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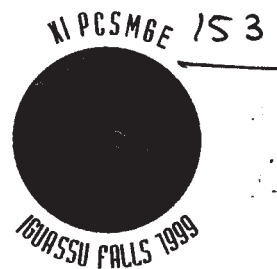
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Ground reinforcement in seismic areas

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ABSTRACT: The yield design theory provides a rational tool for the evaluation of the seismic bearing capacity of shallow foundations. It has been successfully implemented and tested against case histories for foundation on unreinforced soil. This theoretical framework is extended to the analysis and design of an innovative foundation concept in seismic areas. This concept is based upon the reinforcement of the in-situ soil with stiff, closely spaced inclusions: it makes it possible to improve the seismic bearing capacity of the foundation significantly while being both economical and easy to implement. Moreover, this concept introduces a capacity design philosophy in foundation engineering. Numerical studies and centrifuge model tests have demonstrated the validity of the concept and of the theoretical tools.

1 INTRODUCTION

Following the 1985 Michoacan Guerrero earthquake and the foundation failures reported in Mexico City, a significant amount of work has been devoted to the evaluation of the foundation bearing capacity under earthquake loading. Although restricted to shallow foundations, these studies represent a significant improvement on the previous situation, clarifying some of the key aspects, such as relative contribution of the inclination and eccentricity of the structural loads on the one hand, and of the inertia forces within the soil, on the other hand.

The general theoretical framework for the evaluation of the seismic bearing capacity has been set forth in these studies (Pecker, Salençon, 1991; Salençon, Pecker, 1995a, b; Pecker et al., 1996; Auvinet et al., 1996; Paolucci, Pecker, 1997a, b): introducing the concept of a bounding surface in the loading parameters space to define the set of allowable loads, recognizing the limitations of a pseudo-static approach and developing a methodology to compute the permanent, irreversible displacements. These results have been validated by comparison with observed foundations behaviors during earthquakes, experimental results on model tests and sophisticated finite element analyses.

These theoretical tools are the basis of the analysis of a new, innovative (at least in seismic areas), design concept implemented to improve the seismic bearing capacity of a shallow foundation (Pecker & Salençon 1998). This scheme, which is presently being implemented for the foundations of the Rion Antirion bridge in Greece (Pecker & Teyssandier 1998), besides improving the bearing capacity, introduces the capacity design philosophy in foundation engineering.

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2 GENERAL FRAMEWORK FOR THE EVALUATION OF THE FOUNDATION BEARING CAPACITY

The dynamic bearing capacity of foundations can be examined from two different viewpoints:
- The probably most rigorous approach would be to develop a comprehensive model (finite element model) including both the soil and the structure. Obviously, if the analysis is meant to be significant, a realistic non-linear constitutive soil model must be used and initial stresses need to be correctly estimated. Owing to this constraints, to computer limitations and also to the fact that development of a global model requires competence in geotechnical engineering, as well as in structural engineering, in soil-structure interaction and in numerical analysis, such an approach is seldom used in every day practice. In addition, it is actually not well-suited for the development of design which requires that various alternatives be tested before achieving a final design.
- The alternative approach, which represents the state of practice, is to uncouple the evaluation of dynamic loads (a structural engineer task) from the verification of the bearing capacity (a geotechnical engineer task). This is a so-called substructure approach, with the following limitations, which, up to now, have not clearly been assessed:
. the evaluation of the dynamic loads is based on a linear elastic analysis of the soil-structure system; at most, some degrees of non-linearities can be introduced in an approximate manner, but how the dynamic loads are affected by the yielding of the foundation is usually not evaluated. Recently, Paolucci (1997) has shown that the base shear force transmitted by the supertructure may differ significantly from that predicted from a classical linear elastic soil-structure interaction analysis, if soil yielding is accounted for;
. the bearing capacity is checked using a pseudo-static approach, in which only the maximum loads acting on the foundations are considered.

This is clearly the approach which is favored in aseismic design building codes, like Eurocode 8 - Part 5, which states that *"the bearing capacity of the foundation should be calculated using appropriate graphs and formulae which include the load inclination and eccentricity arising from the inertia forces of the structures as well as the possible effects of the inertia forces in the supporting soil itself"*. It is further required that the design action S_d be smaller than or equal to the design strength:

$$S_d \le R_d \tag{1}$$

In the preceding equation, the design action must be interpreted as any combination of the design vertical force N_d, horizontal force T_d, overturning moment M_d (and possibly inertia forces in the soil mass):

$$S_d = F(N_d, T_d, M_d) \tag{2}$$

and the design strength as the bearing capacity equation:

$$R_d = \psi\left(\frac{C}{\gamma_m}, \frac{\tan\phi}{\gamma_m}, B, L\right) \tag{3}$$

where C and ϕ are the soil cohesion and friction angle respectively, γ_m is a material partial safety factor, B and L define the geometry of the foundation.

Equation (1) explicitly requires that at any time, the design action be smaller than the design strength and precludes the possibility of having the available resistance of the soil-foundation system being exceeded for short periods of time.

As pointed out by Newmark (1965) in his classical work on the seismic stability of earth dams, short periods of exceedance of the available resistance are not associated with a general failure of the structure but are accompanied by the development of permanent, irreversible displacements. This philosophy has been successfully applied to the seismic design of earth dams and gravity retaining walls and should be extended to shallow foundations.

It could be incorporated in code-like format simply through a slight modification of equation (1)

$$F\left(N_d, T_d, M_d\right) \leq \frac{1}{\gamma_c} \psi\left(\frac{C}{\gamma_m}, \frac{\tan\phi}{\gamma_m}, B, L\right) \qquad (4)$$

where γ_c is a partial "safety" factor which can take values greater than 1.0 to allow for small exceedance of the available resistance. Clearly, the value of γ_c depends on the failure mode of the structure, on its consequences with regards to the overall behavior and on the soil behavior, whether it exhibits a post peak softening or a ductile behavior.

2.1 Determination of the equation for the bounding surface

The yield design theory (Salençon 1983, 1990), which belongs to the category of limit analysis methods, constitutes the appropriate theoretical framework for the evaluation of the system available resistance through equation (1). Alike any limit analysis method, the derivation of upper and lower bound solutions allows to bracket the exact solution and, possibly, to determine it exactly when both bounds coincide. In the following, only the kinematic approach is used to derive an upper bound estimation of the bearing capacity; for the validity of the derived solution, one can refer either to Salençon & Pecker (1995a, b) or to Ukrichton et al. (1998).

The kinematic approach of the yield design theory states that, for any virtual, kinematically admissible, velocity field $\underline{U}$, the following inequality holds:

$$P_e\left(\underline{U}\right) \leq P_m\left(\underline{U}\right) \qquad (5)$$

where P_e is the work of the external forces (N, T, M) in velocity field $\underline{U}$ and P_m is the maximum resisting work of the system in that velocity field which is fully defined from the system geometry and the constitutive materials strengths.

Equation (5) is similar to equation (1) and constitutes the theoretical background for limit state design calculations (Salençon 1992). Consideration of different kinematic mechanisms, like those presented in figure 1 for a cohesive soil, allows the right hand side of inequality (5) to be minimized and the system resistance to be approximated (by excess).

Once the best approximation is obtained, inequality (5) defines a surface, called the bounding surface, which delimits the set of allowable loads for the system: any combination (M, N, T) falling inside the surface is a stable state of forces, whereas those combinations falling outside cannot be withstood by the system.
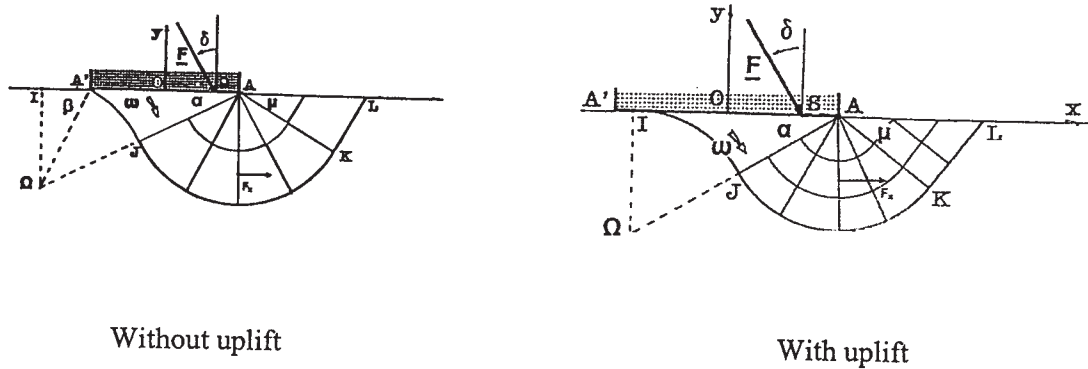

Without uplift

With uplift

Figure 1. Kinematic mechanisms - Cohesive soil.

Figure 2 presents a three-dimensional view of the bounding surface determined for a cohesive soil without tensile strength; only the upper part corresponding to $M \geq 0$ is shown.
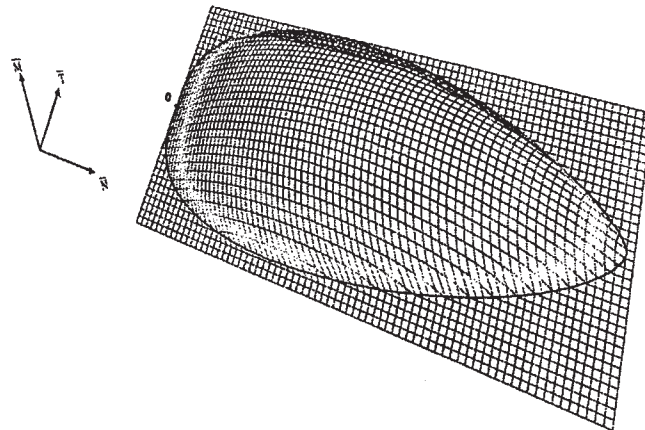

Figure 2. Skeletal view of the bounding surface for a cohesive soil without tensile strength.

Simplified analytical formulae are available to define the bounding surface for a cohesive or a cohesionless soil (Pecker 1997). It is worth noting that experimental evidence of the existence of a bounding surface has been given by Butterfield & Gottardi (1994) and Kitazume & Terashi (1994).

2.2 Determination of permanent displacements

One possibility which is offered by the theory, to propose tentative values for the "safety factor" γ_c in equation (4), is the computations of permanent displacements which take place once the bounding surface is reached. The soil foundation system is assumed to behave as a rigid perfectly plastic system, for which the bounding surface defined previously is adopted as the boundary for the apparition of plastic deformations.

This method is an extension of Newmark's original rigid blocks method to deformable bodies. It has been successfully applied to actual case histories of foundation failure (Auvinet et al. 1996; Pecker et al. 1995).

Under the assumptions spelled above, this method permits a rigorous definition of failure in terms of unacceptable permanent displacements.

3 NEW CONCEPTS IN FOUNDATION ENGINEERING

Although the concepts introduced in the previous paragraph lead to a more rational approach of the seismic bearing capacity of foundations and often result in a significant cost saving of the design, the bearing capacity of foundations may still be of concern in difficult environmental conditions characterized by poor soil conditions and high seismic intensities.

Under such circumstances, alternative foundation designs must be investigated and their relative merits, in terms of economy, feasibility and technical soundness, must be weighted before a final choice is made. When shallow foundations prove to be unsatisfactory or inadequate, a classical alternative is to resort to piled foundations, although piled foundation failures during or after earthquakes have also been reported.

The general framework outlined in the previous paragraph may however be used to design shallow foundations in such a way that:
- permanent displacements are allowed,
- the development of these permanent displacements do not impede a proper functioning of the structure; this can be achieved by a careful control on the failure mode.

To illustrate these possibilities, the theory is applied to a new concept in foundation engineering (Pecker & Salençon 1998), which, to the best of the authors' knowledge, had never been proposed or implemented before in seismic areas. This concept is presently being designed and implemented for a large bridge structure in Greece, the Rion Antirion bridge (Pecker & Teyssandier 1998).

It consists of reinforcing the existing soil strata with stiff inclusions at a close spacing and to lay a shallow foundation on top of the reinforced soil through a transition, gravel layer.

For illustration purposes, figure 3 presents an example of this concept: the foundation is a gravity caisson, 90 m in diameter, and the inclusions consist of hollow steel cylinders, 2 m in diameter, 20 mm thick and 25 m long, spaced at as a square grid of 7 m x 7 m below and outside the footprint of the foundation.
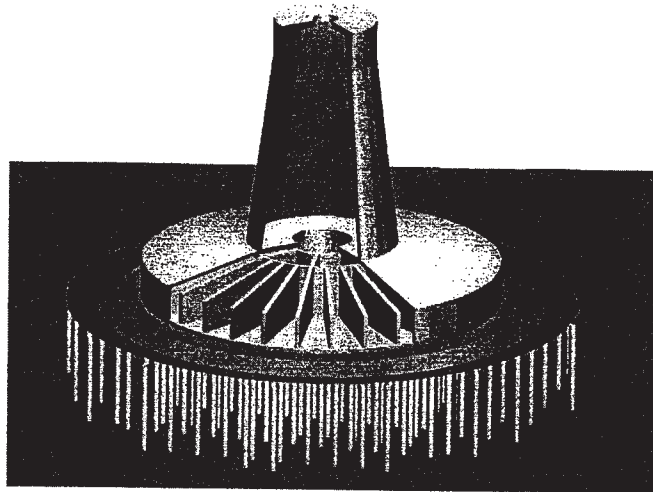

Figure 3. View of a reinforced soil foundation (Dumez-GTM).

Although the foundation looks like a piled foundation, it does not at all behave as such since no connection exists between the inclusions and the raft which allows the foundation to uplift or to slide with respect to the soil; the density of inclusions is far more important and the length smaller than usually employed in piled foundations.

Besides the merits of its simplicity and economy, this technique allows for the implementation of a seismic design philosophy very similar to the capacity design principles used in structural engineering.

3.1 Theoretical analysis of a reinforced soil

The yield design theory which has been used for the evaluation of the bearing capacity on an unreinforced soil can be extended to account for the presence of inclusions or nails (de Buhan & Salençon 1993). A mixed modeling approach is used for the reinforced soil in which the soil is modeled as a 2D continuum and the inclusions as beams. The kinematic mechanisms shown in figure 1 have been adopted to account for the presence of inclusions (Pecker, Salençon, 1998).

The contributions of all the inclusions to the maximum resisting work $P_m(\underline{U})$ in equation (5) are added to the maximum resisting work of the soil (right hand side of equation [5]) and minimization is performed on the geometric parameters defining the mechanisms to find the best upper bound.

These calculations can easily be programmed on a PC; numerical minimization on the five geometric parameters defining the kinematic mechanisms is efficiently performed with a downhill simplex method (Nelder & Mead 1965).

3.2 Example of application

Let us take for illustration purposes the example presented in figure 3. The soil profile below the foundation consists of a purely cohesive clay layer with a shear strength increasing linearly with depth:

$$Su = 35 + 2.8\,z \qquad (6)$$

where Su is expressed in kPa and z is the depth below the ground surface in m. The normal force acting on the foundation (dead weight) is equal to 860 MN, corresponding to a uniform vertical stress of 135 kPa.

Without the inclusions, the cross-section of the bounding surface by the plane N = 860 MN is shown as a dotted line in figure 4 in the lower left corner.
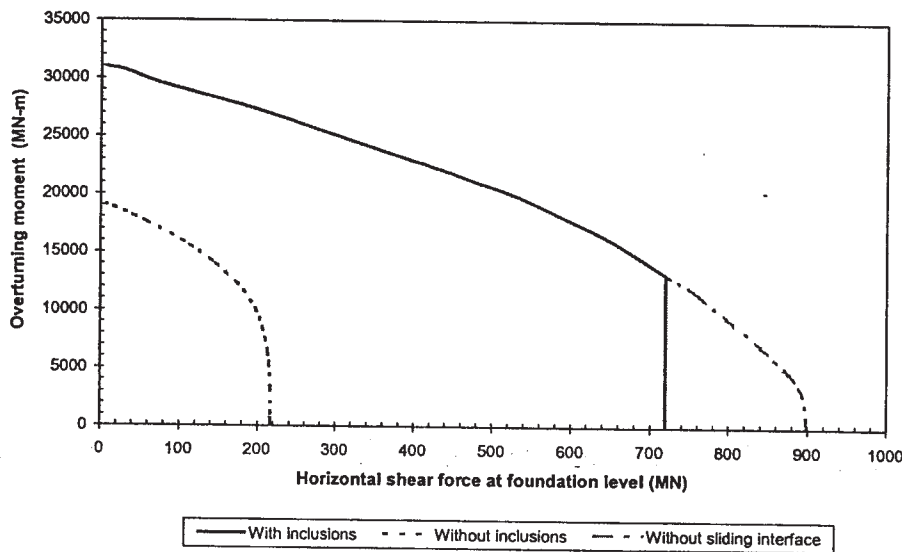

Figure 4. Bounding surface for a reinforced soil.

If the reinforced scheme described in the previous paragraph is implemented, the bounding surface is considerably expanded, as represented by the solid line. The maximum allowable horizontal shear force, corresponding to the vertical ascending line to the right of the figure is associated to horizontal sliding at the soil - foundation interface which occurs in the transition gravel layer placed on top of the inclusions; therefore:

$$T = N \tan \phi = 860 \tan 40° = 721 \text{ MN} \qquad (7)$$

Starting from the point (M = 0, T = 721 MN) and moving along the bounding surface, it is seen that sliding at the interface does occur until the overturning moment reaches a value of 15 000 MN.m; for higher values of the overturning moments, rotational mechanisms prevail and the maximum allowable horizontal force decreases.

3.3 Experimental and numerical validation of the concept

The concept and theoretical evaluations presented above have been validated through model tests performed in a centrifuge (Garnier, Pecker, 1999) and through non-linear finite element analyses.

3.3.1 Numerical analyses

Non-linear finite element analyses using the computer code DYNAFLOW (Prevost 1981) have been run under monotonic increasing loads up to failure. An elastoplastic von Mises constitutive model with kinematic hardening for the clay, and a Mohr Coulomb model for the ballast layer, have been

used. Special contact elements with a limited shear force capacity and no tensile strength connect the raft and the inclusions to the soil. All the runs confirmed, within ± 10%, the limit loads computed through limit analysis.

In addition, the failure mechanisms found in the finite element analyses are alike the kinematic mechanisms shown in figure 1.

3.3.2 *Centrifuge model tests*

This totally innovative concept clearly calls for experimental validation. As sophisticated as they can be, the theoretical and numerical tools do not have the capability of modeling all the details of the behavior of this complex scheme during an earthquake. Centrifuge model tests were therefore undertaken with a threefold objective:
- to validate the theoretical predictions of the ultimate bearing capacity of the foundation under monotonically increasing shear force and overturning moment;
- to identify the failure mechanism of the foundation under these combined loads;
- to assess the behavior of the foundation under various cyclic load paths.

All the tests have been carried out at 100 g on models at a scale of 1/100 of a prototype foundation; the footing diameter is 0.30 m, the inclusions lengths 8.5 cm and their diameters 6.7 mm.

The soil material has been sampled on site, at the location of the bridge and reconsolidated in the container (0.90 m in diameter).

The models are instrumented to measure:
- pore pressure at different locations below the foundation,
- soil settlements
- applied loads (shear force and overturning moment)
- bending moments in the inclusions.

The soil properties were measured in flight (at 100 g) with a cone penetrometer actuator.

Two series of tests have been run:
- the first one is intended to check the validity of the theory for monotonic loads; therefore tests set up departing, even significantly, from the foreseen one have been investigated;
- the second one is intended to check the behavior of the proposed reinforcement scheme under cyclic loads. The typical load paths are shown in figure 5.
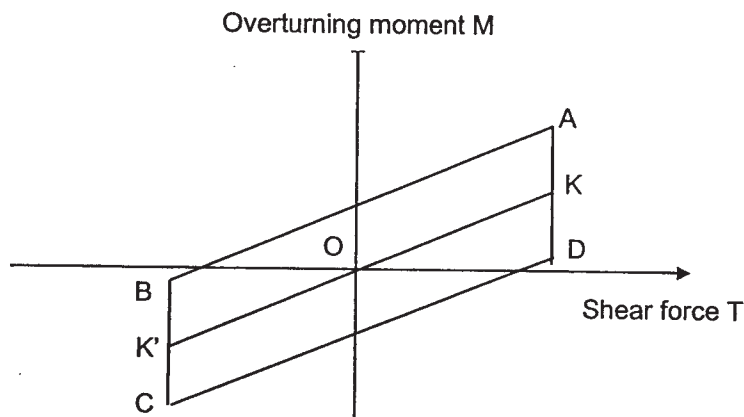

Figure 5. Typical load path

For either the three tests carried out in Nantes or for the five tests carried out in Bordeaux in another centrifuge facility, figure 6 presents the comparison between the computed failure loads (yield design theory) and the measured ones. In Nantes, the failure loads (triangles in figure 6), were measured after cyclic loading, whereas in Bordeaux (diamonds in figure 6), they were measured without prior cyclic loads. Figure 6 shows that the trends are correctly predicted and that a very good agreement is achieved for the tests in Nantes, whereas failure loads are undestimated by the theoretical analysis for the tests in Bordeaux, as a result of significant changes in the initial geometry of the system near the

end of the test (soil bulging in front of the model). For those tests, if the "failure loads" are taken at the start of the changes in geometry, the agreement between predictions and measurements comes out very good.
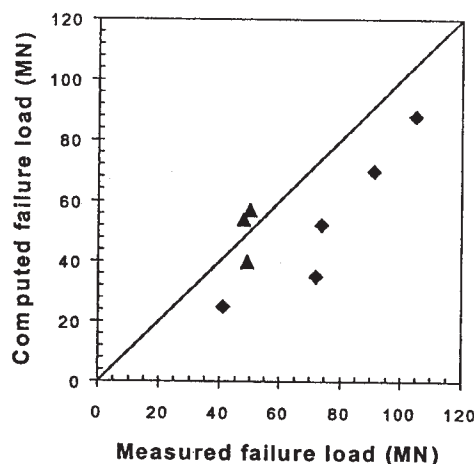

Figure 6. Comparison between measured and computed ultimate loads.

Finally, the cyclic tests have shown that the system exhibits a significant energy dissipation capacity with the formation of fat hysteresis loops during cyclic loading. The equivalent damping ratio computed from the hysteresis loops of figure 7 is equal to 18%, whereas the theoretical predictions from the finite element analyses are of the order of 15% at the same load level.

In addition, even at very high cyclic load levels (75% to 80% of the failure load), the system exhibits very small degradation; the hysteresis loops stabilize after a few number of cycles (figure 7); the permanent excess pore pressure is only a small fraction (30%) of the initial hydrostatic pressure.
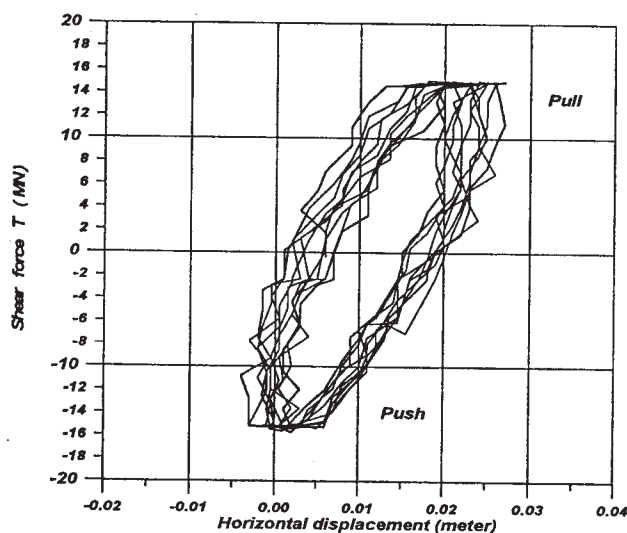

Figure 7. Horizontal displacement of the foundation vs. cyclic shear force T.

4 CAPACITY DESIGN PHILOSOPHY IN FOUNDATION ENGINEERING

The capacity design philosophy used in structural engineering consists in establishing a suitable strength hierarchy between the components of the system (Paulay 1993). The structural system is rationally and deterministically chosen so as to be able to mobilize energy dissipating regions which will have ample reserve deformation capacity to accommodate significant departures from the initial

estimates. Paulay notes that *"the strategy invites the designer to tell the structure where plastic hinges are desirable or convenient and practicable at the ultimate limit state and to proscribe plastification in other regions"*.

Clearly, this statement is relevant to the proposed reinforcement concept (Pecker, 1998).

Referring to figure 4, besides significantly improving the resisting capacity of the foundation, the reinforcement concept enforces this design philosophy:

- without reinforcement, the maximum horizontal force corresponding to a sliding at the gravel - clay interface, decreases from the beginning for increasing overturning moments; this decrease becomes more significant for overturning moments larger than 7 000 MN.m and the failure mechanism involves rotation of the foundation from the very beginning;

- with reinforcement, pure sliding prevails over a large range of overturning moments (up to 15 000 MN.m in that case, more than twice the previous value). In addition, would the interface have a larger strength capacity, the vertical line at 720 MN would move to the right and the bounding surface would be represented by the dashed line joining the horizontal axis at 900 MN; in such a case, the domain of the allowable forces is extended, but as soon as the bounding surface is reached, failure modes involve a rotation of the foundation.

This shows that, the effect of the combined gravel layer and soil reinforcement is not only to improve the bearing capacity, but moreover, to enforce and control the failure mode:

- the fuse provided by the gravel layer (which is a well-controlled material) plays the role of the energy dissipating region: it limits the maximum shear force at the interface, dissipates energy by sliding and forces the foundation "to fail" according to a failure mode which is not detrimental to its overall behavior,

- the reinforcement increases the strength capacity with respect to undesirable failure modes, like rotational failure modes especially for tall structures. In addition, it provides an efficient means of dissipating energy, as evidenced by the results of cyclic centrifuge tests.

This increase in the foundation bearing capacity and the shape of the bounding surface is a function of the reinforcement scheme. Therefore, with a proper choice of the inclusions strength, spacing and length, the capacity design principle described above can be enforced for a wide range of load-moment combinations.

5 CONCLUSIONS

Based on the yield design theory, a rational approach to the evaluation of the seismic bearing capacity of shallow foundations has been developed. This approach accounts for the essential features of the problem: the loading parameters (N, T, M and soil inertia forces) are treated as independent parameters, leaving to the designer the choice of the most approximate combination of them; failure is no longer defined with reference to a pseudo-static safety factor, and a methodology for computing the permanent displacements has been derived and tested against case histories.

This approach has been extended to a new design concept for foundation engineering in seismic areas. This concept based on an in-situ reinforcement of the existing soil with stiff, closely spaced, inclusions overlaid by a well-controlled gravel layer allows for the use of shallow foundations, even in difficult environmental conditions (poor soil conditions, high level of seismicity). Even more important is the fact that this foundation concept enforces a capacity design philosophy in foundation engineering. It looks therefore very promising for increasing the safety of structures and presents the advantage of being simple and rather economical.

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