# X A dynamic macro-element for performance-based design of foundations

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**Abstract** The work is concerned with the development of an original macro-element model for shallow foundations within the context of performance-based design of structures. The macroelement can be viewed as a link element placed at the base of the structure that reproduces in a simplified yet coherent way the non-linear interaction phenomena arising at the soil-footing interface during dynamic excitation. As such, it offers an efficient prediction of the maximum and permanent displacements at the foundation level by identifying the non-linear mechanisms that produce them. These are: a) the sliding mechanism along the soil-footing interface, b) the irreversible soil behaviour mechanism and c) the foundation uplift mechanism. These non-linear mechanisms are introduced within the macro-element model in a fully coupled way. In its present state of development the model can be used for strip and circular footings in purely cohesive or purely frictional soils.

## X.1 General context – motivations

Since Newmark's fundamental remark (Newmark 1965) that action exceeding resistance during transient loading does not necessarily mean failure, displacement and performance-based design have been established as major trends in modern earthquake engineering. Most of the developments made to date, have mainly focused on structural aspects without consideration of soil-foundation-structure interaction. Nonetheless, the latter has proved to influence the response of the overall structure significantly, both in terms of safety and in terms of serviceability. Non-linear phenomena arising at the foundation level will usually function as desirable isolation mechanisms for the superstructure unless they lead to excessive displacements and rotations. In certain cases, taking this positive effect into account may prove necessary for achieving a viable design. On the other hand, uncontrolled displacement/rotation accumulation at the foundation level may constitute a severely detrimental agent for the structure. Prescribing acceptable limits and possessing tools for an efficient prediction of displacements and rotations lies in the heart of performance-based design applied to soil structures and, in particular, to foundations.

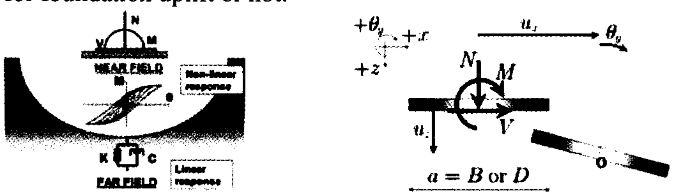
Current engineering practice offers two possibilities for displacement/rotation evaluation at the foundation level: either a detailed FEM model (encompassing the superstructure, the foundation and at least part of the foundation soil) or a simplified Newmark type of analysis. The first option stands as the most rigorous from a mechanical point of view, but it is also the most difficult: it implies more or less a FEM model with consideration of phenomena such as soil plasticity, interface sliding, loss of soil-footing contact (geometric non-linearity), absorbing boundaries *etc.* Such features increase significantly the complexity and the uncertainties of modelling, especially for 3D foundation configurations, and require significant modelling skills and computational resources. The second option is evidently much simpler; it is bound however to a number of significant limitations: it presupposes a heuristically defined failure mechanism and a known force history applied on the foundation. Since this force history is usually computed from a linear soil structure interaction analysis, it does not account for foundation yielding that actually modifies the force history at the foundation level. Moreover, it can only provide post-yield displacements/rotations of the foundation and neglects pre-yield displacements that may be significant for frequent (small) excitations.

## X.2 Dynamic macro-element

The notion of dynamic macro-element constitutes an alternative and a compromise to the two aforementioned methods. The macro-element can be thought of as a link element attached at the base of the structure. Its scope is to reproduce the non-linear effects that arise along the soil-footing interface during dynamic soil-structure interaction. It preserves the simplicity of the structural model by encapsulating all the non-linear mechanisms within a unique constitutive law affected to the introduced link element. Force evaluation at the foundation level is performed incrementally and it is fully coupled with the superstructure response. This approach presents the advantage of translating complexity onto developing a sufficiently sophisticated constitutive law for the foundation macro-element which is written once for all. This element can then be used as a typical link/spring element in structural FEM modelling.

*Modelling principle.* Figure X.1 schematically presents the main modelling principle for the foundation macro-element. It consists in dividing the soil domain in two distinct virtual fields: the near field and the far field. The near field is the part of the soil domain in the vicinity of the foundation and it is considered that all non-linearity in the system is concentrated in it, whereas the far field remains linear. This technical decomposition allows introducing the well established notion of foundation dynamic impedance describing far field behaviour. Foundation impedance accounts for wave propagation, both from the incoming and scattered wave field, and provides the linear visco-elastic component of the macroelement constitutive law.

Focus is hitherto put on describing non-linear near field response for shallow perfectly rigid footings. Chatigogos *et al.* (2009a) provide an overview of the existing literature on the topic and introduce a macroelement model for shallow circular footings on cohesive soils. In Chatzigogos *et al.* (2009b) the proposed macroelement is further generalized to encompass the most usual soil and footing-soil interface conditions. These include both cohesive and frictional soils, two-dimensional or three-dimensional foundation geometries and interface conditions allowing for foundation uplift or not.




(a) (b)

**Fig. X.1** (a) Modelling principle for foundation macroelement and (b) Forces and displacements for planar loading.

Macroelement model development consists in a two-step procedure: initially description of each nonlinear mechanism that contributes to the overall response independently from one another; then introduction of the surface of ultimate loads of the foundation (calculated separately in the Yield Design theory context) as a means to calibrate coupling between the non-linear mechanisms. In its present state of development, the model comprises three nonlinear mechanisms: a) the mechanism of sliding along the soil-footing interface, b) the mechanism of yielding in the vicinity of the footing due to soil irreversible behaviour and c) the mechanism of uplift as the footing may get detached from the soil surface. For simplicity, planar loading and cohesive soil conditions are considered in the following. For frictional soil conditions one can refer to Chatzigogos *et al*, (2009b). The model is formulated in terms of dimensionless parameters with the dimensionless forces assembled into a vector $\underline{Q}$ and dimensionless displacements into a vector $\underline{q}$ as follows:

$$\underline{Q} = \begin{bmatrix} Q_N & Q_V & Q_M \end{bmatrix} = \frac{1}{aN_{\max}} \begin{bmatrix} aN & aV & M \end{bmatrix} \tag{X.1}$$

$$\underline{q}^T = \begin{bmatrix} q_N & q_V & q_M \end{bmatrix} = \frac{1}{a} \begin{bmatrix} u_z & u_x & a\theta_y \end{bmatrix} \tag{X.2}$$

In (X.1) and (X.2), $a$ is a characteristic dimension of the footing (width $B$ for strip footings, or diameter $D$ for circular footings) and the dimensional forces and displacements are as in Figure X.1b. The quantity $N_{\max}$ represents the maximum vertical centered force that can be supported by the foundation.

*Uplift mechanism.* The uplift mechanism is described with a nonlinear elastic model that respects its fully reversible and non-dissipative character. In fact, footing detachment introduces a non-linearity of geometric nature: as the footing is uplifted, the soil-footing contact area is diminished, which leads to a reduction of the apparent stiffness of the foundation. This reduction is reproduced by means of an appropriately calibrated tangent elastic stiffness matrix $\underline{\underline{K}}$, function of the level of elastic displacements in the system:

$$\dot{\underline{Q}} = \underline{\underline{K}}\left(\underline{q}^{el}\right)\dot{\underline{q}} \tag{X.3}$$

The tangent elastic stiffness matrix is determined through finite element analyses for strip footings conducted by Crémer *et al.* (2002) and for circular footings conducted by Wolf and Song (2002). Under certain simplifying assumptions and using the above results, Chatzigogos *et al.* (2007) have shown that the tangent elastic stiffness matrix describing uplift on an elastic soil may be written as follows:

$$\begin{pmatrix} \dot{Q}_N \\ \dot{Q}_V \\ \dot{Q}_M \end{pmatrix} = \begin{bmatrix} K_{NN} & 0 & K_{NM} \\ 0 & K_{VV} & 0 \\ K_{MN} & 0 & K_{MM} \end{bmatrix} \begin{pmatrix} \dot{q}_N \\ \dot{q}_V \\ \dot{q}_M \end{pmatrix} \tag{X.4}$$

In (X.4), elements of $\underline{\underline{K}}$ are defined though the following relationships:

$$K_{NN} = K_{NN}^0 \tag{X.5}$$

$$K_{VV} = K_{VV}^0 \tag{X.6}$$

$$K_{NM} = K_{MN} = \begin{cases} \varepsilon K_{NN}\left(1 - \dfrac{q_{M,0}^{el}}{q_M^{el}}\right) & si \quad \left|q_M^{el}\right| > \left|q_{M,0}^{el}\right| \\ 0 & si \quad \left|q_M^{el}\right| \le \left|q_{M,0}^{el}\right| \end{cases} \tag{X.7}$$

$$K_{MM} = \begin{cases} \gamma\delta K_{MM}\left(\dfrac{q_{M,0}^{el}}{q_M^{el}}\right)^{\delta+1} + \varepsilon^2 K_{NN}\left(1-\dfrac{q_{M,0}^{el}}{q_M^{el}}\right)^2 & \text{if } \left|q_M^{el}\right| > \left|q_{M,0}^{el}\right| \\ K_{MM}^0 & \text{if } \left|q_M^{el}\right| \leq \left|q_{M,0}^{el}\right| \end{cases} \qquad \text{(X.8)}$$

$$q_{M,0}^{el} = \pm\frac{Q_N}{\alpha K_{MM}^0} \qquad \text{(X.9)}$$

In the above expressions, $K_{NN}^0, K_{VV}^0, K_{MM}^0$ represent the real part of the foundation impedance for quasistatic loading. The quantity $q_{M,0}^{el}$ represents the rotation angle of the foundation at the instant of uplift initiation. For an elastic soil, this quantity is linear with respect to the applied vertical force on the foundation. The parameters $\alpha$, $\beta$, $\gamma$, $\delta$ and $\varepsilon$ are numerical parameters that depend on footing geometry. Their values for strip and circular footings are given in Table 1.

**Table X.1** Numerical values for the parameters in equations (X.5)-(X.9)

| Geometry | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $\varepsilon$ |
|---|---|---|---|---|---|
| STRIP | 4.0 | 2.0 | 1.0 | 1.0 | 0.50 |
| CIRCULAR | 6.0 | 3.0 | 2.0 | 0.5 | 0.75 |

*Soil plasticity mechanism.* The soil plasticity mechanism is described through a bounding surface hypoplastic formulation following Dafalias and Hermann (1982). The yield surface of classical plasticity is replaced by a bounding surface $f_{BS}$: in the interior of this surface a continuous plastic response is obtained as a function of the distance between the actual force state represented by the loading point $\underline{Q}$ and an image point **P** on the bounding surface, defined through an appropriately chosen mapping rule (cf. Figure X.2a). As the loading point moves towards the bounding surface, the plastic response becomes more and more pronounced until a plastic flow is eventually produced when the loading point reaches the bounding surface: this situation corresponds to bearing capacity failure of the foundation. We can thus identify the bounding surface $f_{BS}$ with the ultimate surface of a footing resting on a cohesive soil with a perfectly bonded interface (no uplift or sliding allowed). Gouvernec (2007) has presented numerical results offering a detailed determination of this surface for various footing shapes. An extremely simple approximation that proves sufficient for practical applications is obtained by considering that this ultimate surface is an ellipsoid centered at the origin:

$$f_{BS} = Q_N^2 + \left(\frac{Q_M}{Q_{M,max}}\right)^2 + \left(\frac{Q_V}{Q_{V,max}}\right)^2 - 1 = 0 \qquad \text{(X.10)}$$

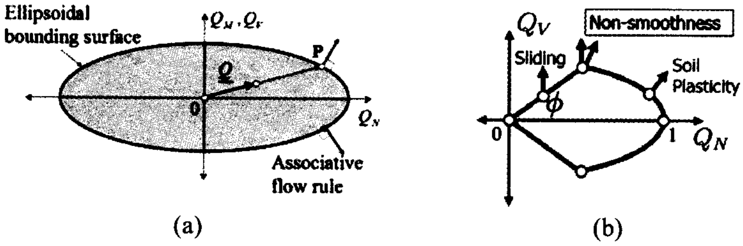



**Fig. X.2** (a) Ellispoidal bounding surface in the space of force parameters and mapping rule and (b) coupling of interface sliding and soil plasticity in the plane $Q_N - Q_V$

The functional form in (X.10) remains approximately independent of footing geometry and soil heterogeneity as shown by Bransby and Randolph (1998). The only parameters that change are $Q_{V,max}$ and $Q_{M,max}$, which define the maximum horizontal force and moment respectively: they occur for a zero vertical force. The quantity $N_{max}$ (necessary for the definition of $Q_i$) is retrieved from solutions presented by Salençon and Matar (1982). $Q_{V,max}$ is obtained by the condition of sliding along the interface. Finally, $Q_{M,max}$ is obtained for strip and circular footings from solutions presented by Bransby and Randolph (1998) and Gouvernec (2007) respectively. Table 2 provides values of these parameters for three values of soil heterogeneity expressed through the dimensionless parameter $k = a\nabla c / c_0$, where $a$ is the characteristic footing dimension, $\nabla c$ the cohesion gradient with respect to depth and $c_0$ the cohesion at the soil surface.

**Table X.2** Parameters for the definition of the bounding surface (*cf.* equation X.10)

| | Geometry | $N_{\max}$ | $Q_{V,\max}$ | $Q_{M,\max}$ |
|---|---|---|---|---|
| $k = 0$ | Strip | $5.14c_0a$ | 0.195 | 0.111 |
| | Circular | $6.05c_0(a^2\pi/4)$ | 0.165 | 0.111 |
| $k = 2$ | Strip | $8.01c_0a$ | 0.125 | 0.119 |
| | Circular | $7.63c_0(a^2\pi/4)$ | 0.131 | 0.115 |
| $k = 6$ | Strip | $10.29c_0a$ | 0.097 | 0.131 |
| | Circular | $9.68c_0(a^2\pi/4)$ | 0.103 | 0.129 |

*Sliding mechanism.* In case of a frictional interface obeying the Mohr-Coulomb strength criterion and characterized by a friction angle $\phi$, sliding of the footing has to be addressed as well. Consideration of frictional interface will induce Mohr-Coulomb branches in the $Q_N$-$Q_V$ space, as it is shown in Fig. 2b. The global domain of admissible force states will thus be obtained by the intersection of the bounding surface and the Mohr-Coulomb branches. This domain is convex but non-smooth.

Non-smoothness is treated within the multi-mechanism plasticity framework. For the examined case, two plastic mechanisms are introduced: the associated bounding surface hypoplastic model presented above (related to the soil response) and a non-associated perfectly plastic model related to the interface response. For the numerical implementation of multi-mechanism plasticity, the algorithm developed by Prévost and Keane (1990) is used.

*Coupling between mechanisms and consideration of ultimate surface of foundation.* The three non-linear mechanisms are combined together in a fully coupled way. Sliding and soil plasticity interact through the multi-mechanism plasticity formulation. Plasticity and uplift are coupled through dependence of the uplift initiation parameter on the vertical force, which in turn derives from the elastoplastic response of the system. Moreover, Crémer *et al.* (2002) have shown that in the presence of plastic soil behavior, uplift initiation is no longer linear with respect to $Q_N$, as is shown in Figure 3b. To address this additional uplift-plasticity coupling effect, equation (X.9) is replaced by a nonlinear relationship of the form:

$$q_{M,0}^{el} = \pm \frac{Q_N}{\alpha K_{MM}} e^{-\zeta Q_N} \tag{X.11}$$

In (X.11), $\zeta$ is a numerical constant that is calibrated with respect to the foundation ultimate surface.

The above non-linear mechanisms have been formulated independently from one another. However, these formulations are implicitly based on the notion of foundation ultimate surface. The foundation ultimate surface defines the domain of all possible combinations of loads that can be supported by the foundation. It can be determined on the basis of geometry and strength criteria of the system and independently of any constitutive law for the constituents of the system. This is achieved within the context of the Yield Design theory as for example in Chatzigogos *et al.* (2007) who presented the ultimate surface of a circular footing on a cohesive soil under general loading with possible uplift. A representation of this surface is shown in Figure 3a. Knowledge of the foundation ultimate surface is essential in the sense that it is identified with the hypoplastic bounding surface in the absence of uplift and also because it guides parameter calibration (parameter $\zeta$ and parameters of plasticity model) for uplift-plasticity coupling. The condition it supplies is that the toppling limit shown in Figure 3b has to be identified with the boundary of the foundation ultimate surface (and certainly should not exceed it).

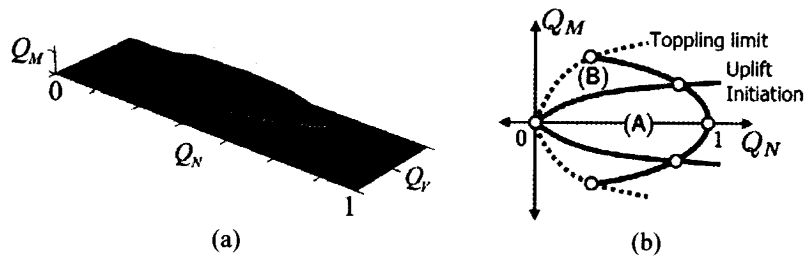



**Fig. X.3** (a) Foundation ultimate surface (cf. Chatzigogos et al. 2007) and (b) Uplift-plasticity coupling in the plane $Q_N - Q_M$

## X.3 Numerical application

Results pertaining to the validation of the presented model with respect to experimental and numerical results have been presented in Chatzigogos *et al.* (2009a, b). A simple numerical application of the model is presented herein in order to demonstrate its versatility for non-linear dynamic soil-structure interaction analyses. The macroelement is used to model a typical bridge pylon subjected to horizontal earthquake excitation as is shown in Figure X.4. The pylon model exhibits 4 degrees of freedom: 1 horizontal translation of the superstructure and 3 dofs for the translations and rotation of the foundation, which are reproduced by the macroelement. Three analyses are conducted with different base conditions considered at the foundation level: a) elastic analysis, b) elastic analysis with activation of uplift mechanism only and c) full elastoplastic analysis with uplift.


(a) (b)

**Fig. X.4** (a) Typical bridge pylon and (b) simplified model for non-linear dynamic analyses with macroelement.

The results of the three analyses are given in Figure X.5, which provides the obtained displacement time histories at the footing centre and the force-displacement diagrams at the foundation level.

Consideration of uplift on an elastic soil (Figure X.5b) reveals the reversible and non-dissipative character of this mechanism and leads to an initial reduction of the maximum moment applied on the footing. One can also notice the coupling between the rotation and the vertical displacement (heave) as the footing is uplifted. The maximum horizontal force is not affected since this force parameter is not coupled with uplift unless plastic behaviour is considered.

In the case of full elastoplastic response with uplift (Figure X.5c), a significant reduction is computed both for the maximum moment and the maximum horizontal force on the footing. This emphasizes the "isolation" effect for the foundation offered by the consideration of non-linearities, which are anyway present in the real structure. The price to pay is larger maximum and permanent displacements. As a matter of fact, in the context of displacement based approaches, forces are no longer of interest because the model gives direct access to the (peak and residual) displacements. In the examined case, it is interesting to notice the accumulation of vertical settlement as the system is excited horizontally: this is an immediate consequence of the strong coupling between all degrees of freedom within the adopted plasticity formulation. It is an important feature of real behaviour that cannot be captured by other types of simplified foundation models such as Winkler spring models.

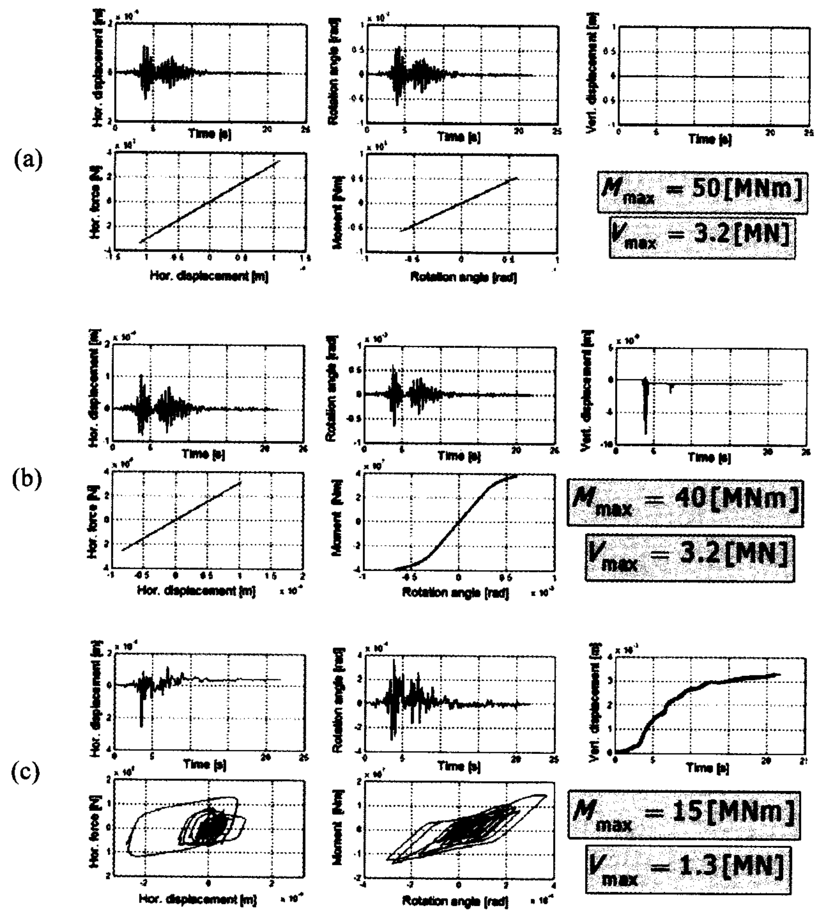



**Fig. X.5** Macroelement used for modelling a typical bridge pylon subjected to horizontal seismic excitation: (a) linear elastic base conditions, (b) elastic response with uplift and (c) full elasto-plastic response with uplift.

Additionally, it is interesting to notice that accumulation of permanent displacement is taking place continuously and not only when ultimate resistance is exceeded as in a Newmark type model. The macroelement is thus capable of efficiently reproducing both pre- and post-yield displacements of the footing. Finally, the model simulates the cycles of energy dissipation at the foundation level (notice the difference in cycle shape between moment and horizontal force due to presence of uplift affecting rocking response) allowing for a combined consideration of ductility demand in both the superstructure and the foundation.

## X.4 Concluding Remarks

The presented macroelement model is intended to serve as a practical tool for performance-based design of shallow foundations. It can facilitate permanent displacement evaluation at the foundation level within standard FEM structural modelling and offers the flexibility of activating/deactivating, at will, independent nonlinear mechanisms that may take place at the foundation level during dynamic soil-structure interaction. It exhibits sufficient generality to be applicable to most usual shallow foundation configurations and in parallel preserves simplicity that facilitates numerical implementation. Finally, it may prove useful in tackling new research questions such as assessing direct-displacement based design methodologies with consideration of SSI as in the spirit of Priestley *et al.* (2007).

REFERENCES


Bransby MF, Randolph MF (1998) Combined loading of skirted foundations. Géotechnique 48(5): 637-655

Chatzigogos CT, Pecker A, Salençon J (2007) Seismic bearing capacity of a circular footing on a heterogeneous cohesive soil. Soils and Foundations 47(4): 783-798

Chatzigogos CT, Pecker A, Salençon J (2009a) Macroelement modeling of shallow foundations. Soil Dynamics and Earthquake Engineering 29(5): 765-781

Chatzigogos CT, Figini R, Pecker A, Salençon J (2009b) A macroelement formulation for shallow foundations on cohesive and frictional soils. International Journal for Numerical and Analytical Methods in Geomechanics (submitted for publication)

Crémer C, Pecker A, Davenne L (2001) Cyclic macro-element for soil-structure interaction: material and geometrical non linearities. International Journal for Numerical and Analytical Methods in Geomechanics (25): 1257-1284

Dafalias YF, Hermann LR (1982) Bounding surface formulation of soil plasticity. In: Pande GN, Zienkiewicz OC (Eds) Soil Mechanics-transient and cyclic loading, Wiley, 173-218

Gouvernec S (2007) Failure envelopes for offshore shallow foundations under general loading, Géotechnique 57(9): 715-728

Newmark (1965) Effects of earthquakes on dams and embankments. Géotechnique 15(2): 139-160

Prévost JH, Keane CM (1990) Multi-mechanism elasto-plastic model for soils. ASCE Journal of Engineering Mechanics 116(8): 1255-1263

Priestley MJN, Calvi GM, Kowalsky MJ (2007) Displacement-based Seismic Design of Structures. IUSS Press, Pavia, Italy.

Salençon J, Matar M, (1982) Capacité portante des fondations circulaires. Journal de Mécanique théorique et appliquée 1(2): 237-267

Wolf JP, Song C (2002) Some cornerstones of dynamic soil-structure interaction. Engineering Structures 24: 13-28