COMMUNICATION

P. de Buhan and J. Salençon**

Yield Strength of Reinforced Soils as Anisotropic Media

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ABSTRACT A new comprehensive approach to the overall strength of reinforced soils as anisotropic media is outlined within the framework of the yield design homogenization method. Starting from the general definition of the macroscopic strength condition for a periodically reinforced soil, the particular case of a purely cohesive soil reinforced with strips is more thoroughly studied. The corresponding criterion, which proves to be of the cohesive anisotropic type, is explicitly formulated and then applied to the exact calculation of the bearing capacity of a footing on a reinforced half-space.

Introduction

Most classical engineering methods devised for analysing the stability of reinforced earthworks come up against difficulties taking the composite nature of the constitutive soil into account. Indeed, the direct implementation of these methods in order to evaluate the failure loads for this kind of structure usually requires assumptions to be made about the way the reinforcing inclusions behave when interacting with the surrounding native soil. In any case much more complex computational methods than those commonly used for homogeneous structures are needed to deal with the strong heterogeneity of the soil.

The purpose of the following contribution is to present a yield design homogenization method aimed at building up an alternative simplified approach to solve such a problem. This method proceeds from the intuitive idea that, from a macroscopic point of view – that is, as far as overall properties of the reinforced soil structure such as its collapse load are concerned – the reinforced soil is likely to be perceived as a homogeneous medium. It seems obvious that, on account of the existence of preferential orientations due to the introduction of the reinforcements, one may expect such a homogenized material to exhibit anisotropic strength characteristics.

Macroscopic strength criterion of a reinforced soil: a theoretical approach

If we assume that the reinforcing inclusions are distributed throughout the mass of the soil following a regular pattern (as is frequently the case in practice), the reinforced soil may be modelled as a two phase composite periodic medium. It can be shown in that case **(14)(4)(8)** that the determination

* Laboratoire de Mécanique des Solides, Ecole Polytechnique, 91128 Palaiseau Cedex, France.

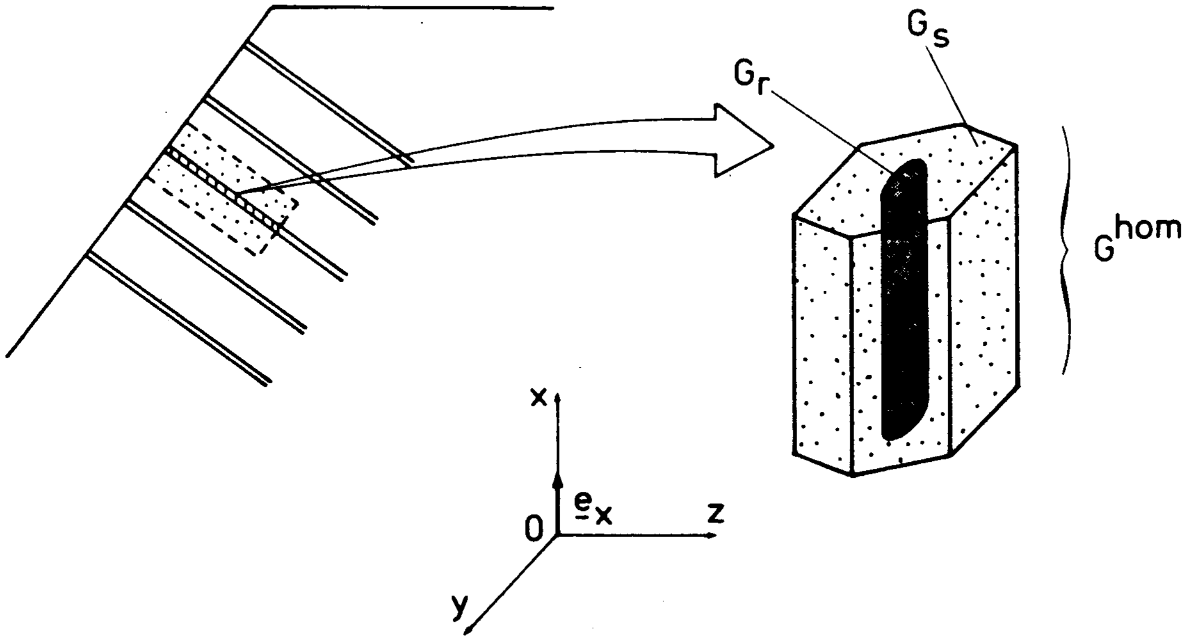


Fig 1 Representative cell for a periodically reinforced soil

of the macroscopic strength criterion of the reinforced soil reduces to the solution of a yield design problem defined over the representative cell which describes the internal structure of the composite soil (Fig. 1).

Denoting by G_s (respectively G_r) the strength domain of the soil (respectively, of the reinforcing material), that is, the convex set of the allowable stress tensors, and assuming perfect bonding between the soil and the inclusion, the macroscopic strength domain G^{hom} is defined as the set of the 'macroscopic' stresses $\boldsymbol{\Sigma}$ such that one could display all over the representative cell a stress field $\boldsymbol{\sigma}$ satisfying the following requirements:

$\boldsymbol{\Sigma} = \langle \boldsymbol{\sigma} \rangle$, where $\langle \cdot \rangle$ stands for volume average over the cell;

$\boldsymbol{\sigma}$ is periodic;

$\operatorname{div} \boldsymbol{\sigma} = 0$ and $[\![\boldsymbol{\sigma}]\!] \cdot \mathbf{n} = 0$ in the case of stress discontinuities;

$\boldsymbol{\sigma}(\mathrm{x})$ belongs to the domain G_s or G_r depending on whether point $\mathbf{x}$ is located in the soil or in the reinforcing inclusion.

According to the general definition, it appears that a lower bound approximation of G^{hom} will be obtained through restricting $\boldsymbol{\sigma}$ to piecewise constant stress field, that is

$$\boldsymbol{\sigma}(\mathrm{x}) = \begin{cases} \boldsymbol{\sigma}^s & \text{in the soil} \\ \boldsymbol{\sigma}^r & \text{in the inclusion} \end{cases}$$

Consequently, any macroscopic stress $\boldsymbol{\Sigma}$ which can be written in the form

$$\boldsymbol{\Sigma} = (1-\eta)\boldsymbol{\sigma}^s + \eta\boldsymbol{\sigma}^r \tag{1}$$

(η is the voluminal fraction of the reinforcing material), with

$$\boldsymbol{\sigma}^s \in G_s, \quad \boldsymbol{\sigma}^r \in G_r \tag{2}$$

and, referring to an $0xyz$ co-ordinate system where the $0x$ axis is taken parallel to the direction of the inclusion

$$\sigma^s_{ij} = \sigma^r_{ij} \quad \text{for every } (i,j) \neq (x,x) \tag{3}$$

will satisfy the macroscopic strength criterion of the reinforced soil. The conditions (3) are necessary to ensure the continuity of the stress vector acting upon the interface between the soil and the inclusion, since this interface is a cylindrical surface parallel to the $0x$ axis.

From now on, the analysis will be focused on the case when the cross sectional area of the inclusion is becoming small in comparison with that of the representative cell, so that $\eta \ll 1$, while the strength characteristics of the constitutive reinforcing material are considerably greater than those of the surrounding soil. Mathematically, this particular configuration which corresponds, for instance, to soil reinforcement by metallic strips, can be obtained by making the parameter η tend to zero, while the strength domain G_r increases according to

$$G_r = G_o/\eta$$

where G_o is a fixed domain in the stress space.

Performing the same reasoning as in the case of a two-dimensional multilayered reinforced soil (**3**), it can be easily shown that the lower bound estimate of G^{hom} defined by equations (1), (2), and (3) reduces to

$$\left\{\begin{array}{l}\boldsymbol{\Sigma} = \boldsymbol{\sigma}^s + \sigma\, \mathbf{e}_x \otimes \mathbf{e}_x \\ \qquad \boldsymbol{\sigma}^s \in G_s, \quad \sigma_o^- \leq \sigma \leq \sigma_o^+\end{array}\right\} \subset G^{hom} \tag{4}$$

where the parameter

$$\sigma_o^+ \text{ (respectively, } \sigma_o^-) = \text{Max (respectively, Min)}\{\sigma; \sigma\, \mathbf{e}_x \otimes \mathbf{e}_x \in G_o\}$$

represents the tensile (respectively, compressive) strength of the reinforcing strips per unit transverse area.

As a matter of fact, it can be proved, by making use of the yield design kinematic approach, that this lower bound estimate of the macroscopic strength domain G^{hom} amounts to the exact solution when η becomes small. So that finally the macroscopic strength criterion of a unidirectionally strip-reinforced soil, which may be conveniently expressed by means of a convex yield function f^{hom}, can be written as follows

$$f^{hom}(\boldsymbol{\Sigma}) \leq 0 \Leftrightarrow \boldsymbol{\Sigma} \in G^{hom}$$

where f^{hom} is defined from f_s, the yield function of the soil alone, by

$$f^{hom}(\boldsymbol{\Sigma}) = \underset{\sigma_o^- \leq \sigma \leq \sigma_o^+}{\text{Min}} \; f_s(\boldsymbol{\Sigma} - \sigma\, \mathbf{e}_x \otimes \mathbf{e}_x) \tag{5}$$

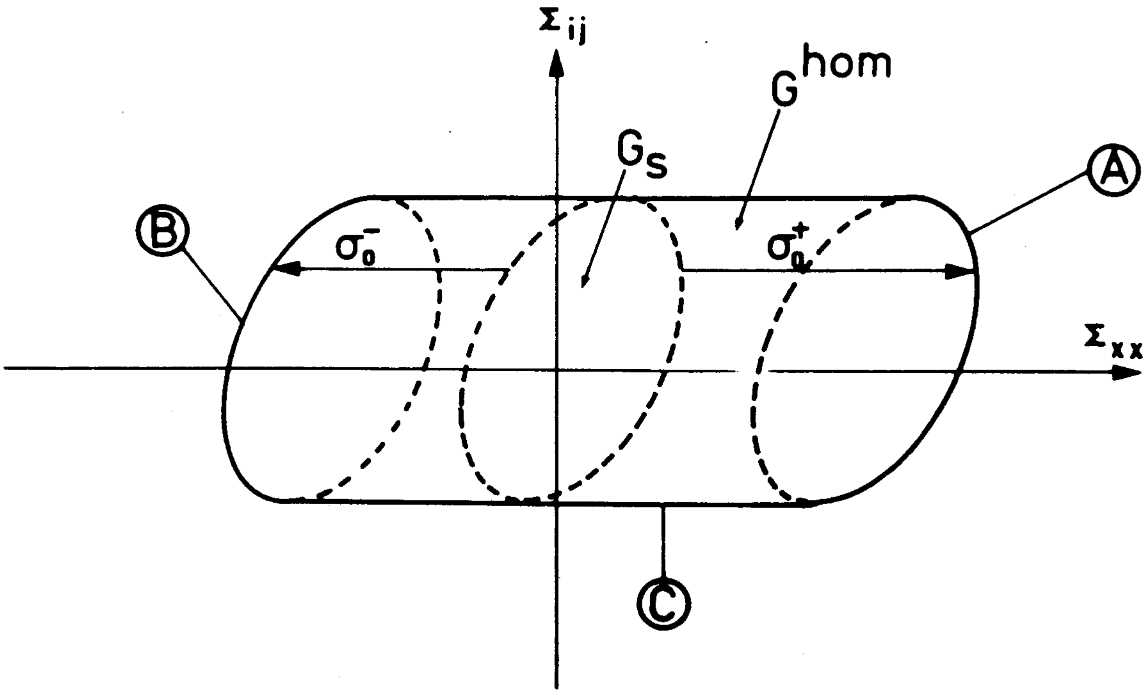


Fig 2 Geometrical representation of the macroscopic strength domain G^{hom}

This result deserves some comments.

(1) The overall increase in strength of the soil due to the incorporation of the reinforcements is clearly evidenced from equation (5), since, taking $\sigma = 0$, we get

$$f^{hom}(\mathbf{\Sigma}) \leq f_s(\mathbf{\Sigma})$$

and then

$$G_s \subset G^{hom}$$

(2) The yield design homogenization method which has been chosen here as a theoretical framework, leads to the same formulation of the macroscopic strength criterion as those previously obtained in **(7)** and more recently in **(12)** and **(13)** through heuristic considerations. The investigation of these authors was mainly based on the assumption that the reinforcing strips (or fibres) are just behaving as uniaxial carrying elements inside the soil (or the matrix).

In the six-dimensional space of stresses $\mathbf{\Sigma}$, the parametric equation

$$f_s(\mathbf{\Sigma} - \sigma\, \mathbf{e}_x \otimes \mathbf{e}_x) = 0, \quad \text{with } \sigma_o^- \leq \sigma \leq \sigma_o^+ \tag{6}$$

is represented by the boundary surface of G_s moved along the Σ_{xx}-axis by the distance σ. Therefore the macroscopic yield locus, i.e., the boundary surface delimiting G^{hom}, can be easily constructed as the convex envelope of the family of surfaces defined by equation (6) (Fig. 2).

Three possible failure modes of the reinforced soil can be seen from this geometrical representation, the boundary surface of G^{hom} being divided into three different regions, denoted by A, B, and C as depicted in Fig. 2.

Region A corresponds to the set of macroscopic solicitations $\mathbf{\Sigma}$ for which both constituents come up to failure, the strips undergoing their maximum tensile load $\sigma = \sigma_o^+$. Its equation writes $f_s(\mathbf{\Sigma} - \sigma_o^+ \mathbf{e}_x \otimes \mathbf{e}_x) = 0$ together with the condition $\partial f_s/\partial \Sigma_{xx} \geqslant 0$.

The second failure mode occurs when the strips reach their ultimate compressive strength: $\sigma = \sigma_o^-$. It is represented by region B whose equation is

$$f_s(\mathbf{\Sigma} - \sigma_o^- \mathbf{e}_x \otimes \mathbf{e}_x) = 0, \quad \text{with } \partial f_s/\partial \Sigma_{xx} \leqslant 0$$

The last failure mode (region C, which is a cylindrical surface parallel to the Σ_{xx}-axis) is obtained for the macroscopic stresses, $\mathbf{\Sigma}$, verifying

$$f_s(\mathbf{\Sigma} - \sigma \mathbf{e}_x \otimes \mathbf{e}_x) = 0$$

with

$$\sigma_o^- \leqslant \sigma \leqslant \sigma_o^+$$

and

$$\partial f_s/\partial \Sigma_{xx} = 0$$

It corresponds to the yielding of the soil alone, while the strips remain within failure.

Moreover, since failure by buckling of the strips is likely to occur for a compression $-\sigma$ significantly lower than the compressive strength of the strips derived from that of the constitutive material, it seems advisable to adopt $k\sigma_o^-$ as a lower limit for σ, where k is a scalar factor lying between $k = 0$ (compressive strength equal to zero) and $k = 1$.

Strip reinforced soils as artificially orthotropic media

From now on the soil will be supposed to obey an isotropic strength criterion. In that case the corresponding macroscopic strength criterion appears to be transversely isotropic around the direction $0x$ of the reinforcement. Indeed, it follows from definition (5) that the macroscopic yield function may be written in the form of an isotropic function of the arguments $\mathbf{\Sigma}$ and $\mathbf{e}_x \otimes \mathbf{e}_x$, which, referring to a classical result, clearly accounts for the macroscopic transverse isotropy of the reinforced soil.

As an illustrative example of this property, it may be worth deriving the macroscopic uniaxial tensile strength Σ^+ of the reinforced soil defined from

$$f^{hom}(\Sigma^+ \mathbf{e}_\theta \otimes \mathbf{e}_\theta) = 0$$

($\mathbf{e}_\theta$ is a unit vector inclined at an angle θ with respect to the $0x$ axis; Fig. 3) in the particular case when the soil obeys a Tresca yield condition

$$f_s(\boldsymbol{\sigma}^s) = \sigma_1^s - \sigma_2^s - 2C_s \leqslant 0$$

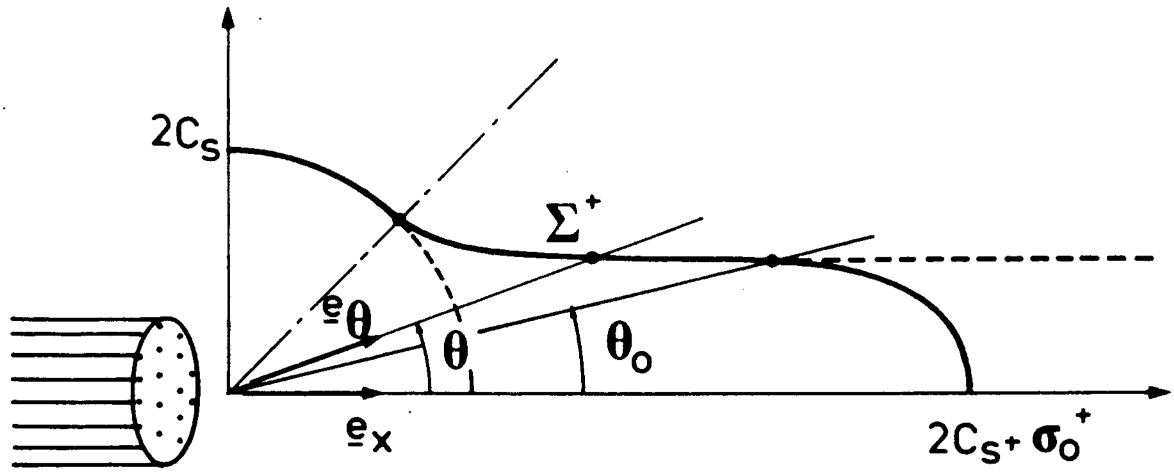


Fig 3 Yield strength anisotropy in simple tension for a purely cohesive reinforced soil

($\sigma_1^s \geq \sigma_2^s$ = maximum and minimum principal stresses; C_s = cohesion of the soil).

The following analytical expressions are obtained

$$\Sigma^+(\theta) = \begin{cases} \sigma_o^+ \cos 2\theta + \sqrt{\{4C_s^2 - (\sigma_o^+ \sin 2\theta)^2\}} & 0 \leq \theta \leq \theta_o \\ 2C_s/\sin 2\theta & \theta_o \leq \theta \leq \pi/4 \\ 2C_s & \pi/4 \leq \theta \leq \pi/2 \end{cases} \tag{7}$$

where θ_o is defined by

$$\cot 2\theta_o = \sigma_o^+/2C_s$$

together with the obvious relations

$$\Sigma^+(\theta) = \Sigma^+(-\theta) = \Sigma^+(\pi - \theta)$$

Figure 3 displays a typical curve in the form of a polar diagram giving the variation of Σ^+ as a function of the orientation θ. One may notice first that, for a given value of the dimensionless ratio $\sigma_o^+/2C_s$ which characterizes the degree of reinforcement, Σ^+ decreases from $(\sigma_o^+ + 2C_s)$ when $\theta = 0$ to $2C_s$ when $\theta \geq \pi/4$, i.e., the tensile strength of the reinforced soil reduces to that of the original soil. This last result is mainly due to the particular form of the Tresca strength condition which has been adopted for the soil. As a matter of fact it can be easily seen that for $\theta \geq \pi/4$ one can always exhibit in the three-dimensional space a failure plane parallel to the reinforcing strips and making an angle of $\pi/4$ with the direction of the tensile solicitation. Of course, the conclusion would have been different in the case of a von Mises criterion (7).

Conversely, if the orientation θ is kept fixed, Σ^+ increases with the degree of reinforcement $\sigma_o^+/2C_s$ until it reaches its maximum value $2C_s/\sin 2\theta$ and then remains constant for $\sigma_o^+/2C_s \geq \cot 2\theta$ unless $\theta = 0$. In the limit case of 'infinitely strong' reinforcement (i.e., $\sigma_o^+/2C_s \rightarrow +\infty$), the macroscopic tensile strength simply gives

$$\Sigma^{+}(\theta)=\begin{cases}2C_s/\sin 2\theta & \text{if} \quad 0\leqslant\theta\leqslant\pi/4\\ 2C_s & \text{otherwise}\end{cases}$$

(Dashed line in Fig. 3.)

Similar conclusions can be drawn for the uniaxial compressive strength, replacing σ_0^+ by σ_0^- in equation (7).

A two dimensional anisotropic strength criterion

As we intend to apply the yield design homogenization procedure to reinforced soil structures subjected to 'plane strain' loading conditions in a parallel direction to the reinforcing inclusions (namely in the $0xy$ plane), more particular attention will be paid by now to the investigation of the corresponding two-dimensional macroscopic strength criterion.

Description of the strength criterion

Starting from the definition of the fully three-dimensional macroscopic strength condition, it can be easily shown that the associated 'plane strain' yield function writes in just the same form as (5) with, for a purely cohesive soil

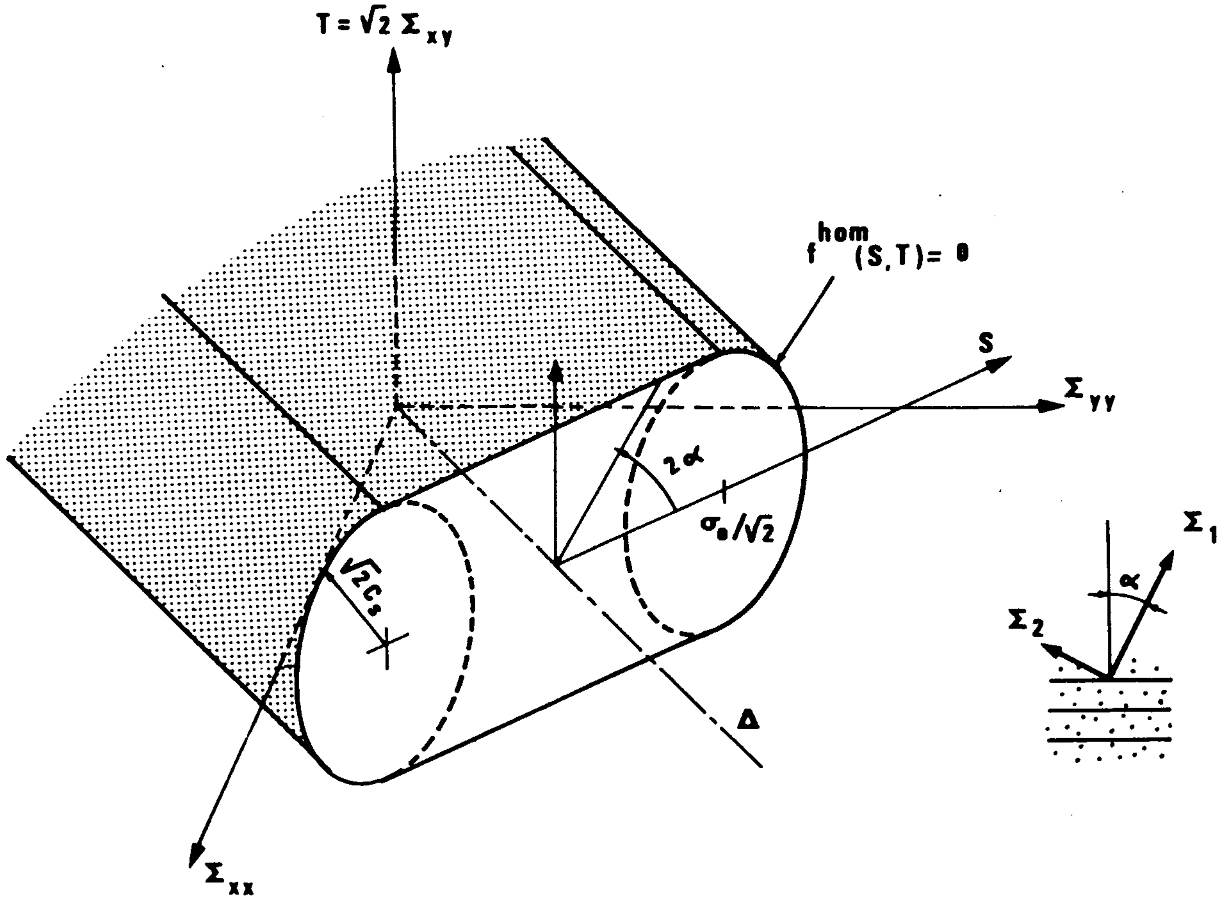


Fig 4 Representation of the strength domain G^{hom} in the space of stresses (Σ_{xx}, Σ_{yy}, $\sqrt{2}\Sigma_{xy}$)

$$f_s(\boldsymbol{\Sigma} - \sigma\, \mathbf{e}_x \otimes \mathbf{e}_x) = (\Sigma_{yy} - \Sigma_{xx} + \sigma)^2/2 + 2\Sigma_{xy}^2 - 2C_s^2 \tag{8}$$

where Σ_{xx}, Σ_{yy}, and Σ_{xy} are the three independent components of the two-dimensional stress in the (x, y) plane which will be denoted by $\boldsymbol{\Sigma}$ from now on.

This expression shows that the macroscopic strength criterion does not involve the hydrostatic component of the stress $\boldsymbol{\Sigma}$ (i.e., $\operatorname{tr}\boldsymbol{\Sigma} = \Sigma_{xx} + \Sigma_{yy}$), so that the boundary of G^{hom} is a cylindrical surface parallel to the straight line Δ which represents the isotropic states of stress ($S = T = 0$) (Fig. 4).

Assuming, for simplicity's sake, that $\sigma_o^+ = -\sigma_o^- = \sigma_o$, the computation of the minimum of function f_s with respect to the parameter σ as indicated by equation (5), leads to the following analytical expressions for the yield function f^{hom}

$$f^{hom} = T^2 - 2C_s^2 + \begin{cases} 0 & \text{if} \quad |S| \leq \sigma_o/\sqrt{2} \\ (S - \sigma_o/\sqrt{2})^2 & \text{if} \quad S \geq \sigma_o/\sqrt{2} \\ (S + \sigma_o/\sqrt{2})^2 & \text{if} \quad S \leq -\sigma_o/\sqrt{2} \end{cases} \tag{9}$$

where

$$S = (\Sigma_{yy} - \Sigma_{xx})/\sqrt{2}$$

and

$$T = \sqrt{2}\Sigma_{xy}$$

The cross-section of the boundary of G^{hom} by any deviatoric plane orthogonal to the line Δ may be constructed as the convex envelope of two circles with a radius equal to $\sqrt{2}C_s$ and centred, respectively, at points ($S = \pm\sigma_o/\sqrt{2}$, $T = 0$).

That strength criterion can be conveniently expressed in terms of the principal stresses $\Sigma_1 \geq \Sigma_2$ of $\boldsymbol{\Sigma}$ in the $0xy$ plane together with the inclination α of major principal stress Σ_1 to the $0y$ axis (Fig. 4).

Indeed, substituting the classical expressions

$$S = (\Sigma_1 - \Sigma_2)\cos 2\alpha/\sqrt{2}$$
$$T = (\Sigma_1 - \Sigma_2)\sin 2\alpha/\sqrt{2}$$

into the different expressions of function f^{hom} given by equation (9), we get after some calculations

$$f^{hom}(\boldsymbol{\Sigma}) \leq 0 \Leftrightarrow \Sigma_1 - \Sigma_2 \leq 2C^{hom}(\alpha) \tag{10}$$

with, for $0 \leq \alpha \leq \pi/4$

$$C^{hom}(\alpha) = \begin{cases} \dfrac{\sigma_o}{2}\cos 2\alpha + \sqrt{\left(C_s^2 - \dfrac{\sigma_o^2}{4}\sin^2 2\alpha\right)} & \text{if} \quad \tan 2\alpha \leq \dfrac{2C_s}{\sigma_o} \\ C_s/\sin 2\alpha \quad \text{otherwise} \end{cases} \tag{11}$$

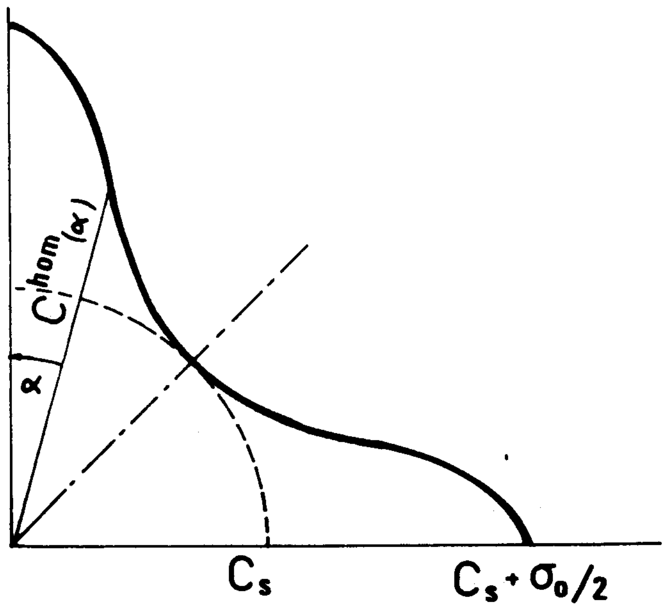


Fig 5 Polar diagram for the anisotropic cohesion $C^{hom}(\alpha)$ of the reinforced soil

and

$$C^{hom}(\alpha) = C^{hom}(-\alpha) = C^{hom}(\pi/2 - \alpha)$$

Figure 5 shows the variations of the 'anisotropic cohesion' C^{hom} as a function of the angle α in the form of a polar diagram. This cohesion proves to be maximum for $\alpha = 0$ or $\pi/2$, that is when the principal stresses are coinciding with the privileged directions of orthotropy for the homogenized reinforced soil. It decreases to a minimum value equal to the cohesion C_s of the unreinforced soil itself when $\alpha = \pi/4$. As can be seen in Fig. 4, it is worth pointing out that, referring to polar coordinates (ρ, θ) in a deviatoric plane, the equation of the macroscopic yield locus reduces to

$$\rho(\theta) = C^{hom}(\theta/2)$$

Comparison with other proposed criteria

The preceding formulation of the macroscopic strength condition makes the comparison with analogous anisotropic strength criteria previously proposed by several authors possible, provided they can be expressed in terms of an anisotropic cohesion. Indeed, in that case, the corresponding yield curves may be plotted in the (S, T) deviatoric plane, as shown in Fig. 6, where three different strength criteria have thus been compared with the macroscopic criterion. In each case, the parameters which define the proposed criterion have been adjusted in such a way that all the curves intersect the axes at the same points.

Curve No. 1 refers to the analytical formula suggested by Bishop (1) and

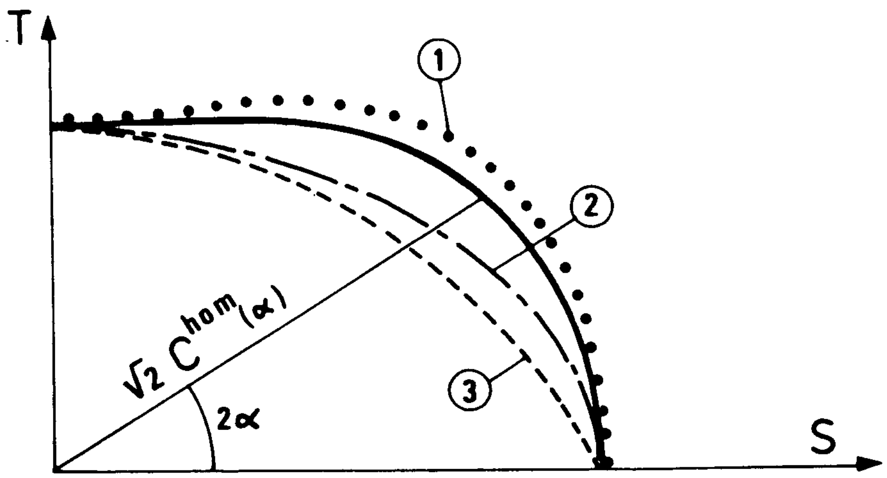


Fig 6 **Comparison with other strength criteria proposed for anisotropic materials**

Salençon (**11**) in order to describe the natural anisotropy of clayey soils. This formula reduces here to

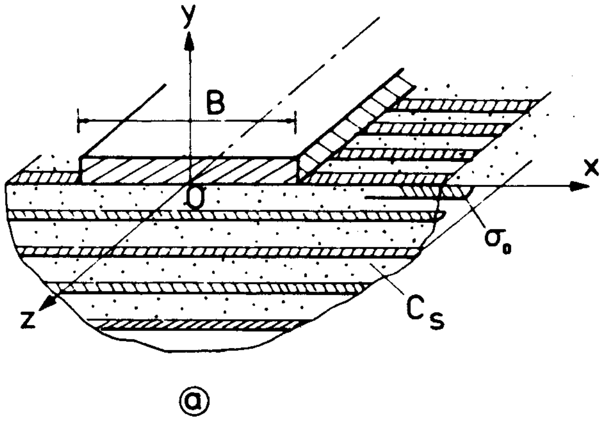
$$C_1(\alpha) = C_s + (\sigma_o/2)\cos^2 2\alpha$$

Curve No. 2, which proves to be elliptic, is derived from the criterion proposed by Boehler and Sawczuk (**2**) as a generalization of the 'plane strain' Tresca or von Mises criterion based on the concept of anisotropy tensor.

Curve No. 3, which is an arc of a circle centred on the T axis, has been drawn according to the 'maximum shear stress' criterion formulated by Nova and Sacchi (**9**) for orthotropic media.

Bearing capacity of a footing lying on a reinforced soil

As an application of the theoretical approach presented here, this last section is devoted to the problem of the bearing capacity of a strip footing (width B) lying on a purely cohesive soil (cohesion C_s) reinforced by uniformly distributed reinforcements perpendicular to the axis of the footing (Fig. 7(a)). Provided that the characteristic length of the reinforcement (e.g., the horizontal or vertical spacing between the strips) is small enough when compared with the overall size of the structure (the width of the footing), the yield design homogenization theory states that such a problem, which is *a priori* fully three dimensional, since the strength properties of the constitutive soil vary with z, reduces to the solution of a two dimensional homogenized problem defined in the $0xy$ plane (Fig. 7(b)). Dimensional analysis applied to the yield design solution of that latter problem, when the strength criterion of the constitutive material is the one investigated in the preceding section, shows that the extreme value of the loading parameter Q (axial force per unit length along $0z$) denoted by Q^* gives the form



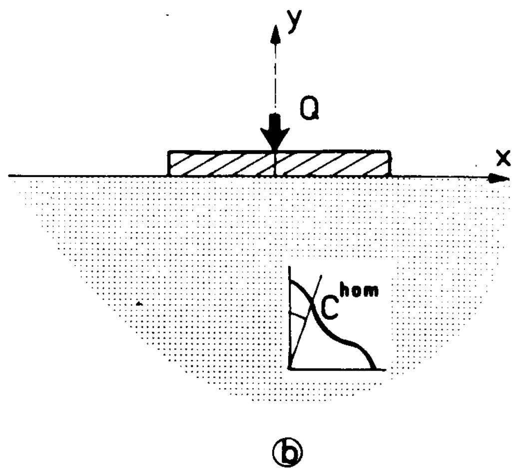


Fig 7 (a) Bearing capacity of a footing on a strip reinforced half-space
(b) Two dimensional homogenized problem

$$Q^* = BC_sN^*(\sigma_0/C_s) \tag{12}$$

where N^* is a dimensionless factor depending on the ratio σ_0/C_s.

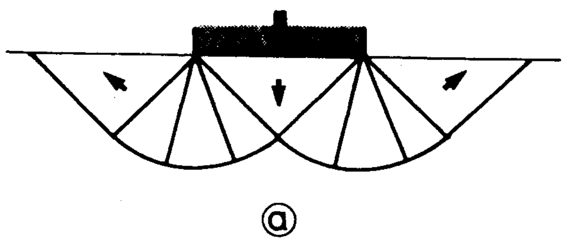
The exact determination of N^* is then carried out by performing both yield design approaches (**10**). Leaving aside all intermediate computational developments which may be found in (**4**), the obtained result will be presented below.

Using the velocity field associated with the classical Prandtl's failure mechanism (Fig. 8(a)) the implementation of the kinematic yield design approach leads to the following upper bound estimate for N^*

$$N^* \leqslant (\pi + 2) + 2\sigma_0/C_s$$

Likewise, superimposing the stress field sketched in Fig. 8(b) upon any allowable stress field (i.e., satisfying the equilibrium equations together with the strength condition at any point) equilibrating the value of N corresponding to a purely cohesive unreinforced soil ($N^* = \pi + 2$), the following lower bound estimate is derived

$$N^* \geqslant (\pi + 2) + 2\sigma_0/C_s$$



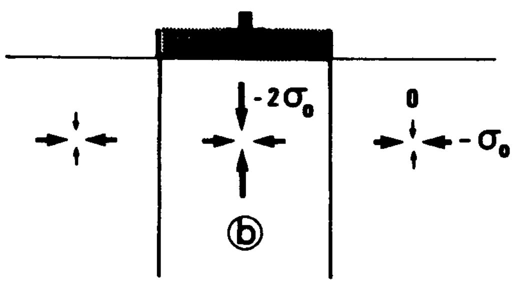


Fig 8 (a) Prandtl's kinematic field
(b) Stress field

from which one gets the very simple formula

$$N^* = (\pi + 2) + 2\sigma_o/C_s$$

Furthermore, it can be easily established that when the compressive strength of the strips is taken equal to $-k\sigma_o$, the exact value of N^* becomes

$$N^* = (\pi + 2) + (1 + k)\sigma_o/C_s$$

The increase of the bearing capacity induced by the reinforcement of the soil is thus shown to be directly proportional to the ratio σ_o/C_s. It may be worth pointing out that the proportionality factor is significantly lower than what would have been obtained from crude approximation of the reinforced soil as a purely cohesive isotropic medium with cohesion $(\sigma_o/2 + C_s)$

$(1 + k)$ compared with $(1 + \pi/2)$

Concluding remarks

The homogenization method has been shown an appropriate theoretical framework for the collapse design of reinforced soil structures, since it makes possible to account for the macroscopic strength anisotropy of the constitutive composite soil explicitly. Besides, this method proves to be fully applicable when failure by lack of adhesion between the soil and the reinforced inclusions has to be considered (**4**).

Concerning the specific, though important, case of strip reinforced soils, a similar investigation has been performed for a cohesionless soil obeying a Coulomb failure condition, thus leading to a fairly adequate model for 'reinforced earth' (**5**). The theoretical results obtained through this model have proved to be in good agreement with available experimental data.

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