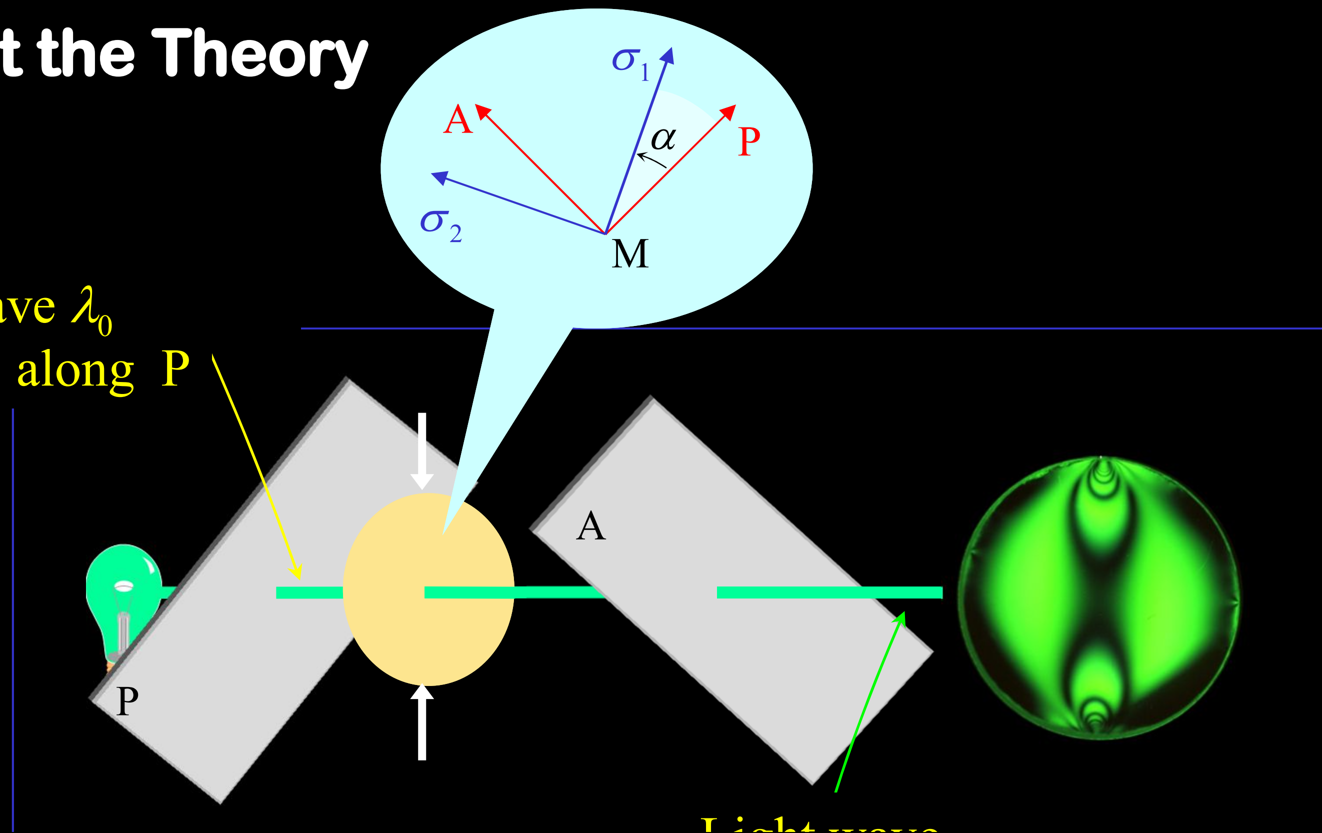
A Glance at the Theory

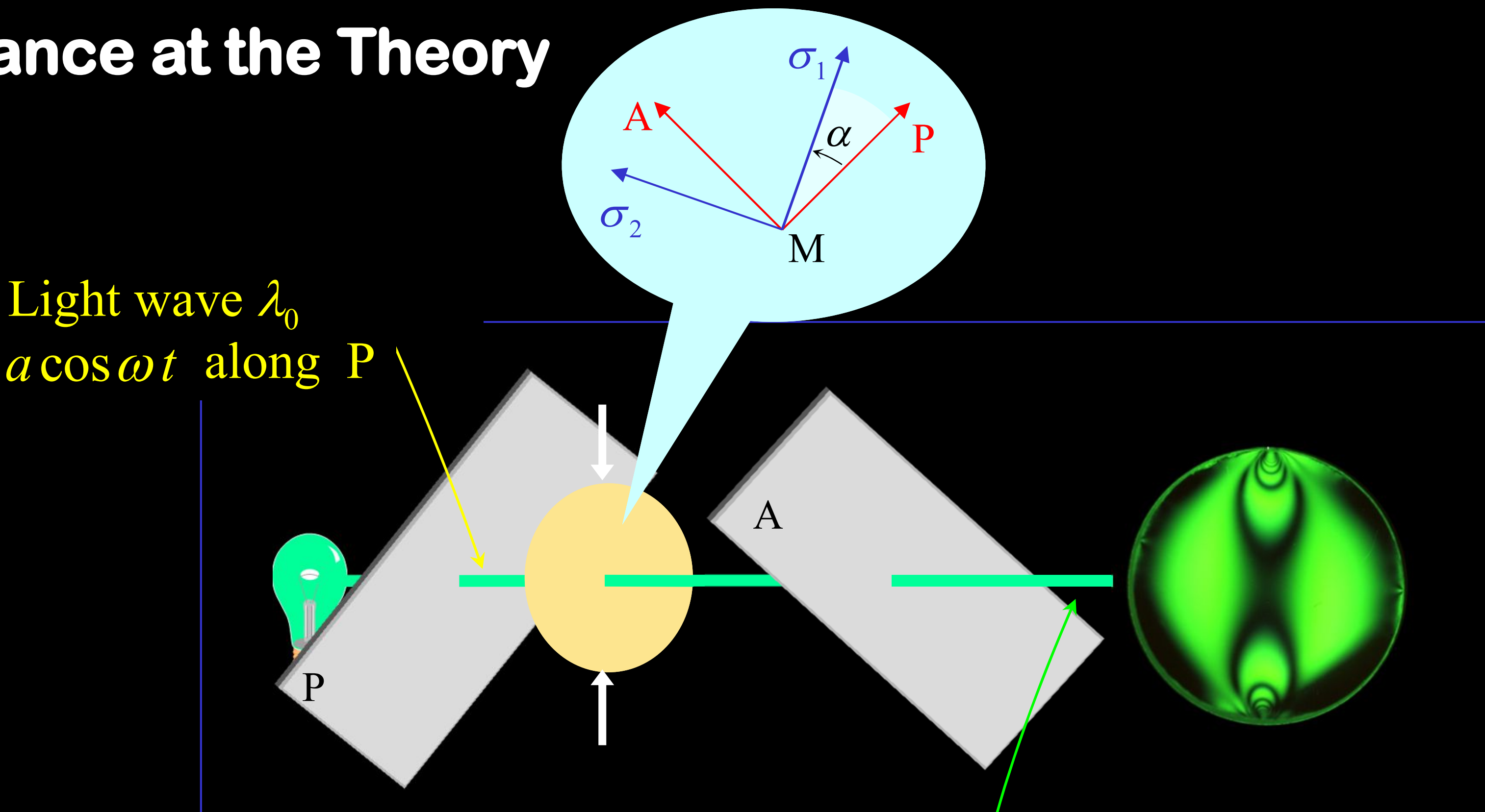


Light wave

$$a \sin 2\alpha \sin \frac{\pi C(\sigma_1 - \sigma_2) e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along } A_A$$

From Brewser Law (1816), the refraction-index difference is proportional to the principal-stress difference $(n_1 - n_2) = C(\sigma_1 - \sigma_2)$

A Glance at the Theory

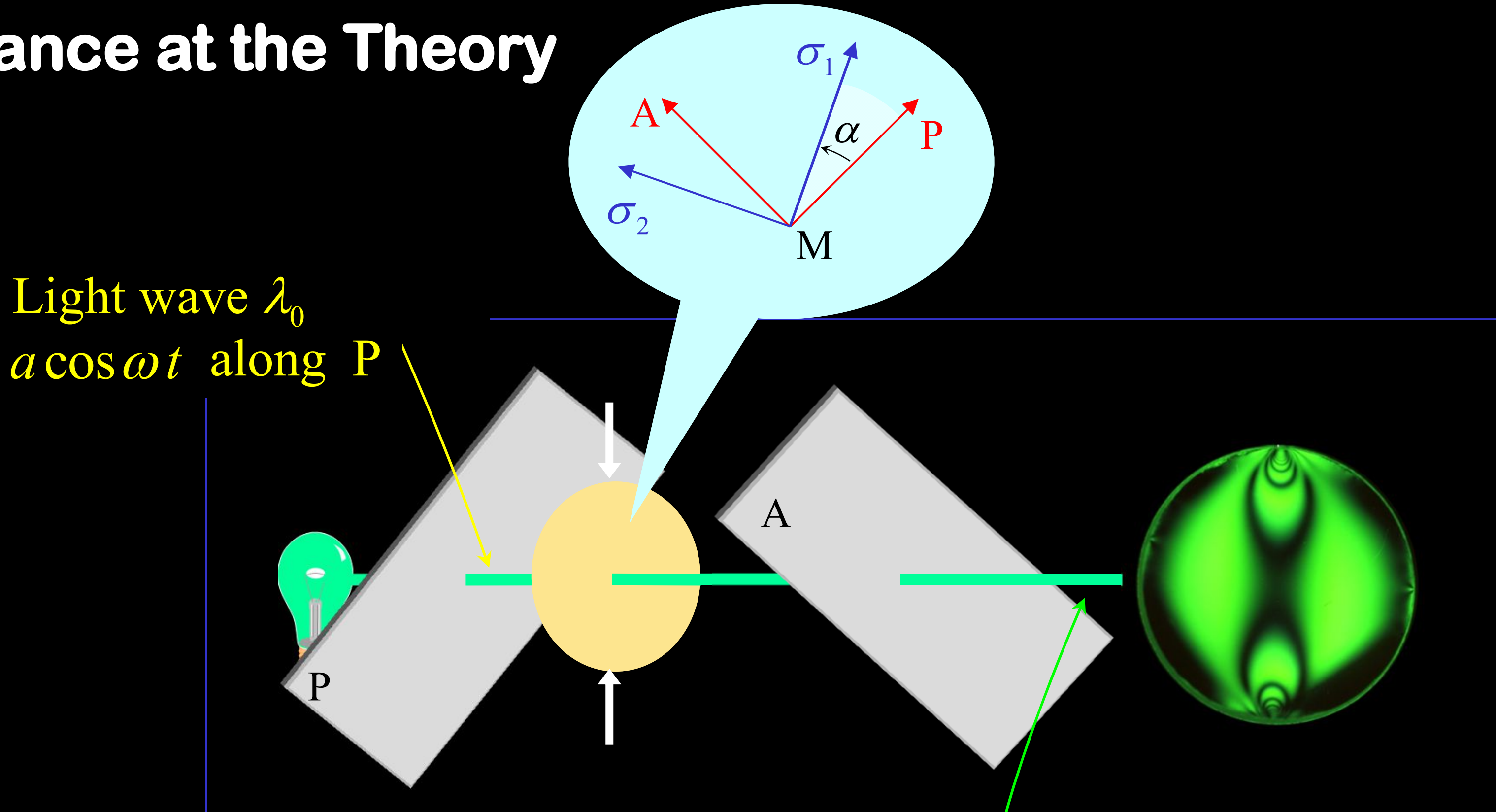


$$a \sin 2\alpha \sin \frac{\pi C (\sigma_1 - \sigma_2) e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

- Black fringes: if $\alpha = 0$ or $\alpha = \pi / 2$ or $\sigma_1 = \sigma_2, \ \forall \lambda_0$

A first type of black fringes is obtained when the principal axes of the stress tensor coincide with the directions of the polariser and analyser. These fringes are called isoclinics.

A Glance at the Theory

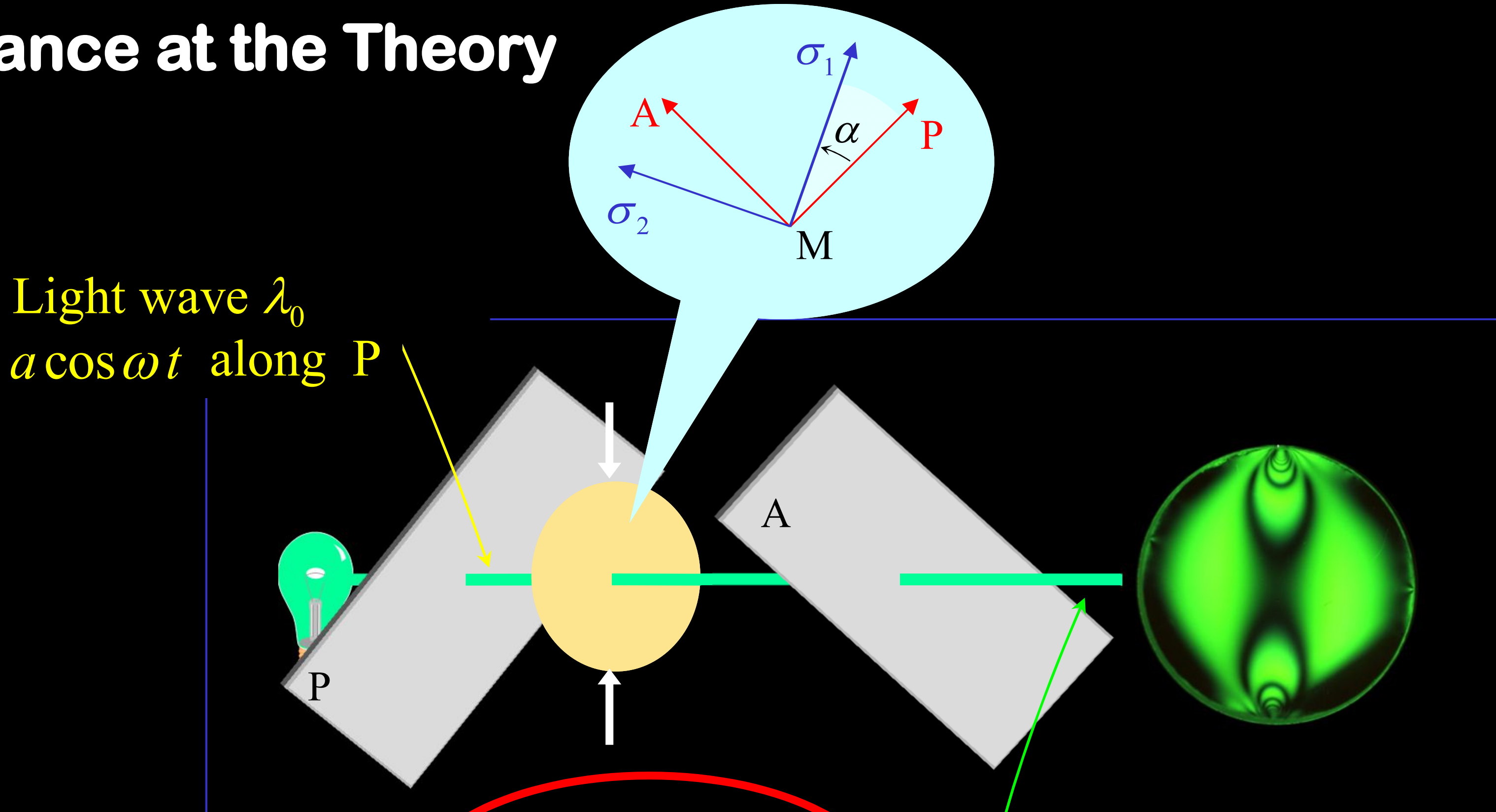


$$a\sin 2\alpha \sin\frac{\pi C(\sigma_1-\sigma_2)e}{\lambda_0}\sin\left(\omega t-\frac{\pi e}{\lambda_0}(n_1+n_2)\right) \text{ along A}$$

- Black fringes: if $\alpha = 0$ or $\alpha = \pi/2$ or $\sigma_1 = \sigma_2, \ \forall \lambda_0$

They are independent of the wave length. They vary with the rotation of the polariscope (polariser-analyser) but for the points where the principal stresses are equal to each other.

A Glance at the Theory



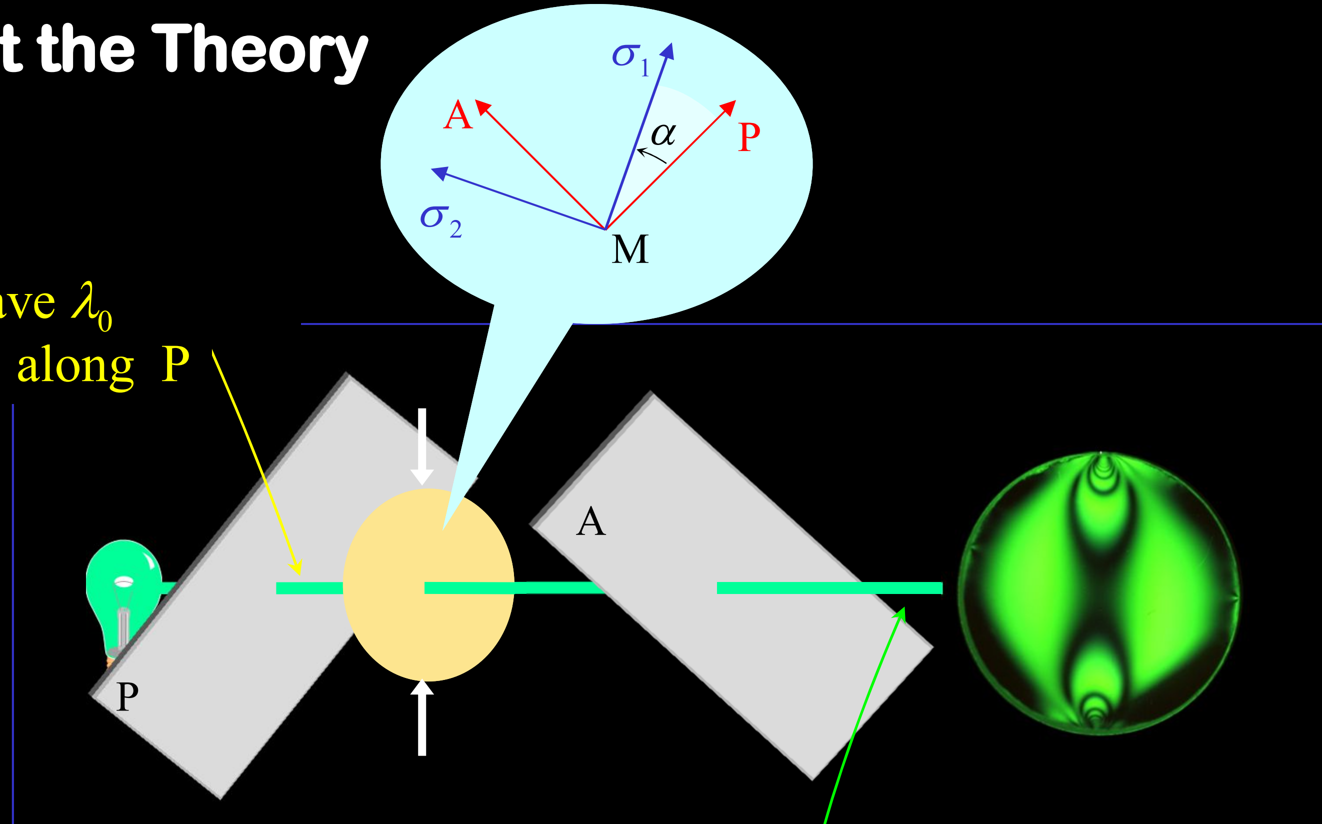
$$a \sin 2\alpha \sin \frac{\pi C(\sigma_1 - \sigma_2)e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

- Black fringes: $(\sigma_1 - \sigma_2) = p\,\lambda_0 / C e, \quad p \geq 1$

The second type of black fringes is independent of the orientation of the polariscope. They are obtained when the principal-stress difference is proportional to a constant depending on the wave length.

A Glance at the Theory

Light wave λ_0
$a\cos\omega t$ along P



$$a \sin 2\alpha \sin \frac{\pi C(\sigma_1 - \sigma_2)e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

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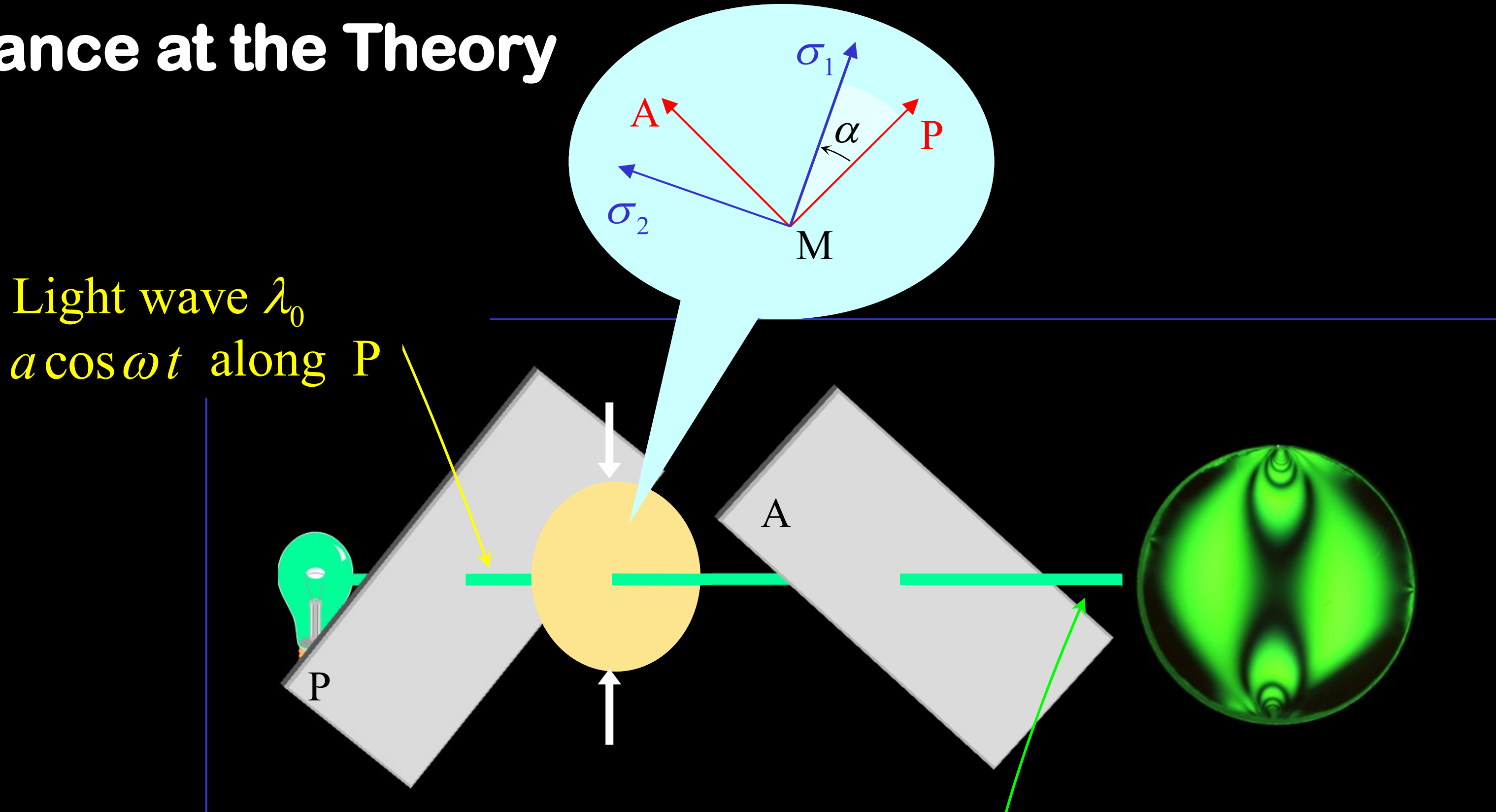
A Glance at the Theory



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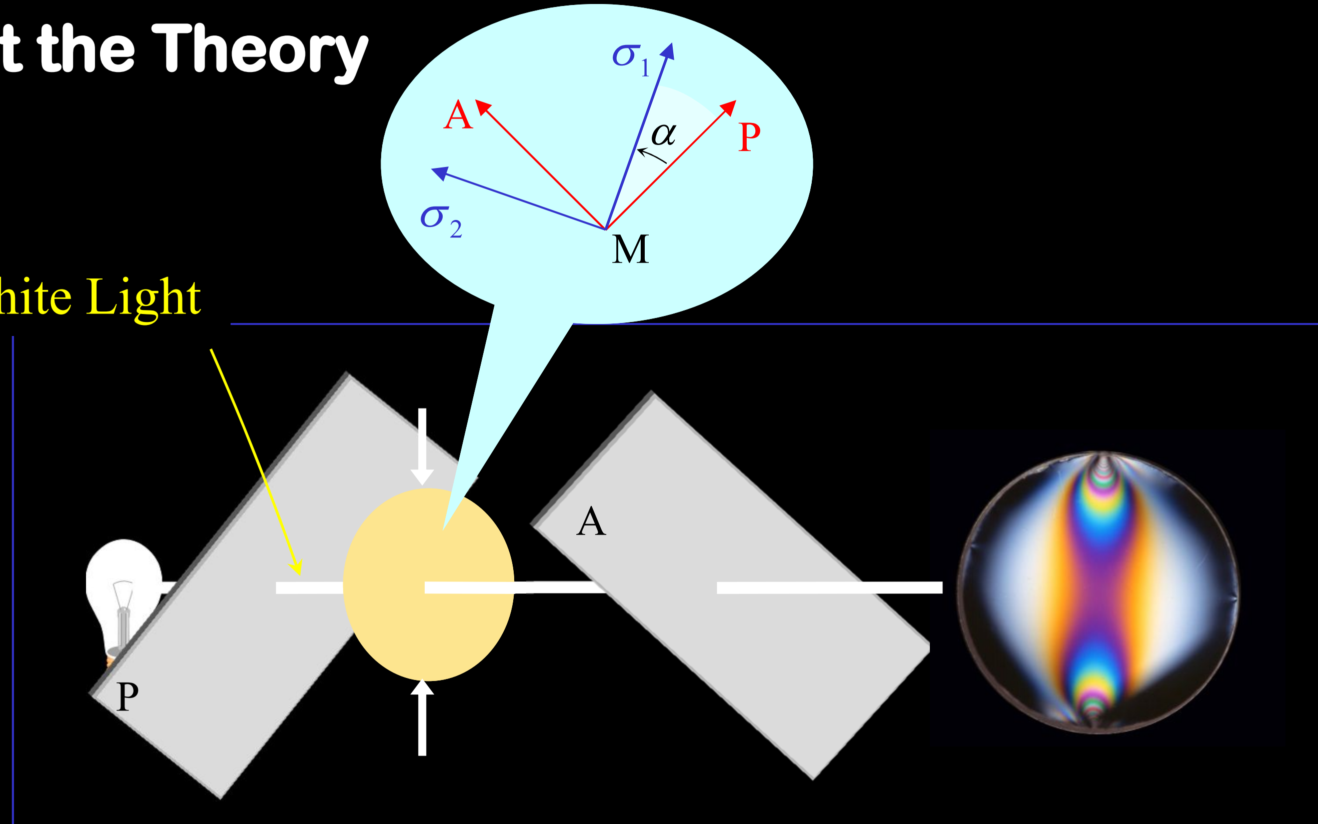
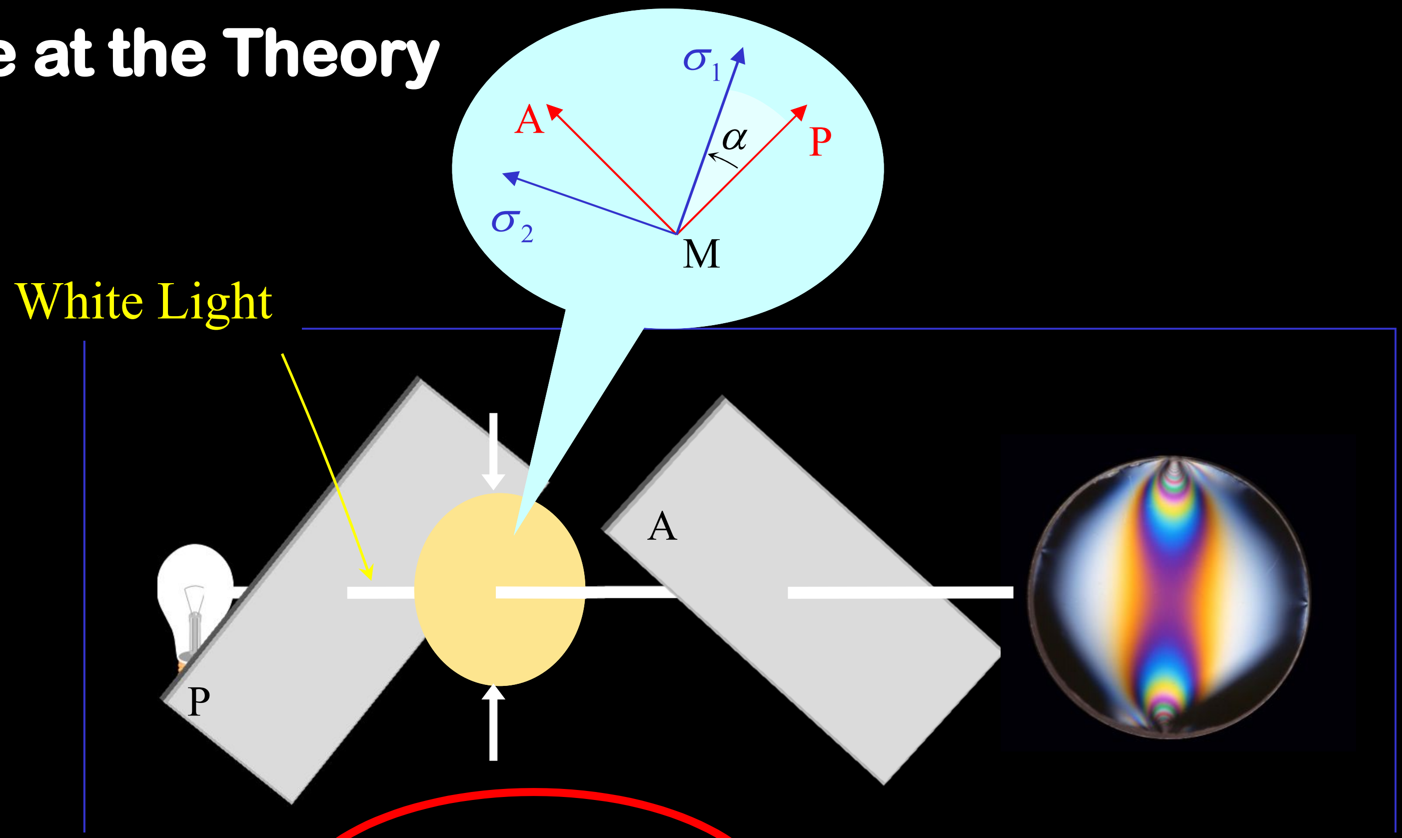


$$a \sin 2\alpha \sin \frac{\pi C(\sigma_1 - \sigma_2) e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

In the case of a white light source the isoclinics are still black fringes because they are independent of the wave length.

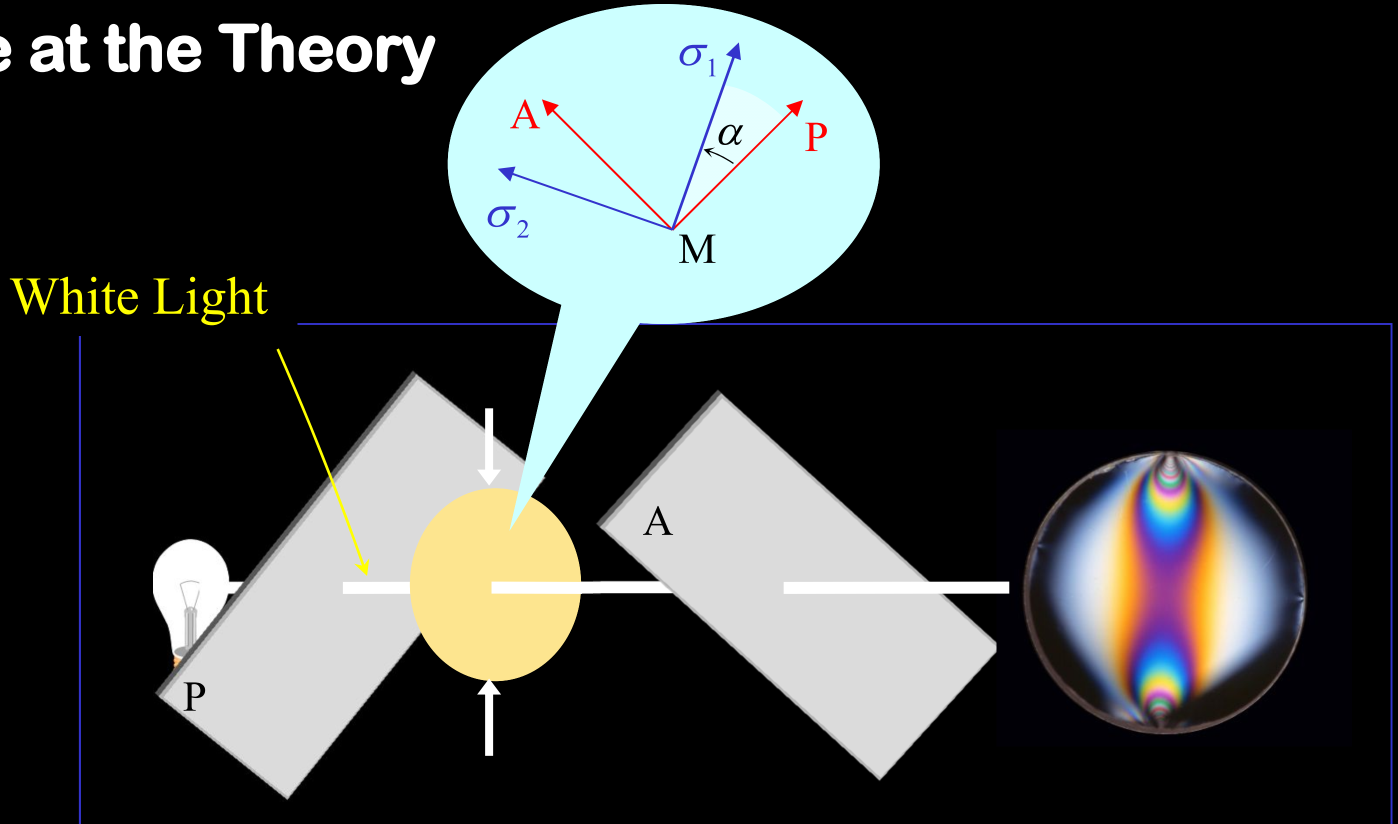
A Glance at the Theory



$$a \sin 2\alpha \sin \frac{\pi C(\sigma_1 - \sigma_2) e}{\lambda_0} \sin\left(\omega t - \frac{\pi e}{\lambda_0}(n_1 + n_2)\right) \text{ along A}$$

A black fringe that is specific to a given wave length becomes a colour fringe. It is called an isochromatic. Along an isochromatic, the principal-stress difference is constant.

A Glance at the Theory



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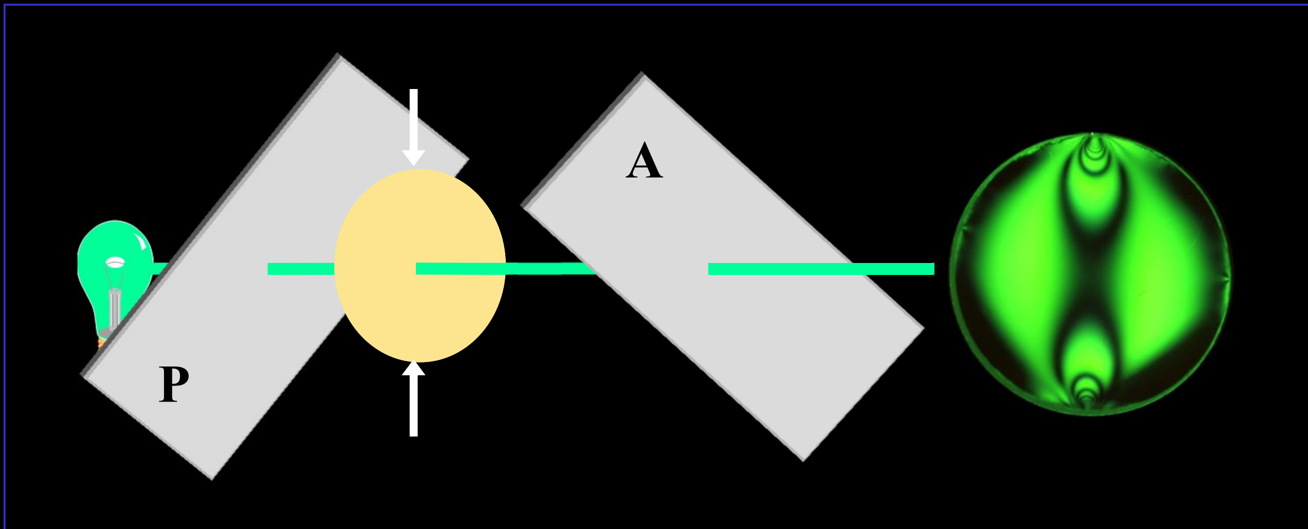
- Black fringes: if $\alpha = 0$ or $\alpha = \pi / 2$ or $\sigma_1 = \sigma_2, \ \forall \lambda_0$
- Colour fringes: $(\sigma_1 - \sigma_2) = p \lambda_0 / C e, \quad p \geq 1$

A Glance at the Theory



Monochromatic Light source

Isoclinics

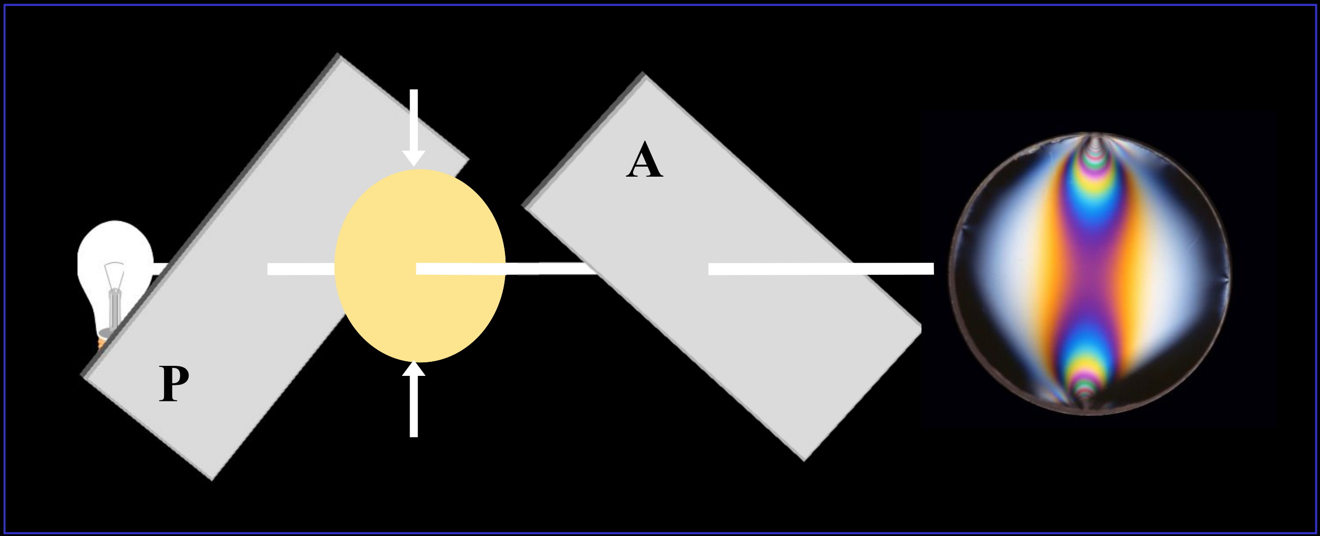
Black Fringes

Principal axes σ_1, σ_2 collinear with *P* and *A*

Rotate with *P*-*A*

Independent of λ

A Glance at the Theory



White Light source

Isoclinics

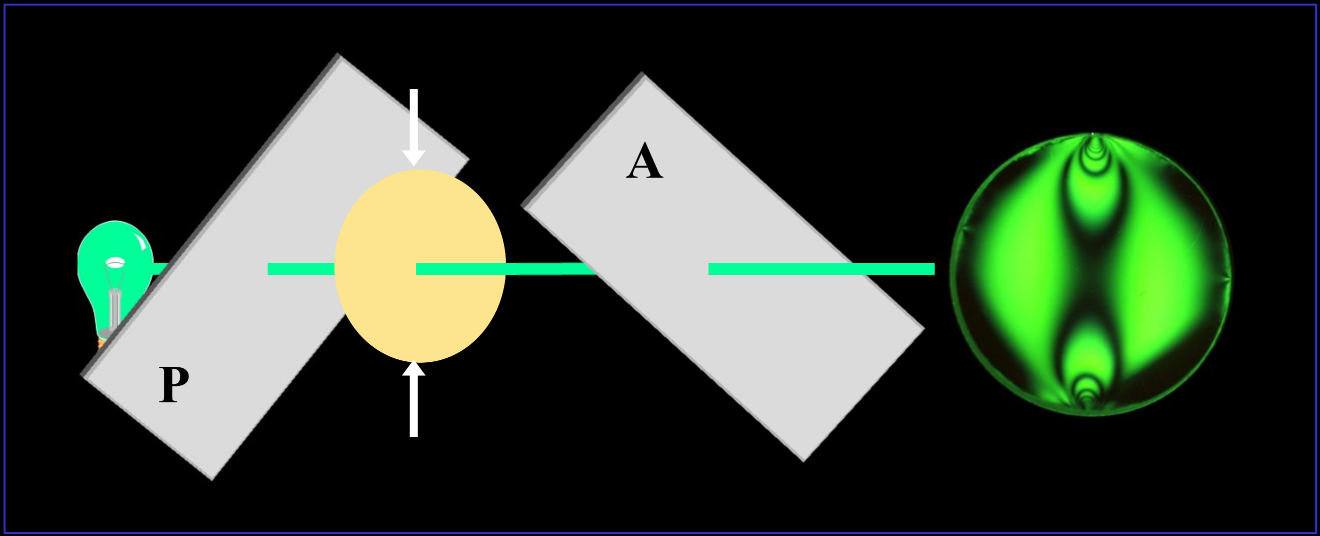
Black Fringes

Principal axes σ_1, σ_2 collinear with *P* and *A*

Rotate with *P*-*A*

Independent of λ

A Glance at the Theory



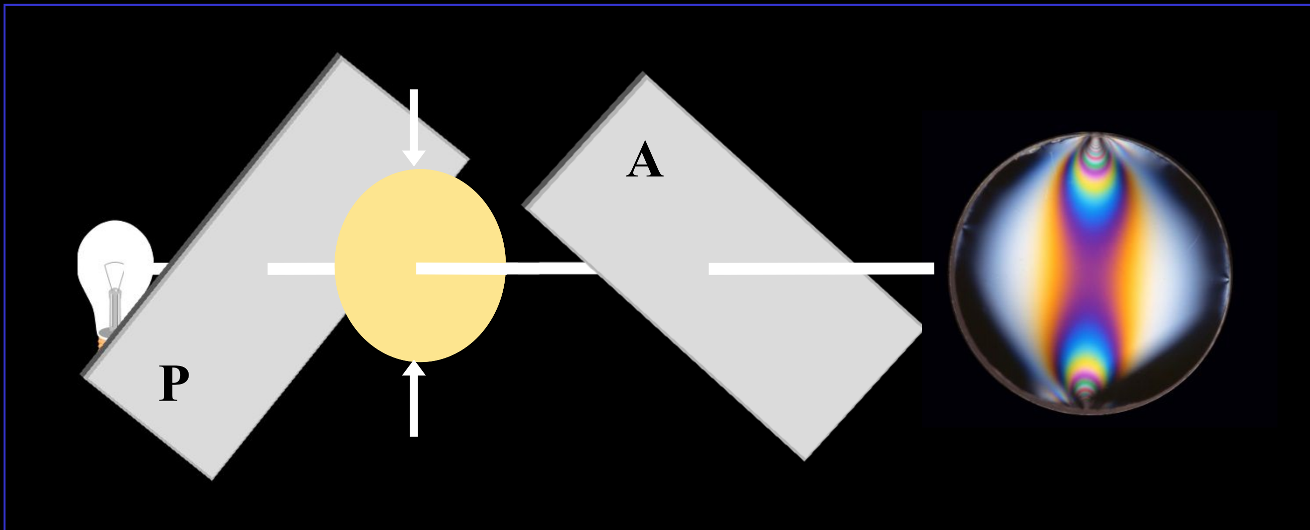
Monochromatic Light source

Black Fringes

$$\sigma_1 - \sigma_2 = C(\lambda)$$

Independent of *P-A*

A Glance at the Theory



White Light source

Colour Fringes

Isochromatic lines

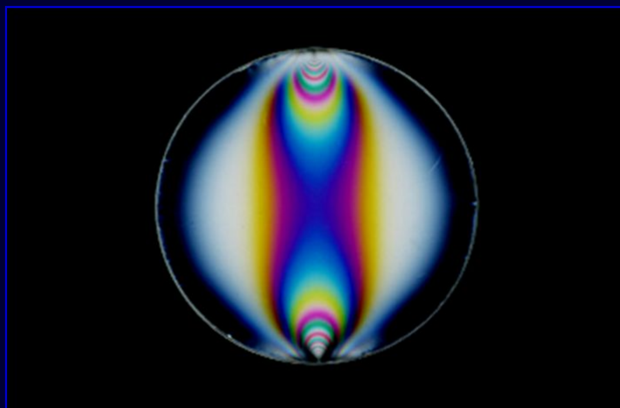
$$\sigma_1 - \sigma_2 = C(\lambda)$$

Independent of *P-A*

Examples

Examples

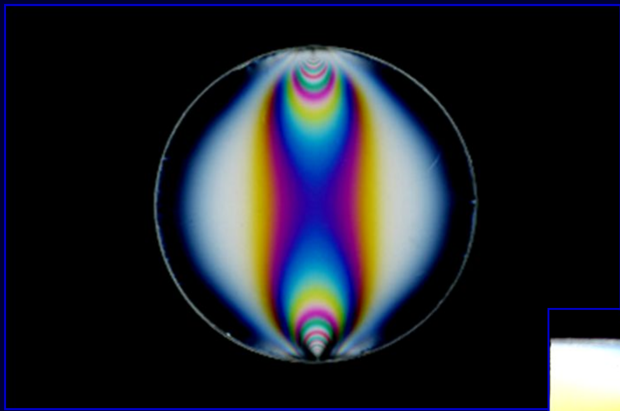
Three experiments performed with a white light source are presented. In the first one the load is kept constant and the evolution of the isoclinic and isochromatic patterns is observed.



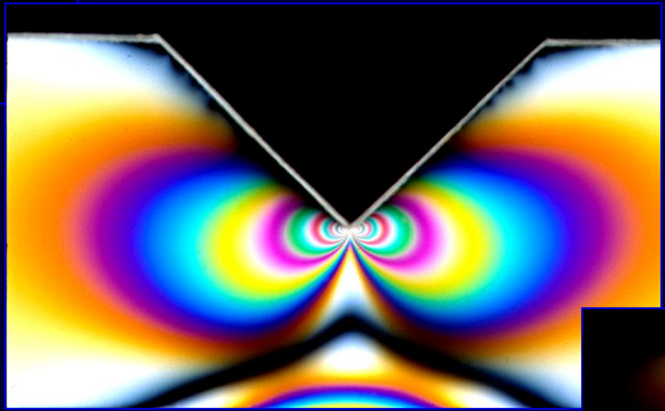
Diametrally loaded circular disk

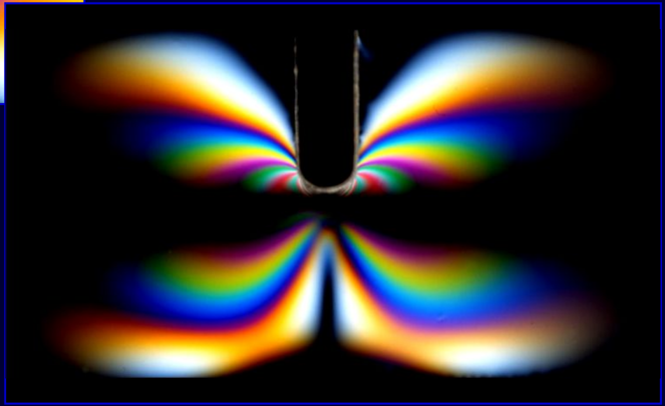
Examples

Three experiments performed with a white light source are presented. In the first one the load is kept constant and the evolution of the isoclinic and isochromatic patterns is observed. In the other ones the orientation of the polariscope is fixed and the magnitude of the load is increased steadily.



Diametrally loaded circular disk





Traction tests on V-notched and U-notched specimens

Thanks for viewing

Jean Salençon
Senior Fellow of IAS