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Piecewise Linear Yield Criteria in Invariant Form[a]

Discussion by Jean Salençon[2]

In his very interesting paper, Haythornthwaite is looking for expressing piecewise linear yield criteria by means of stress invariants. He ocasionaly refers to a text book (7) in which I briefly dealt with stress-invariant form for Tresca's *yield function*.

I stated there that there is no closed polynomial expression equivalent to Tresca's yield function; this one, which compares the maximum shear-stress induced by a stress tensor $\underline{\underline{\sigma}}$ to the yield value τ_0, is expressed by:

$$f(\underline{\underline{\sigma}}) = \max\{\sigma_i - \sigma_j - 2\tau_0 \mid i,j = 1,2,2\} \qquad (22)$$

I added that, for instance, the expression

$$\phi(J_2, J_3) = 4J_2^3 - 27J_3^2 - 36\tau_0^2 J_2^2 + 96\tau^4 J_2 - 64\tau_0^2 \qquad (23)$$

which is the first member of Eq. 19, cannot be taken as an equivalent to the yield function given by Eq. 22. This point may need to be clarified.

As a matter of fact, as pointed out by Prager (6), Eq. 17, $\phi(J_2, J_3) = 0$, is but the result of

$$[(\sigma_1 - \sigma_2)^2 - 4\tau_0^2][(\sigma_2 - \sigma_3)^2 - 4\tau_0^2][(\sigma_3 - \sigma_1)^2 - 4\tau_0^2] = 0 \qquad (24)$$

Therefore Eq. 24 is, indeed, a consequence of

$$f(\underline{\underline{\sigma}}) = 0 \qquad (25)$$

when f is defined by Eq. 22, but it is not of course and unfortunately equivalent to Eq. 25.

This may also be seen when trying to represent the elastic domain of a material ruled by Tresca's criterion; taking $f(\underline{\underline{\sigma}})$ given by Eq. 22 as the yield function, the inequation

$$f(\underline{\underline{\sigma}}) < 0 \qquad (26)$$

will describe the interior of Tresca's well-known hexagonal prism and only that one, whereas taking $\phi(J_2, J_3)$ given by Eq. 23, as the yield function, the inequation

$$\phi(J_2, J_3) < 0 \qquad (27)$$

will lead to that very domain plus six extra regions (Fig. 5). This shows that constraints must be added to Eq. 27 in order to describe the elastic

[a]August, 1983, by Robert M. Haythornthwaite (Paper 18154).

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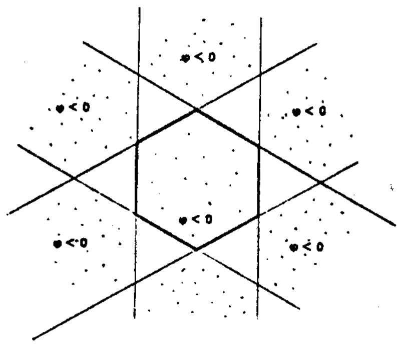

FIG. 5.—$\phi(J_2, J_3) < 0$ Regions in Deviatoric Plane

domain and illustrates the fact that it is impossible to describe a closed curve with vertices by means of a closed polynomial form.

Closure by Robert M. Haythornthwaite,[3] F. ASCE

I concur fully with Salençon's assertion that Eq. 17 in particular and polynomials in general may not represent a complete statement of the yield criterion: a supplementary inequality may also be needed. The writer is confident Prager and Hodge were aware of this when Eq. 17 was first published (6). As is apparent from Salençon's Eq. 24, the polynomial form is the union of three pairs of parallel lines and a means must be found to isolate the central hexagon in Salençon's Fig. 5. This is done by selecting the root for which

$$J_2 \leq \frac{4}{3}\tau_0^2 \qquad (28)$$

In a formal sense, Eq. 28 is a necessary part of the yield criterion statement. An alternative suggested in the paper is to employ a numerical procedure to select the smallest positive root.

With the exception of Eqs. 16, 17, and 21, which were obtained by squaring other expressions, the yield criteria presented in the paper are single valued, so that the admissible zones may be defined by replacing the equality signs by $\leq$. Thus they are exceptions to the rule that cubic and higher order order polynomial forms are in general multi-valued. They exemplify one significant advantage that accrues from the introduction of the invariant $|\tau_1\ \tau_2\ \tau_3|$.

Errata.—The following correction should be made to the original paper:

Page 1020, Eq. 16: The last term in the equation should read $-64\ \tau^6$

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