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**AN INTERACTIVE COMPUTER SOFTWARE FOR THE YIELD DESIGN  
OF REINFORCED SOIL STRUCTURES  
UN LOGICIEL INTERACTIF DE CALCUL À LA RUPTURE  
DES OUVRAGES EN SOLS RENFORCÉS**

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**Abstract** : This paper outlines the main features of a computer software (STARS) specifically devised for analysing the stability of reinforced soil retaining structures within the framework of the yield design theory. The considerable speed of numerical computations associated with a high degree of conviviality make it possible to perform parametric studies, such as the one presented hereafter, assessing the way the shear and bending strengths of the reinforcements should be taken into account in design calculations.

**Résumé** : Cette contribution décrit les principales caractéristiques d'un logiciel de calcul (STARS) spécialement conçu pour l'analyse de stabilité des ouvrages de soutènement en sols renforcés dans le cadre de la théorie du Calcul à la Rupture. La rapidité des calculs numériques associée à une grande convivialité du programme rendent possibles des études paramétriques, telles que celle présentée ci-après, examinant la façon dont il convient de prendre en compte les résistances à l'effort tranchant et au moment de flexion des renforcements dans les calculs de dimensionnement.

I- PRINCIPLE OF THE METHOD AND PRACTICAL IMPLEMENTATION

According to the yield design theory ([1],[2]), and more specifically to its kinematic approach, a reinforced soil retaining structure such as that sketched in Fig. 1, will be proved definitely unstable as soon as one can exhibit a virtual velocity field  $\underline{v}$ , also called "failure mechanism", for which the work done by the given external forces (e.g. weight of the soil, loads applied to the structure) exceeds the maximum resisting work developed by the soil and the reinforcements on account of their respective strength capacities. Considering a particular class of failure mechanisms where a volume OAB of reinforced soil is given a rigid body motion with a velocity  $\omega$  about a

point  $\Omega$ , one may write :

$$\Gamma(\Omega, AB) = \frac{W_r(\Omega, AB, \omega)}{W_e(\Omega, AB, \omega)} < 1 \rightarrow \text{Instability of the structure} \quad (1)$$

where  $W_e(\Omega, AB, \omega)$  is the work of the external forces, and  $W_r(\Omega, AB, \omega)$  denotes the maximum resisting work. The factor  $\Gamma(\Omega, AB)$ , which is independent of  $\omega$ , is called the factor of confidence associated with the rotational failure mechanism under consideration. In the case of an unreinforced homogeneous soil of friction angle  $\phi$ , it has been shown ([1], [2]) that,  $\Omega$  being kept fixed, the minimum of  $\Gamma(\Omega, AB)$ , and hence the best estimate for the stability of the structure, is obtained when  $AB$  is an arc of logspiral of angle  $\phi$  and focus  $\Omega$ . It follows that, denoting by  $\Gamma(A, \theta_1, \theta_2)$  the corresponding value of  $\Gamma(\Omega, AB)$ , where  $\theta_1$  and  $\theta_2$  are the angular parameters defining the position of  $\Omega$ , the minimum value of  $\Gamma(A, \theta_1, \theta_2)$  with respect to  $\theta_1$  and  $\theta_2$  is such that

$$\Gamma(A) = \min_{\theta_1, \theta_2} \Gamma(A, \theta_1, \theta_2) < 1 \rightarrow \text{Instability of the structure} \quad (2)$$

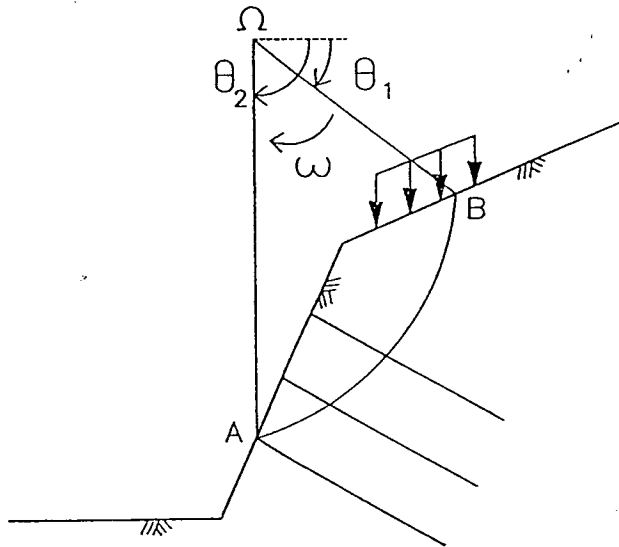


Fig. 1. Rotational failure mechanism of a reinforced soil retaining structure.

Remark : It should be pointed out that the above definition of the factor of confidence does fit into the new way of thinking based on the ultimate limit state concept recently introduced for the design of reinforced soil structures [3]. It differs from the usual notion of safety factor adopted in the traditional approach, which

can actually be interpreted as a partial safety coefficient on the sole strength parameters of the soil.

STARS is a computational programme based upon the use of such rotational failure mechanisms within the framework of the yield design theory. Since it refers to the concept of maximum resisting work, it follows that, unlike most classical stability analysis methods, notably the well-known "method of slices", it does not need the introduction of complementary assumptions, and leads therefore to clearly interpretable (namely upper bound) estimates for the stability of structures.

In its most updated version, this programme can deal with a wide range of reinforced structures, involving several types of reinforcements (nails, tie-backs, geotextiles etc.), and subjected to concentrated or distributed loads applied to the boundary surface, including under seismic conditions accounted for through a pseudo-static analysis. In case of a soil made up of several superimposed layers with different friction angles  $\phi_i$ , the line bounding the rotating volume is constructed by connecting several arcs of logspiral with the same focus, each of them being characterized by the angle  $\phi_i$  of the corresponding soil layer it intersects [4].

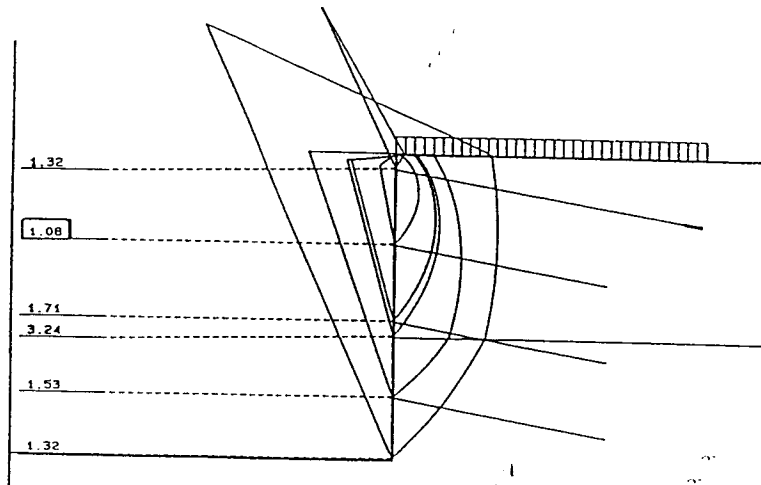


Fig. 2. Yield design of a reinforced excavation.

Fig. 2 shows an example of structure designed with the help of this programme (further details may be found in [4]). It concerns a 10m deep vertical cut excavated in a two-layer soil, with a uniformly distributed load applied on its upper surface. Since the unreinforced structure is obviously unstable (the factor of confidence remains far below unity), a reinforcement scheme consisting of three

parallel rows of inclusions (nails) and one layer of tie-backs emerging just below the upper corner has been proposed. The results of the computations performed are reported in Fig. 2 which, for each family of failure mechanisms passing either through the toe of the excavation, or emerging just above a layer of reinforcement, displays the most critical one with the associated factor of confidence. It should be pointed out that, contrary to what might have been expected, the lowest factor of confidence (1.08) does not correspond to a failure mechanism emerging at the toe of the structure (the corresponding factor is 1.32), but to a failure line coming out on the facing of the excavation at a depth of 3m. This result underlines the fact that, restricting the minimization to failure mechanisms passing through the toe, may lead to considerably overestimating the stability of the structure.

The swiftness of computations, along with a great conviviality (all data can be checked through graphic visualization, and changed immediatly if necessary, at any stage of the design process), allows any user of this programme to work out the optimum reinforcement scheme of any structure within a minimum time. By way of illustration, the computations of the above described example, which involves exploring nearly 4000 failure mechanisms, are carried out in much less than a few minutes, even when operating on a personal computer.

## II- AN EXAMPLE OF PARAMETRIC STUDY.

Besides its outstanding performances as a handy computational tool for the engineering design of reinforced soil structures, STARS provides an invaluable means for undertaking otherwise tedious parametric studies, such as the one presented hereafter concerning the role of shear and bending strengths of the reinforcements.

### II.1 Theoretical background

In any rotational failure mechanism, the *maximum resisting work* is computed as the sum of two terms, one relating to the contribution of the soil, while the other one concerns the reinforcing inclusions :

$$W_r(\Omega, AB, \omega) = W_r^{\text{soil}} + W_r^{\text{reinf}} \quad \text{with} \quad W_r^{\text{reinf}} = \sum_{k=1}^n \pi_k \quad (3)$$

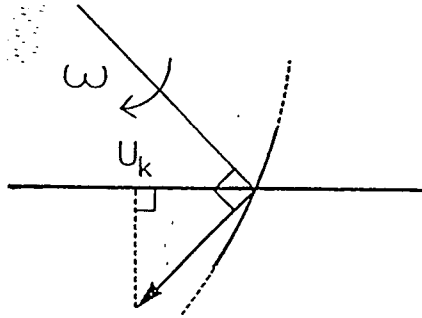
$\pi_k$  ( $k=1, \dots, n$ ) represents the *maximum resisting work* developed by the  $k^{\text{th}}$  layer of reinforcement in the velocity discontinuities induced by the rotational failure mechanism at the intersecting point between AB and the reinforcement (Fig. 3). In the case of reinforcements,

which exhibit resistance to axial tensile forces only, the expression of  $\pi_k$  simply reduces to :

$$\pi_k = N_o \langle U_k \rangle \quad (4)$$

where  $N_o$  is the tensile strength of the reinforcement and  $\langle U_k \rangle$  denotes the positive part of the axial component along the reinforcement of the velocity jump generated by the failure mechanism.

Fig. 3



A more general expression is available [5] when the shear and bending strengths of the reinforcement are to be taken into account through a strength condition such as [6] :

$$(N/N_o)^2 + (V/V_o)^2 + |M/M_o| - 1 \leq 0 \quad (5)$$

where  $N$ ,  $V$  and  $M$  are the axial, shear forces and bending moment respectively, and  $N_o$ ,  $V_o$ ,  $M_o$  their ultimate values. Expression (4) of  $\pi_k$  is obtained for  $M_o = V_o = 0$ , the compressive axial strength being set equal to zero.

Furthermore, a comprehensive theoretical analysis [5] has shown that in order to account for the possibility of a failure mode of the reinforcements by flexion across a shear zone, only a fraction of their shear resistance is to be introduced by means of the following additional condition :

$$|V| \leq \mu V_o \quad (6)$$

$\mu$  is a dimensionless parameter ranging between 0 and 1 which compares a characteristic transversal length of the inclusions (their diameter in the case of a circular cross section) with the thickness of the shear zone intersecting the reinforcements in a flexional failure mode. A typical value of  $\mu$  is 0.1 .

Fig. 4 represents the domain corresponding to the combination of the strength conditions (5) and (6) in the

non dimensional coordinates  $N/N_0, V/V_0, M/M_0$ .

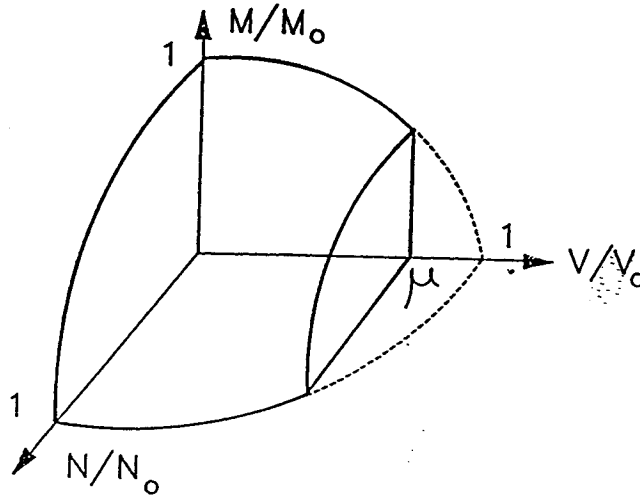


Fig. 4. Strength domain of a reinforcement.

The corresponding maximum resisting work  $\pi_k$  has been computed, then introduced in the computer code STARS.

## II.2 Results of the parametric study [7]

The charts represented below refer to the retaining structure of Fig. 5. The strength parameters of the soil are the cohesion  $C$  and the friction angle  $\phi$ , while the strength of the reinforcements is characterized by  $N_0, V_0, M_0$  and  $\mu$ . Varying the latter, while all the other parameters are held constant, the factor of confidence  $\Gamma$  derived from the computations through STARS using logspiral failure mechanisms, appears as a function of the following non dimensional parameters :

$$r = N_0 / (Ch) \quad ; \quad \alpha = M_0 / (N_0 h) \quad ; \quad \text{and } \mu$$

since the shear strength  $V_0$  is taken equal to  $N_0/2$ .

Actually, those computations show that the influence of  $\alpha$  is hardly noticeable, so that the factor of confidence depends merely on  $r$  and  $\mu$  :

$$\Gamma = \Gamma(r, \mu)$$

Figures 6-a, b, c which relate to  $\phi = 0^\circ, 20^\circ$  and  $40^\circ$  respectively, give the variations of the ratio  $\Gamma(r, \mu) / \Gamma(r, \mu = 0)$  with respect to  $r$  and  $\mu$ . This ratio

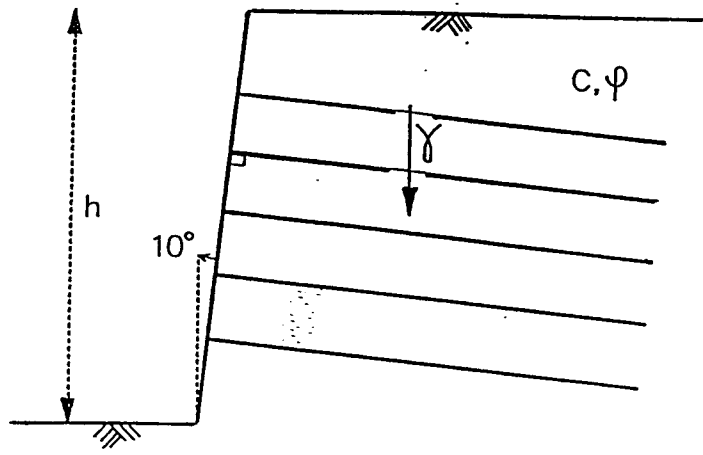


Fig. 5. Structure investigated in the parametric study.

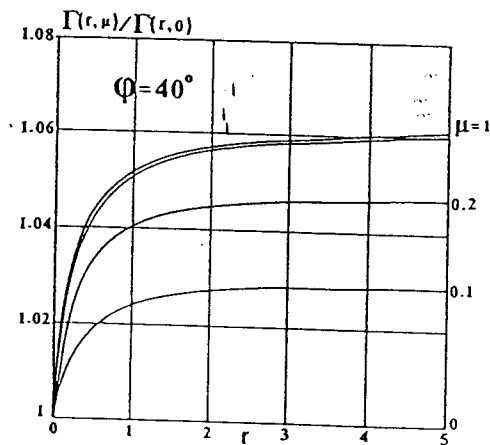
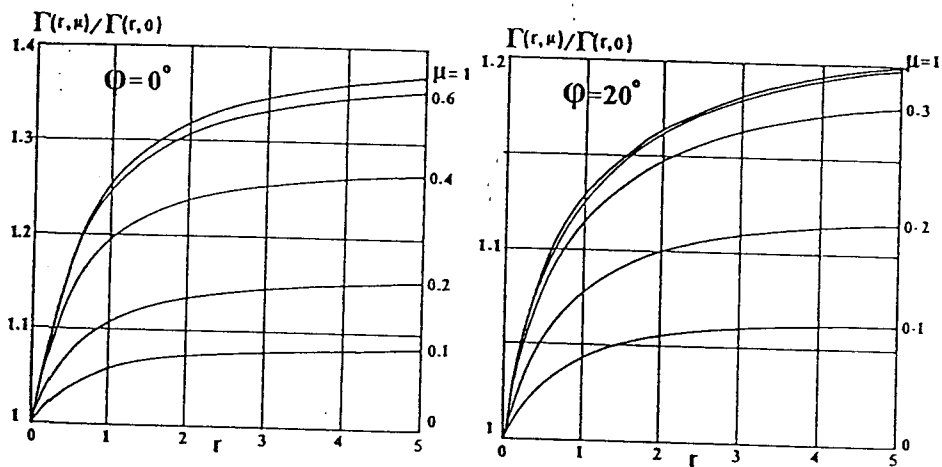


Fig. 6. Charts of  $\Gamma(r, \mu)/\Gamma(r, \mu=0)$ .



accounts for the percentage of increase in stability which can be expected when adopting the reduced shear strength  $\mu V$ , instead of completely neglecting it in the simplified analysis usually performed. They lead to the following conclusions.

- At first, they confirm what could be deduced from the theory of yield design (and from common sense !), namely that, for the same kind of failure mechanisms, ignoring the shear and bending strengths of the reinforcements always results in a conservative estimate of the stability of the structure.

- The relative improvement of the factor of confidence due to the shear strength capacities increases with  $r$  and  $\mu$ . The increase of the factor of confidence undergoes a significant decline with the friction angle  $\phi$  of the soil. As an example, for  $r=3$  and  $\mu=0.1$ , it may reach 10% for a purely cohesive soil ( $\phi=0^\circ$ ), whereas it reduces to less than 6% for  $\phi=20^\circ$ , and to less than 3% for  $\phi=40^\circ$ . This last result, though obtained on a particular example, seems of great practical importance since it validates, from the theoretical and computational points of view, the simplified analysis which are performed most often.

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