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Stability analysis of reinforced soil retaining structures using the yield design theory

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Introduction

In the design of earth retaining structures, geotechnical engineers are primarily concerned with the problem of their stability. Sticking to the spirit of Coulomb's approach (ref.1), the yield design theory (refs.2,3) provides a unified comprehensive framework for dealing with such stability analyses. The basic principle of the method consists in checking the compatibility of the equilibrium requirement of a structure under prescribed loading conditions with the material strength characteristics. Unlike most currently used classical methods (such as the well-known "method of slices" for slope stability analyses), the implementation of the yield design approach does not need the introduction of complementary assumptions and results therefore in clearly interpretable estimates for the stability of soil structures.

Starting from the presentation of the method on the case of a homogeneous soil structure, it is shown that the method is fully applicable to structures made up of several soil layers exhibiting different weight and strength characteristics. Furthermore, any kind of reinforcement of the structure (nails, tie-backs, geotextiles etc.) can be easily taken into account in the analysis through the so-called "mixed modelling" of the composite reinforced soil. All these recent developments of the initial theory have been put in the concrete form of a computational program, whose efficiency is illustrated on the design of a practical example.

Principle of the yield design reasoning

Consider for illustrative purpose the elementary example of a vertical cut of height h, subjected to its own weight γ. The constitutive soil is assumed to be homogeneous and to obey a Mohr-Coulomb strength condition with cohesion C and friction angle φ (Fig. 1).

According to the yield design reasoning, this structure will be termed "safe" or "stable" if a system of internal forces (i.e. a stress field) can be exhibited which complies with the equilibrium equations under the load

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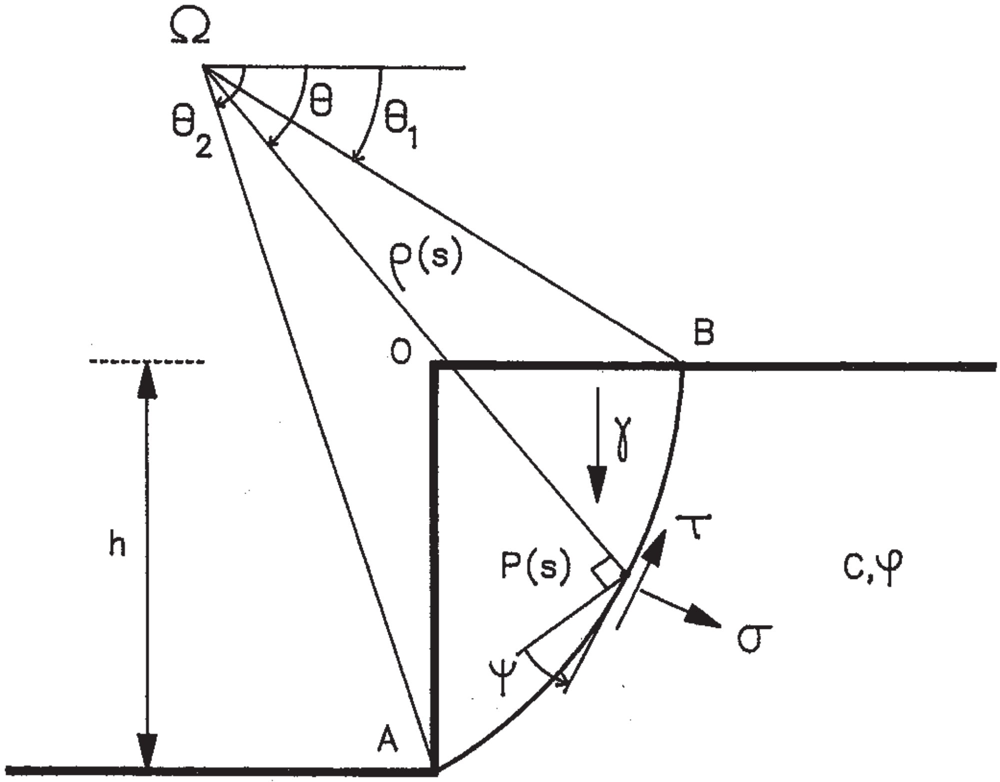

Fig.1 Stability analysis of a vertical cut

parameter γ and with the strength condition of the soil, throughout the whole bulk of the soil. It turns out that the stability of such a structure is governed by the dimensionless parameter $\gamma h/C$ with the following equivalence.

$$\text{stability of the structure} \leftrightarrow \gamma h/C \leq (\gamma h/C)^* \tag{1}$$

where $(\gamma h/C)^*$ denotes the critical value of the parameter $\gamma h/C$ beyond which the failure of the structure will occur.

It is convenient to define the factor of confidence of the structure as

$$\Gamma^* = (\gamma h/C)^* / (\gamma h/C)$$

so that the above statement may be put in the form

$$\text{stability of the structure} \leftrightarrow \Gamma^* \geq 1 \tag{2}$$

The problem being dealt with from a two dimensional standpoint ("plane strain" problem), let OAB be any volume of soil separated from the rest of the structure by an arbitrary line AB. It clearly appears that for the structure to be safe, in the sense stated above, it is necessary that the moment of the weight of OAB with respect to any point Ω (clockwise "driving" moment M^{Ω}_{w}) be balanced by the anticlockwise "resisting" moment developed about the same point by the distribution of normal and shear stress (σ, τ) acting upon OAB along AB, provided the strength condition of the soil

$$|\tau| \leq C - \sigma \tan\varphi \tag{3}$$

be satisfied everywhere (tensile normal stresses are reckoned positive).

In other terms, introducing the concept of maximum resisting moment M^{Ω}_{r} defined as

$$M^{\Omega}_{r} = \int_{A}^{B} \mathcal{M}^{\Omega}_{r}(s)\,ds \tag{4}$$

where at every point P(s) of AB

$$\mathcal{M}^{\Omega}_{r}(s) = \rho(s) \sup_{(\sigma,\tau)} \{\sigma \sin\psi + \tau \cos\psi;\ (\sigma,\tau) \text{ satisfying } (3)\} \tag{5}$$

with ψ denoting the angle between the direction normal to the radius ΩP of length ρ, and the tangent to AB at point P (Fig. 1), it comes out

$$\text{stability of the structure} \rightarrow M^{\Omega}_{w} \leq M^{\Omega}_{r}$$

and comparing with (2)

$$\Gamma(\Omega, AB) = M^{\Omega}_{r} / M^{\Omega}_{w} \geq \Gamma^{*} \tag{6}$$

The important conclusion which can be drawn from the previous analysis, is that, given any point Ω and line AB, the ratio between the maximum resisting moment and the driving moment due to the weight of the soil, constitutes an upper bound estimate for the factor of confidence of the structure.

Trying to get the best estimate of this factor would therefore require to minimize $\Gamma(\Omega,AB)$ over all possible points Ω and lines AB. Fortunately, this minimization procedure is greatly simplified by the following result (ref.3).

For a given point Ω, it can be shown that the most critical volume OAB, associated with the minimum estimate $\Gamma(\Omega, AB)$, is such that $\psi=\varphi$. This implies that AB is the arc of logspiral of angle φ and focus Ω passing through the toe A of the cut. Its equation in polar coordinates (ρ,θ) attached to Ω (Fig. 1) writes

$$\rho(\theta) = \rho_1 \exp[(\theta_1 - \theta)\tan\varphi]$$

Consequently, denoting by $\Gamma(\theta_1,\theta_2)$ the corresponding value of the ratio $\Gamma(\Omega,AB)$, where θ_1 and θ_2 are the angular parameters defining the position of Ω, the minimization of $\Gamma(\Omega, AB)$ reduces to searching for the minimum value of $\Gamma(\theta_1,\theta_2)$ with respect to θ_1 and θ_2. This is achieved through a numerical procedure, which in the present case yields .

$$\Gamma = \min_{\theta_1,\theta_2} \Gamma(\theta_1,\theta_2) \cong 3.83\ C/(\gamma h)\ \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \tag{7}$$

This is to be related to a classical result (refs. 2,4). The above analysis may be directly extended to the case when the structure is submitted to any kind

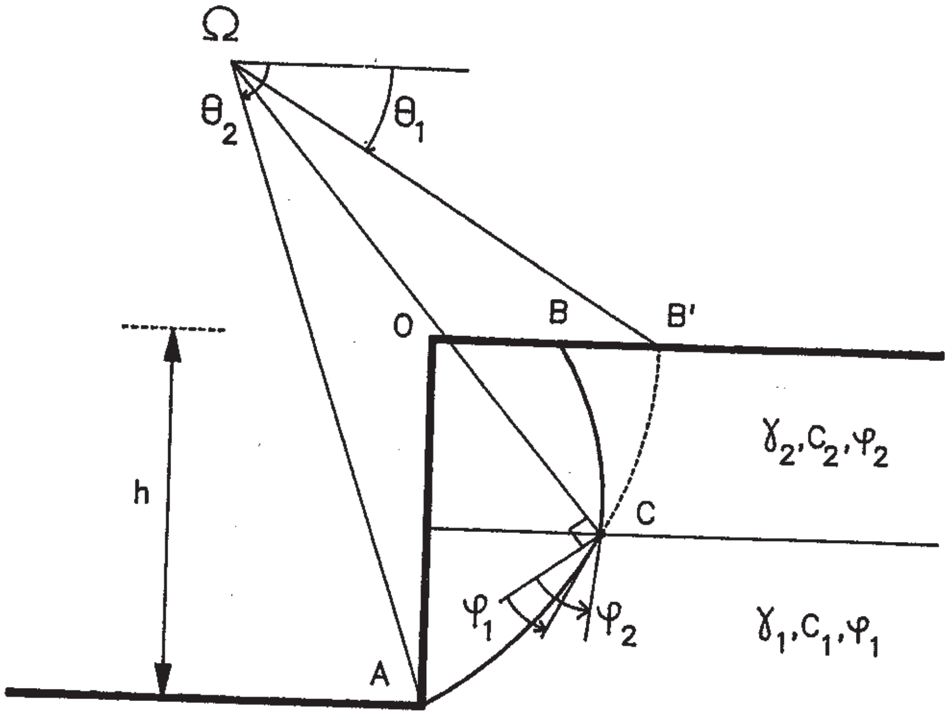

Fig. 2 . Stability analysis of a two-layered vertical cut

of external load applied on its boundary surface. For a given point Ω and line AB , the definition of Γ (Ω, AB) becomes

$$\Gamma(\Omega,AB) = M_r^{\Omega} / (M_w^{\Omega}+M_L^{\Omega}) \tag{8}$$

where M^{Ω}_L represents the moment about Ω of the external loads acting upon OAB. It should be noticed that, depending on the sign of M^{Ω}_L relative to that of M^{Ω}_w, such external forces may have favourable or unfavourable effects on the stability of the volume under consideration, and hence of the structure as a whole.

Likewise, the method remains fully relevant for dealing with structures where several, layers of soils having different characteristics must be taken into account. Assuming for instance the vertical cut to be made of two such layers with a horizontal interface, one may analyze its stability by checking the moment equilibrium of volumes such as sketched in Fig. 2.

AB′ is an arc of logspiral with angle φ_1 (friction angle of the lower layer of soil) and focus Ω defined through angles θ_1 and θ_2, the curve bounding the volume of soil whose equilibrium is examined is constructed by connecting the arc of logspiral AC in the lower part of the structure, with CB which is drawn as the arc of logspiral of angle φ_2 having the same focus Ω as ACB′. It induces therefore a discontinuity of the tangent to the line ACB at point C where it intersects the interface between the two layers. The estimate of the factor of confidence derived from the global equilibrium check of volume

OACB is defined in the same way as in Eq. (6), taking into account different expressions of $\mathcal{M}^{\Omega}{}_{r}(s)$ along AC and CB. The best upperbound estimate of Γ^* will once again be derived from a minimization with respect to the angular parameters θ_1 and θ_2.

Application to the design of reinforced retaining structure

Various reinforcement techniques are increasingly used today in order to enhance the stability of otherwise unstable retaining structures. Adopting a "mixed modelling" of the so obtained reinforced soil (refs.5,6), that is treating the reinforcing inclusions as strips or beams embedded in the soil modelled as a classical 3-D continuum, makes it possible to apply the same reasoning as that used for natural soil structures.

Fig. 3 outlines such a retaining structure in which several arrays of reinforcements have been placed at each stage of the excavation ("soil nailing" technique).

Resuming the analysis carried out on the corresponding homogeneous structure, the moment equilibrium of volume OAB with respect to a point Ω is considered. While the driving moment $M^{\Omega}{}_{w}$ generated by the weight of the soil remains unchanged (provided the weight of the inclusions might be neglected), the maximum resisting moment $M^{\Omega}{}_{r}$ appears as the sum of two terms

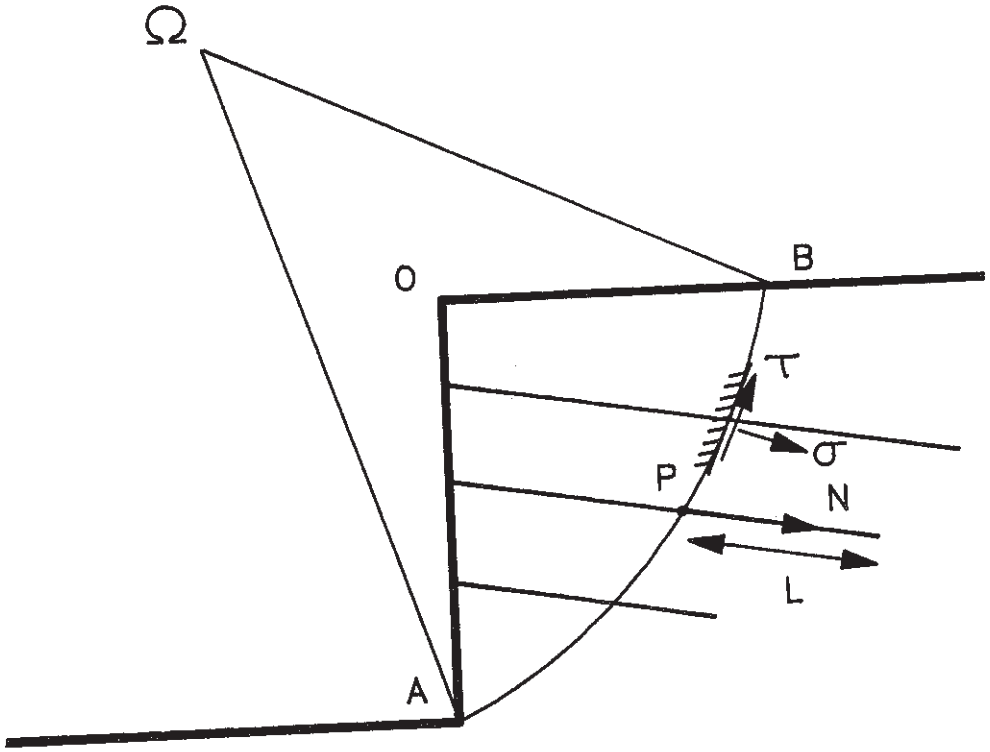

Fig.3. Stability analysis of a reinforced retaining structure

- the contribution due to the distribution of stresses (σ,τ) in the soil along AB, whose expression is given by Eqs. (4) and (5)
- the new term corresponding to the resisting forces developed by the inclusions at their intersection with AB, in account of their strength capacities which are specified as follows.

Failure of the reinforcements may occur according to two modes

- through the breaking of the inclusions themselves
- or by lack of adherence between the soil and the reinforcements due to their limited anchoring length L beyond the failure line AB (Fig. 3).

Either failure mode can be taken into account through a specific condition, namely

- $0 \leq N \leq N_{max}$ which means that the reinforcements can withstand tensile forces up to a maximum value N_{max} corresponding to the yielding of the constitutive material;
- $N \leq f_l L$, where f_l is a lateral friction coefficient characterizing the bonding between the soil and the reinforcement and L the anchoring length.

This deserves two comments

- the adopted strength criterion involves only the axial force in the inclusions, and disregards their resistance to bending and shear forces. Actually, it can be easily shown that such a simplifying assumption is always conversative) as regards the evaluation of the factor of confidence
- the resisting tensile force reckoned for each inclusion is equal to the minimum value between the ultimate load carrying capacity N_{max} and the "pull out" resistance $f_l L$.

$$0 \leq N \leq N_o = \inf\,\{N_{max}, f_\ell L\}\ . \tag{9}$$

The definition of the ratio $\Gamma(\Omega, AB)$ between the maximum resisting moment and the driving moment relative to a volume OAB and a point Ω is

$$\Gamma(\Omega,AB) = \frac{M_r^\Omega(soil) + M_r^\Omega(incl)}{M_w^\Omega + M_L^\Omega} \tag{10}$$

where $M^\Omega{}_r$ (incl) is the maximum resisting moment generated by the inclusions, taking their strength condition (9) into account. The two following situations will be encountered when computing the latter term, depending on the sign of the angle δ between the inclusion, oriented outwards with respect to volume OAB, and vector ΩP (Fig. 4).

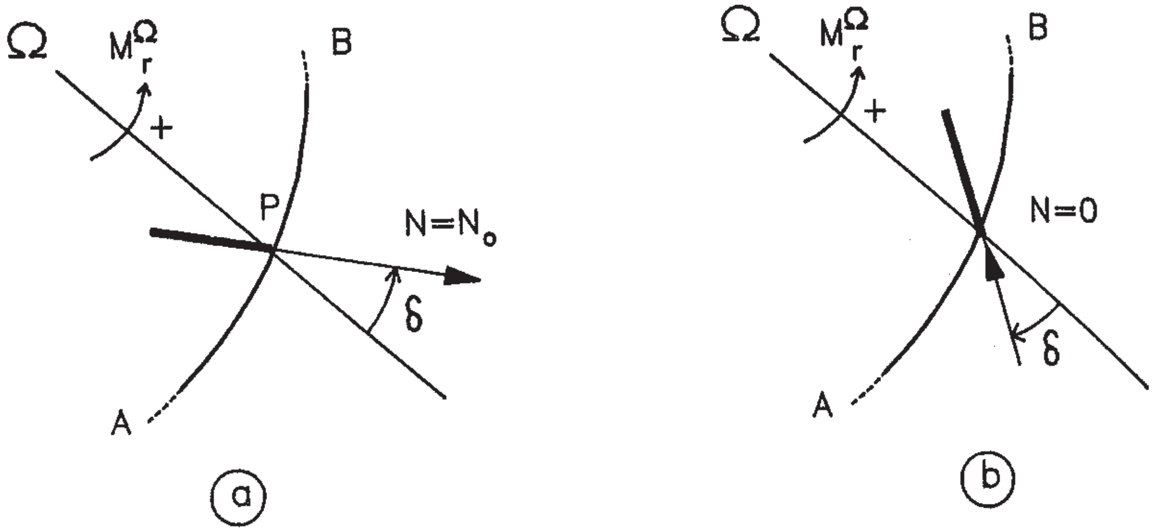

Fig. 4 : Maximum resisting moment generated by a reinforcement

If $\delta>0$ (Fig. 4a) the maximum value of the resisting moment about Ω is obtained for $N=N_0$, that is

$$M_r^\Omega(\mathrm{incl})=\Omega P\ N_o \sin\ \delta \qquad (11)$$

Conversely, if $\delta<0$ (Fig. 4b), the resisting moment is positive when the inclusion undergoes compressive forces, and therefore the maximum resisting moment reduces to zero, since the compressive strength of the reinforcements is neglected (it is commonly acknowledged that failure by buckling is likely to occur in such a situation).

It follows that in both cases $M^\Omega{}_r$ (incl) remains non-negative and therefore results in an increase for the evaluation of the factor of confidence given by Eq. (10), when compared with that corresponding to the non-reinforced case.

Practical implementation

Based upon the above-described theory, a computational program named STARS (for Stability Analysis of Reinforced Soils) (refs.6,7) has been developed for analyzing the stability of reinforced soil retaining structures. In its most recent version, this program can handle a large variety of reinforced structures, involving several soil layers, all kinds of surface loadings (concentrated or distributed) and taking possible seismic conditions into account through a regular pseudo-static analysis. The high speed of numerical computations (even when operating with a personal computer) combined with a great conviviality allows any user of such a program to optimize the reinforcement pattern of a structure within a very short period of time, as it will be illustrated on the following realistic example.

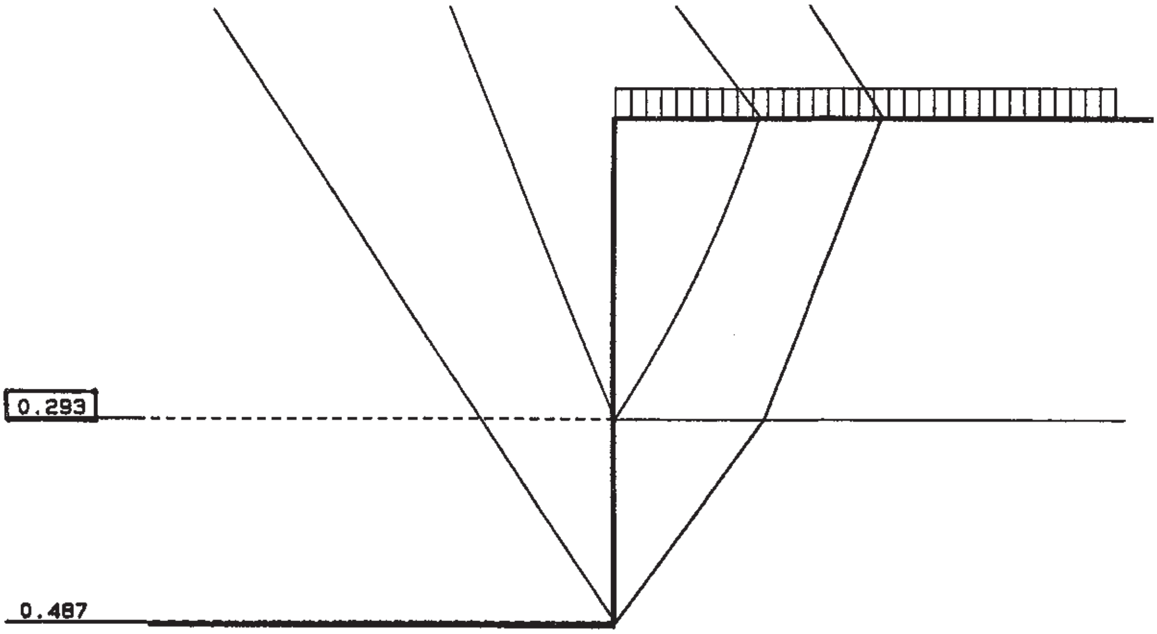

Fig. 5: Stability analysis of the unreinforced structure

The structure under consideration is a 10m high vertical cut subjected to a uniformly distributed load of 10kN/m^2 applied on its upper level (Fig. 5). It is constructed in its upper part (i.e. down to 6 m deep from the upper corner) by a soil with the characteristics γ=18kN/m^3, C = 5kPa and φ=35^0, while those of the lower layer are γ=20kN/m^3, C=30kPa and γ=20^o. As shown by a first

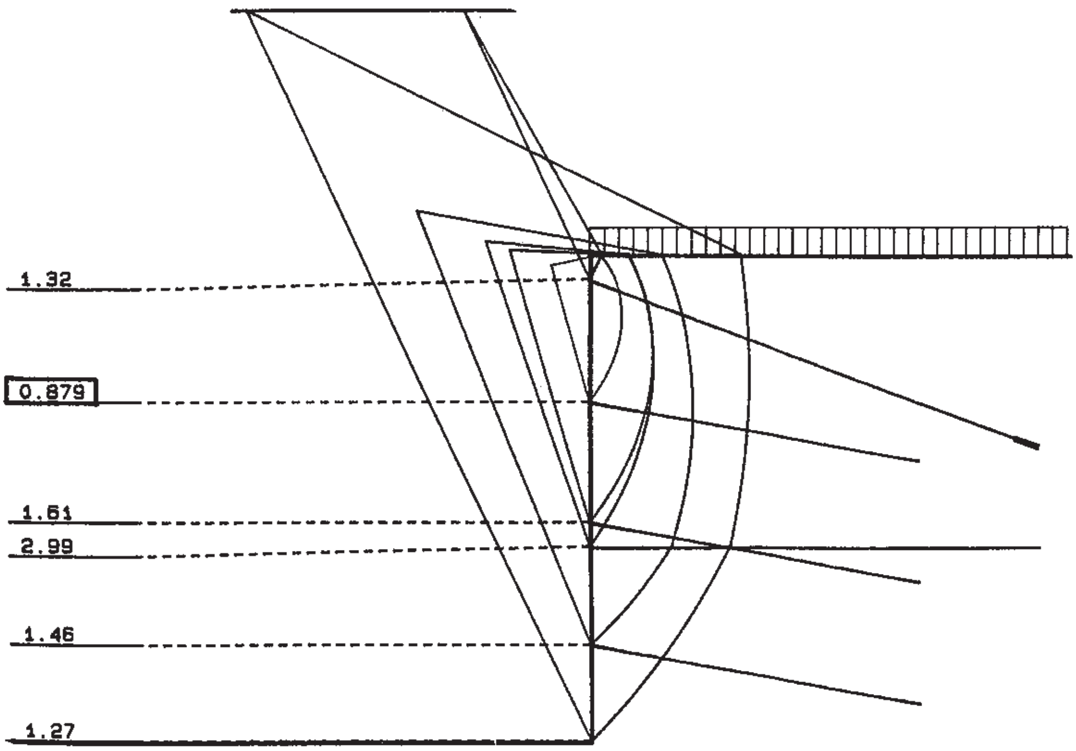

Fig 6: First scheme of reinforcement

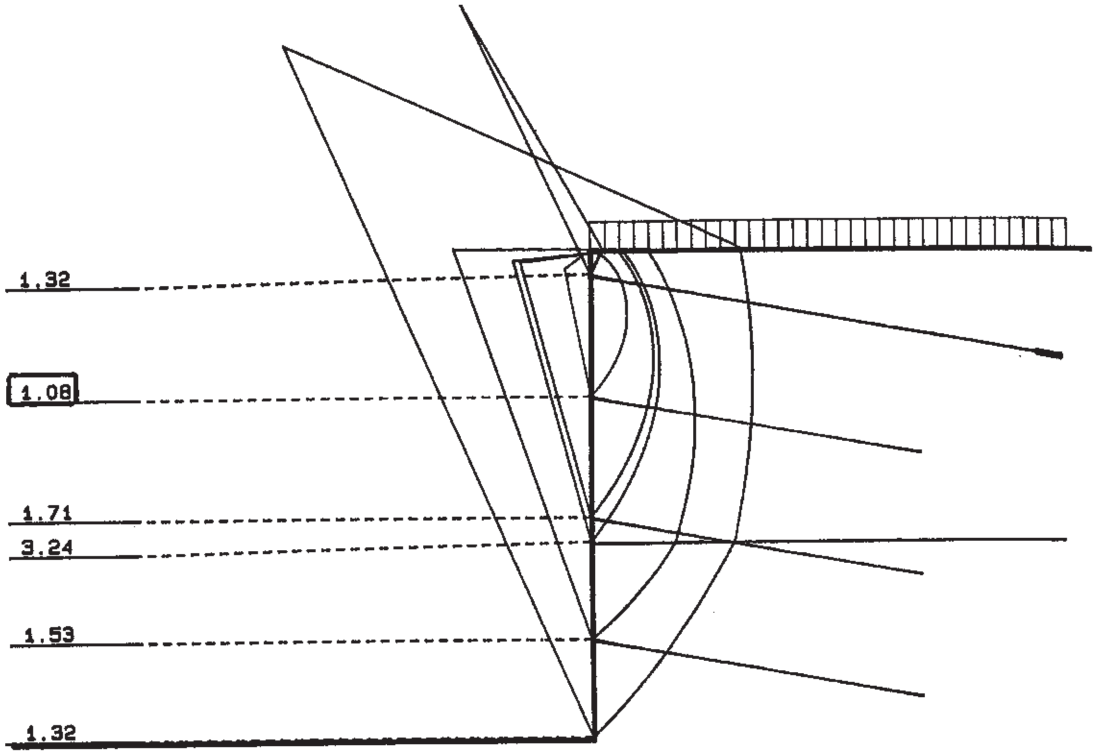

Fig. 7: Modified scheme of reinforcement

calculation, this structure is obviously unsafe, since the best upperbound estimates of its factor of confidence derived from the "global equilibrium check" of volumes bounded by all possible lines made of arcs of spiral passing through either the toe or the emerging point on the interface between layers, remain far below unity (0.49 and 0.29 respectively).

In order to ensure the stability of this excavation, the following reinforcement scheme is proposed (Fig. 6).

Three rows of inclusions (nails) inclined at 10^{o} to the horizontal are placed at respective depths of 3 m, 5.5 m and 8 m, with a regular horizontal spacing of 1 m. The strength parameters of each inclusion as defined earlier, are

N_{max} =100kN; f_l=40kN/m

One layer of tie-backs inclined at 20^{o} and emerging on the facing at 0.5 m below the upper level is fitted, their tension being set equal to 150kN with a spacing of 1.5 m.

The results of the computations are shown in Fig. 6 which, for each family of lines passing either through the toe or emerging above each layer of reinforcement, displays the most critical one with the associated estimate of the factor of confidence. It can be seen that, contrary to what might have been expected, the lowest upperbound estimate (0 . 88) is not given by a line passing through the toe of the excavation, but by a line coming out on the facing at a depth of 3 m. This result emphasizes the fact that, restricting the exploration to those failure lines passing exclusively through the toe could lead to considerably overestimating the stability of the structure (1.27 instead of 0.88 in the present case).

A quite simple modification of the reinforcement scheme may consist in decreasing the inclination of the tie-backs from 20^{o} to 10^{o}. In that case, the calculations presented in Fig. 7 give evidence of a noticeable increase of the estimated factor of confidence from 0.88 to 1.08.

Conclusion

Because it relies upon a clear assessment of the problem along with consistent mechanical arguments for solving it, the yield design approach results in a reliable and versatile method for analyzing the stability of earth retaining structures and especially reinforced ones. Furthermore, it makes it possible to derive quite efficient numerical codes for engineers, as the one presented in this paper. By way of illustration, it should be mentioned that the computations whose outputs are reported in Figs 6 or 7 involve the exploration of nearly 4000 failure lines, which is carried out within a few minutes on any personal computer. One can easily imagine the radical changes that might arise in the design process of structures, when using such numerical tools.

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