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YIELD-STRENGTH OF ANISOTROPIC SOILS

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Natural soils often exhibit a significant mechanical anisotropy. Taking it into account in structure stability analyses requires defining the procedure and the interpretation of experimental tests properly, describing the strength-characteristics of the soil through a mechanically correct criterion, and devising adequate stability analysis methods. Soil mechanical reinforcement has been developped and is now commonly used : the composite material so obtained at the structure level can be modelized as a homogeneous continuum with an anisotropic yield-strength, allowing anisotropic stability analysis methods to be used.

1 - THE ORIGIN OF A CONCEPT

Since he started to improve his environment, man has come up against the problem of the stability of the structures he did build on the ground, sometimes with the very soil of that ground or with borrow material (buildings, bridges, dikes, dams, ...). He has also been concerned with the safety of the natural relief (e.g. slopes, embankments, ...) and with that of mines, pits and quarries he did develop in it. Soil mechanics can therefore be considered as a rather old discipline, but its historical appearance is usually connected with Coulomb's celebrated memoir [1], notwithstanding , as pointed out by Chandler [2], Morton's early contribution to the study of slope stability which does not refer to mechanics.

Coulomb's memoir is devoted both to structures and to soil mechanics : it deals with the stability of pillars and vaults, and with the problem of earth pressure on a retaining wall. As a matter of fact, the range of problems which were to be the main concern of soil mechanics for more than 150 years was defined there : active and passive earth-pressure calculations, stability analyses of slopes, bearing capacity calculations, through yield-design methods. Though we do not intend to produce an exhaustive list, let us recall some famous names in that domain of soil mechanics after Coulomb : Massau, Rankine, Kötter, Caquot, Sokolovski, Mandel.

That approach is based upon some fundamental concepts :
• soil is treated as a *continuous medium*, in spite of its being both particulate as clays, or granular as sands, and polyphasic (particles, water, air) ;
• this medium is defined through a *strength-criterion* which corresponds to failure or yielding when reached ;
• the analysis is performed by investigating if the equilibrium of the studied structure is possible under the applied loads (dead and active) with the limitation of the strength-criterion.

Yield-strength appears then as the first concept introduced by engineers to modelize "soil" as a material through a constitutive law, though incomplete since it is only a limitation imposed on the internal force intensity derived from the risk of failure. The introduction of that concept is certainly to be related to the very patterns of the failure mechanisms. As shown in the highly documented papers by Desrues [4] and Habib [5], one frequently observes that, when failure occurs, the deformation of the material seems to be confined in very thin soil layers (figure 1) which can be treated as surfaces. This localization may be noticed in the failure mechanisms of structures (e.g. landslides or breakings of earth fills), or when performing classical laboratory tests.It has induced engineers to apply intuitive reasoning based on the concept of mobilized strength on a failure plane, which unfortunately reveal some deficiencies from the mechanical point of view.

Anyhow it must be understood that having retained yield-strength as the working concept for stability analyses for more than 150 years shows its good adequation to practical applications.

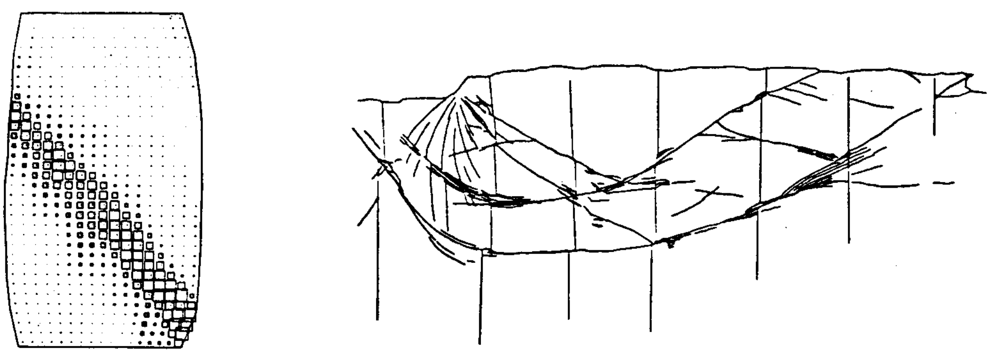

Figure 1 : Localization of deformation at failure : distorsion map for a plane strain compression test (from [4]), plane strain punch indentation test (from[5]).

2 - STABILITY ANALYSIS METHODS

As apparent in the title of the memoir, Coulomb's original study is carried out within the frame of Statics.

Taking as an example the compression of a cylindrical block between the plates of a press assumed to be perfectly smooth (a more precise dëfinition of these boundary conditions is not useful for what follows), the following necessary condition is stated :

Let the specimen be separated in two parts 1 and 2 by an arbitrary plane (P) ; so that failure of the specimen do not occur it is necessary that, whatever (P), the global equilibrium of block 1 or 2 (from the Statics of rigid bodies point of view) be possible under the action of the vertical force Q imposed by the plate, and of the resisting forces developped by the material along the section by plane (P).

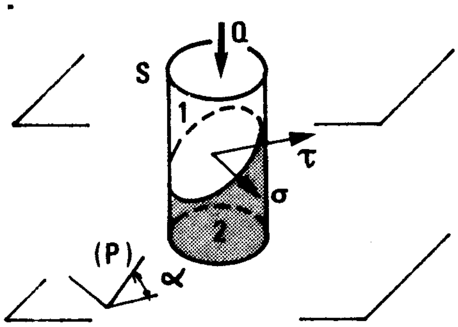

Figure 2 : Compression test of a cylindrical specimen.

The yield-strength of the material being characterized by a cohesion C and a friction angle φ, the Coulomb strength condition indicates that, whatever (P) and whatever the point in (P), the normal stress σ and the shear stress τ must verify the inequality :

(2.1) $$|\tau| \leqslant C - \sigma \tan\varphi$$

(tensile stresses are counted positive). It then follows that, whatever $\alpha > \varphi$, the equilibrium of block 1 is compatible with (2.1) only if :

(2.2) $$Q \leqslant SC\,(1 + \tan^2\alpha) \,/\, (\tan\alpha - \tan\varphi)\ ,$$

a condition on Q whose minimum is obtained when $\alpha = (\pi/4 + \varphi/2)$ and writes :

(2.3) $$Q \leqslant 2SC \tan(\pi/4 + \varphi/2).$$

So it turns out that an upper bound for the loads Q which can be applied to the specimen without causing its failure, is obtained through this reasoning and is given by (2.3).

This example deserves some comments.

• The performed reasoning does not completely make use of the main idea included in the necessary stability condition, namely the compatibility between the studied system equilibrium and the material yield-strength. In fact this condition could be checked within the frame of continuous mechanics instead of restricting the study to a partition of the specimen into two blocks and checking global equilibrium. Therefore it will be necessary to adopt continuous mechanics formulations and to express the Coulomb strength-condition as a criterion on the stress tensor $\underline{\underline{\sigma}}(\underline{x})$ at current point $\underline{x}$. σ_i being the principal values of $\underline{\underline{\sigma}}(\underline{x})$ Coulomb's criterion is derived from (2.1)[1]:

(2.4) $$f(\underline{\underline{\sigma}}(\underline{x})) = \sup_{i,j=1,2,3} \left\{\sigma_i(1 + \sin\varphi) - \sigma_j(1 - \sin\varphi) - 2C\cos\varphi\right\} \leqslant 0$$

which is equivalent to (2.1) expressed at point $\underline{x}$ for any orientation of plane (P).

• Bringing together the schema of figure 2 used for the static reasoning, and the often observed failure mechanisms where deformation seems to be localized and rigid blocks to slip with respect to one another (figure 1) suggests the introduction of an intuitive kinematic reasoning : taking the partition of figure 2 again, it is assumed that failure of the specimen will occur anytime the work of the external forces Q exceeds the work of the resisting forces mobilized on (P). Stated that way, unless the material be purely cohesive ($\varphi = 0$), the method cannot be accepted for it is not mechanically consistent with the original static approach. Nevertheless the idea of a kinematic method must be retained, but it needs being built up by dualizing the static approach through the principle of virtual work.

This points out the fact that the significance of the *yield-strength* of the material must be clearly defined, especially by indicating the stability analysis method it is associated with. Moreover it must be recalled that the yield-strength of a soil is a mechanical characteristic which is to be determined from experimental tests ; therefore the interpretation of those tests must be made in accordance with the method to be used later for the analysis of structures. Asking for a sound mechanical reasoning may seem nothing but banal, yet practical examples, dealing with isotropic or anisotropic soils as well, have shown that this demand has often not been satisfied. Let us add for a fairness' sake that experimental results do not, as a rule, enjoy the same purity as that given by theoreticians to their analyses !

From the mechanical point of view, when setting up and using the concept of yield strength, soil mechanicians can rely on two general theories :

• the theory of representations of isotropic tensorial functions, for what concerns the strength criterion of the soil (cf. many documented studies by Boehler such as [6], and also [7]) ;

• the theory of yield design, when devising consistent static and kinematic approaches for stability analyses.

3 - OUTLINES OF THE YIELD-DESIGN THEORY

3.1 - Setting of the problem

We shall restrict ourselves to a presentation of the main results of the theory of yield design ; comments and discussions may be found in [8].

Given V and S respectively the volume and the boundary of the studied mechanical system ;
$\underline{\underline{\sigma}}$ denotes a stress-field and $\underline{\underline{\sigma}}(\underline{x})$ its value at point $\underline{x}$;
$\underline{v}$ is a velocity-field, $\underline{v}(\underline{x})$ its value at point $\underline{x}$;
$\underline{\underline{d}}$ is the associated strain-rate field ; $[\![\underline{v}(\underline{x})]\!]$ denotes the jump of field $\underline{v}$ at point $\underline{x}$ when crossing the discontinuity surface Σ following its normal $\underline{n}(\underline{x})$.

The system is loaded according to a loading process depending linearly on n scalar parameters Q_j, the components of a loading-vector $\underline{Q}$; $\underline{q}$ is the kinematic vector associated with $\underline{Q}$ when expressing the work of the external forces in a velocity field $\underline{v}$.

The material of the system (which needn't be homogeneous) is only known through its strength characteristics, given at any point $\underline{x}$ of V by a domain $G(\underline{x})$ in $\mathbb{R}^6$ where $\underline{\underline{\sigma}}(\underline{x})$ must stay. As a rule $G(\underline{x})$ contains $\underline{\underline{\sigma}}(\underline{x}) = 0$ and is star-shaped with respect to 0 ; it is often convex. $G(\underline{x})$ is usually given by means of a strength-criterion (such as (2.4)) :

$$f[\underline{x}\ ;\ \underline{\underline{\sigma}}(\underline{x})] \leq 0 \quad (\text{resp.} > 0) \iff \underline{\underline{\sigma}}(\underline{x}) \text{ in (resp. out of) } G(\underline{x}) \tag{3.1}$$

The problem to be solved is to determine whether the system so defined will be "stable" under a given load $\underline{Q}$ of $\mathbb{R}^n$.

3.2 - Static approach "from inside"

As already seen on the example of chapter 2, a necessary stability condition for the system is the

$$\text{compatibility} \begin{cases} \text{equilibrium under } \underline{Q} \\ \text{material strength capacities} \end{cases} \tag{3.2}$$

The following definitions are then adopted :

$$\left.\begin{array}{c}\underline{Q} \text{ is potentially safe} \\ \text{for the system}\end{array}\right\} \iff \left\{\begin{array}{c}\text{system is potentially} \\ \text{stable under } \underline{Q}\end{array}\right.$$
$$\Updownarrow \tag{3.3}$$
$$\exists\ \underline{\underline{\sigma}} \text{ S.A. (statically admissible) with } \underline{Q} \\ \text{such that } \underline{\underline{\sigma}}(\underline{x}) \in G(\underline{x}) \quad \forall \underline{x} \in V$$

The set of all potentially admissible loads is denoted by K. As a consequence of the properties of $G(\underline{x})$, it can be shown that : K contains $\underline{Q} = 0$ and is star-shaped with respect to 0 ; K is convex if $G(\underline{x})$ is convex $\forall \underline{x} \in V$. The loads at the boundary of K in $\mathbb{R}^n$ are called the *extreme loads* for the system.

The static approach is just the application of definition (3.3) to construct points in K. It is clear that this construction can be simplified by taking advantage of the star-shape of K, or of its convexity when valid (figure 3). One shall notice that in the case of a unique positive loading parameter Q, as is fairly often encountered, the static approach will lead to a lower bound for the extreme value $Q^\star$ of Q.

3.3 - Kinematic approach "from outside"

The kinematic approach of K from outside is derived by dualizing (3.3) through the principle of virtual work.

Introducing function $\pi(.)$ defining $G(\underline{x})$ by duality (more precisely : G's convex envelope)

$$\pi\ (\underline{x}\,;\underline{\underline{d}}(\underline{x})) = \text{Sup}\ \{\ \underline{\underline{\sigma}}(\underline{x}) : \underline{\underline{d}}(\underline{x})\ |\ \underline{\underline{\sigma}}(\underline{x}) \in G(\underline{x})\ \}\ , \tag{3.4}$$

and function

$$\pi\ (\underline{x},\ \underline{n}(\underline{x})\ ;\ [\![\underline{v}(\underline{x})]\!]) = \text{Sup}\ \{[\![\underline{v}(\underline{x})]\!] .\ \underline{\underline{\sigma}}(\underline{x}) . \underline{n}(\underline{x})\ |\ \underline{\underline{\sigma}}(\underline{x}) \in G(\underline{x})\}\ , \tag{3.5}$$

makes it possible to define functional $P(\underline{v})$ for any velocity field $\underline{v}$

$$P(\underline{v}) = \int_V \pi\ (\underline{x}\ ;\ \underline{\underline{d}}(\underline{x}))dV + \int_\Sigma \pi\ (\underline{x},\ \underline{n}(\underline{x})\ ;\ [\![\underline{v}(\underline{x})]\!])\ d\Sigma\ . \tag{3.6}$$

We then get the following result :

$$\left\{\begin{array}{l} \forall\ \underline{v}\ \text{ kinematically admissible (K.A)} \\ K \subset \{\ \underline{Q}\ |\ \underline{Q} . \underline{\dot{q}}(\underline{v}) - P(\underline{v}) \leqslant 0\ \} \end{array}\right. \tag{3.7}$$

and the approach of K from outside as sketched on figure 3. In the case of a unique positive loading parameter Q, this kinematic approach leads to an upper bound for $Q^\star$.

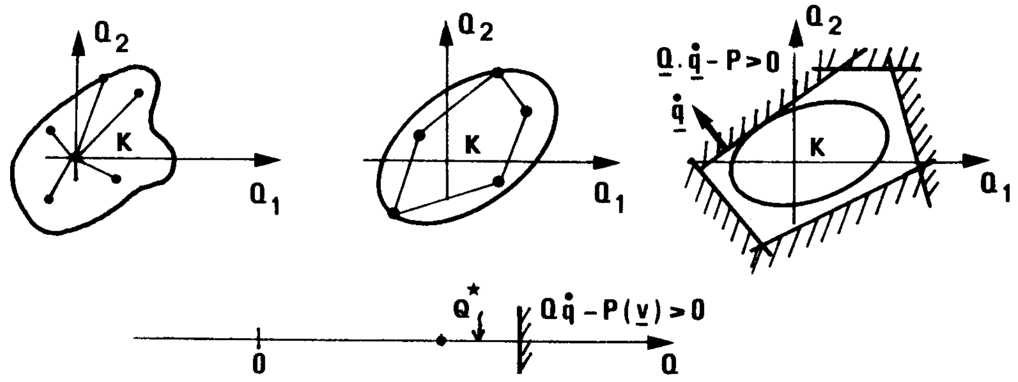

Figure 3 : Approaches from inside and from outside in the theory of yield design.

4 - YIELD-STRENGTH ANISOTROPY OF NATURAL SOILS

4.1 - Introduction

Although most geotechnical materials exhibit anisotropy, be it slight or strong, classical stability analyses have been performed usually under the assumption of isotropic soils. This simplifying hypothesis is justified as long as precise anisotropic yield-strength characteristics cannot be drawn from the identification tests, and as long as the experimental validation of the corresponding stability analyses remains good enough.

From a survey of the available experimental results about the anisotropy of natural soils, Hueckel and Nova [9] state that anisotropy has been observed for rocks (slates, shales, ...), for sands (with elongated grains) and for clays ; yet, as noticed by Ung Sen [10], the anisotropic effects that can be evidenced for the friction angle of sands are most often of the same order of magnitude as the accuracy of the measures. As a matter of fact, except for the works by Boehler et al. dealing with rocks, all the studies of anisotropic yield-strength which have been carried out as far as stability analyses of structures in soil mechanics, are concerned with purely cohesive soils (i.e. with a zero friction angle) ; we shall focus our attention on this kind of materials.

4.2 - Inherent and induced anisotropy

The distinction introduced by Casagrande and Carrillo [11] between inherent and induced anisotropy is now usually referred to. Inherent anisotropy pre-existing in a soil would be evidenced if it were possible to examine that soil without alteration ; induced anisotropy is generated by the testing procedures (sampling and laboratory test conditions ; remoulding for *in-situ* tests). This distinction is worth keeping, for it points out that much care must be taken when realizing the tests and interpreting them ; but it is clear that, at the end, soil mechanicians have to rely on measured characteristics which retain a part of induced anisotropy.

Yield-strength anisotropy of soils originates essentially in the existence of preferential orientations at the fabric level. Those may proceed from the grain or particle shapes (anisotropy is stronger for soils with elongated particles that tend to orientate themselves perpendicularly to the direction of the highest tectonic pressures), from the structure (floculated or loose structure of clays), or from the preferred orientations of heterogeneities (organic matters, change of materials : cf. [12] for the case of diatomite). These considerations, although the scales are different, can be compared with those developped in chap. 6 dealing with reinforced soils where applying the homogenization theory leads to an anisotropic model material (cf. also [13]).

4.3 - Test techniques and experimental results

As far as the principle is concerned, the determination of yield-strength characteristics for an anisotropic soil only requires generating a known and controlled uniform stress fiels in a sample, with the possibility of any orientations of its principal directions with respect to that of the sample.

Classical triaxial compression tests can then be performed on samples drilled out of the ground at various inclinations, but the interpretation of this simple procedure is not so easy, as recalled in [14], due to the non-coincidence of the principal directions of stresses and strains during the experiment as a consequence of anisotropy. Broms et al. [15, 16], Saada et al. [17, 18], have developped hollow cylinder test techniques for anisotropic soils ; for other anisotropic materials Boehler has realized precise experiments evidencing side sway under hydrostatic pressure, that could be worth adapting in soil mechanics.

As has been said before this paper will be concerned with purely cohesive anisotropic soils : for those, in the common range of pressures, the undrained compression triaxial test shows that the applied stresses only appear through their difference $(\sigma_3 - \sigma_1)$ as regards failure. The value of this difference at failure will be plotted as a function of the sample orientation. Due to the already mentioned origin of the anisotropy, such soils are usually transversally isotropic around the axis of the major tectonic pressures which is often vertical. The test results will therefore yield the value of $(\sigma_3 - \sigma_1)$ at failure as a function of α, the inclination of the axis of the sample with respect to the orthotropy axis :

$$(4.1) \qquad \sigma_3 - \sigma_1 = 2\,C\,(\alpha).$$

Most often the deformation of the sample at failure is localized in a plane ; the inclination of that plane may also be measured as a function of α. Duncan and Seed [20] have found this inclination with respect to the sample axis to be practically constant : $\psi = 30°$ (Lo [21] : $\psi = 35°$).

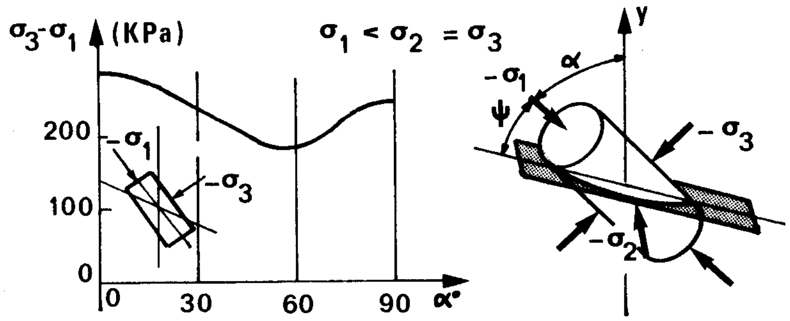

Figure 4 : Triaxial compression test [20].

Figure 5 gives examples of the results obtained through such undrained triaxial compression tests, which are plotted in the form of polar diagrams : the vector radius with a modulus $C(\alpha)$ is inclined at angle α on the vertical direction and indicates the orientation of the sample since the transverse isotropy axis is vertical too.

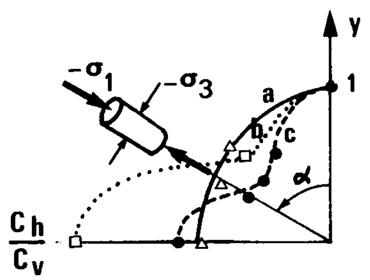

Figure 5 : Polar diagrams : polar representation of $C(\alpha)/C_v$ for the results by Duncan and Seed [22] a, Bishop [23] b, Lo and Milligan [24] c.

Two types of anisotropy can be distinguished from this picture :
Type I (curve a) : normally consolidated soil whose fabric appears homogeneous ; the polar diagram has an "elliptic pattern"
Type II (curves b and c) : those soils demonstrate a minimum cohesion for $\alpha \neq 45°$, which is significantly lower than C_h and C_v corresponding to the vertical and horizontal orientations ; the fabric is oriented, due to high overconsolidation of the soil or to thin layers of stratified materials.

Practically, laboratory tests have produced anisotropy ratios within the ranges

$$0.6 \leqslant C_h/C_v \leqslant 1.6 \quad \text{and} \quad 0.4 \leqslant C_{45}/C_v \leqslant 1.2$$

(actually, in some cases of soils with complex fabrics, one may come across values of $C_{45}/C_v > 1$).

The first formula for describing $C(\alpha)$ was proposed by Casagrande and Carrillo [11] (as it seems, without any experimental support) :

$$C(\alpha) = C_v \cos^2\alpha + C_h \sin^2\alpha \quad ; \tag{4.2}$$

it may be seen that it only accounts for type I anisotropy. From his own experimental results on London clay, Bishop [23] suggested a formula with one parameter more :

$$C(\alpha) = C_v(\cos^2\alpha + K_1\sin^2\alpha)\,[\cos^2 2\alpha + 2K_2\sin^2 2\alpha/(1 + K_1)]$$

(4.3) with

$$K_1 = C_h/C_v \quad \text{and} \quad K_2 = C_{45}/C_v \ .$$

This formula has been found to fit many experimental data fairly well.

5 - STABILITY ANALYSIS OF ANISOTROPIC NATURAL SOIL STRUCTURES

5.1 - Strength-criteria

Yield-design problems of purely cohesive anisotropic soil structures have always been treated as two-dimensional (Lo, Chen, Menzies), using methods which were adapted from those for isotropic soils by introducing the anisotropic cohesion derived from triaxial test experimental data. Therefore the concept of strength criterion of the soil considered as a three-dimensional continuous medium, had never to be referred to in those analyses.

Besides, the studies about strength criteria for anisotropic soils are so numerous that it would be vain to quote them all. The concept of anisotropy-tensor has been used by Caquot [25], Goldenblatt [26], Sobotka [27], and by Boehler and Sawczuk [28] who formulated Mises' and Drucker-Prager's type criteria in the case of anisotropic soils. Nova and Sacchi [29] also made use of that concept after having tried another approach [30]. Relying on the studies by Pipkin, Rivlin, Smith, Spencer, and Wang, about the representations of isotropic tensorial functions, Boehler [31] have produced a most relevant contribution in that field. His three-dimensional analysis makes it possible to study plane problems on sound mechanical bases : the obtained equations have been used by Pastor and Turgeman [32] for finite element calculations.

The data derived from undrained compression triaxial tests, as represented by formula (4.1), are not sufficient to determine a strength criterion valid for any true triaxial stress state ; Salençon and Tristán-López [33] have suggested to construct that criterion under the complementary hypothesis that it should not depend on the intermediate principal stress ([1]): the so-obtaned criterion fulfils the invariance-conditions established by Boehler (cf. [34]), and writes :

$$f(\underline{\underline{\sigma}}(\underline{x})) = \sigma_3 - \sigma_1 - 2C(\alpha) \leqslant 0$$

(5.1) $$\sigma_1 \leqslant \sigma_2 \leqslant \sigma_3 \quad \text{principal stresses,}$$

$$\alpha = (\underline{Oy}, \sigma_1)$$

(tensile stresses are counted positive)

where Oy is the direction of the transverse isotropy axis. With that strength-criterion, yield-design of structures in the case of plane strain parallel to the transverse isotropy axis is proved to reduce to a plane problem the corresponding two-dimensional strength criterion is nothing but (5.1) applied to the two-dimensional stress tensor in the plane ; the later result matches the procedure used in the available methods of stability analysis for anisotropic soils, it confirms that the assumption explicitly made here on the strength-criterion, was implicitly formulated by their authors.

Bishop's formula (4.1) has been retained by Salençon and Tristán-López to express $C(\alpha)$ in (5.1).

5.2 - π- functions

Two problems of yield design in plane strain conditions for anisotropic soils with criterion (5.1, 4.3) will be presented. Solving them through the kinematic approach requires knowing the two-dimensional π - functions for plane strain parallel to the transverse isotropy axis Oy. We get :

- for $\pi(\underline{\underline{d}}(\underline{x}))$:

  let d_1 be the major principal value of two-dimensional $\underline{\underline{d}}(\underline{x})$, and $\beta = (\underline{Oy}, \underline{d_1})$; then :

  $$tr(\underline{\underline{d}}(\underline{x})) \neq 0 \Longrightarrow \pi(\underline{\underline{d}}(\underline{x})) = +\infty$$

  $$tr(\underline{\underline{d}}(\underline{x})) = 0 \Longrightarrow \pi(\underline{\underline{d}}(\underline{x})) = \underset{\alpha}{Max}\ (-2C(\alpha)\ d_1 \cos 2(\beta - \alpha))$$

  i.e. $$\pi(\underline{\underline{d}}(\underline{x})) = C_v\ d_1\ \pi(\beta)\ .$$

- for $\pi(n(x), [\![\underline{v}(\underline{x})]\!])$:

  define unit vector $\underline{t}(\underline{x})$ in the plane by $(n(x), \underline{t}(\underline{x})) = \pi/2$; denote by v_n and v_t the corresponding components of $[\![\underline{v}(\underline{x})]\!]$ and let $\varepsilon = (\underline{Oy}, \underline{n}(\underline{x}))$; then:

  $$v_n \neq 0 \Longrightarrow \pi(\underline{n}(\underline{x}), [\![\underline{v}(\underline{x})]\!]) = +\infty$$

  $$v_n = 0 \Longrightarrow \pi(\underline{n}(\underline{x}), [\![\underline{v}(\underline{x})]\!]) = \underset{\alpha}{Max}\ (C(\alpha)\ v_t\ (\underline{x})\ \sin 2(\varepsilon - \alpha))$$

  i.e. $$\pi(\underline{n}(\underline{x}), [\![\underline{v}(\underline{x})]\!]) = \begin{cases} C_v |v_t| \pi_+(\varepsilon) & \text{if} \quad v_t > 0\ , \\ C_v |v_t| \pi_-(\varepsilon) & \text{if} \quad v_t < 0\ . \end{cases}$$

Functions $\pi(\beta)$ and $\pi_\pm(\varepsilon)$ have been computed in the case of criterion (5.1,4.3).

5.3 - Bearing capacity of a footing on an anisotropic cohesive soil

Davis and Christian [35] appear to have been the first to investigate the effects of cohesive soil anisotropy on the bearing capacity of foundations. As a strength-criterion for the material they adopted a simple polynomial form, depending on three parameters, that generalizes the one previously written by Hill [36]. Thus, they could determine the bearing capacity of strip footings by applying the theory of plane limit equilibriums (method of characteristics) since they remarked that a simple change of variable reduced the equations to those solved by Hill. In their paper Davis and Christian also retained the validity of Bishop's formula for a correct description of the available experimental data, but they noticed that no structure stability analysis had been performed using it.

The determination of the bearing capacity of strip-footings with Bishop's formula has been carried out by Salençon and Tristán-López [33] using both approaches of para. 3.2 & 3.3 ; they constructed static and kinematic solutions depending on several parameters which were optimized (figure 6).

With the notations of figure 6, the loading parameter for this problem is the axial force Q applied to the footing (per unit transversal length). The extreme value of this positive parameter is $Q^\star$ which writes (after a dimensional analysis) :

$$Q^\star = B\ C_v\ \varphi(K_1, K_2) \tag{5.2}$$

where φ is a scalar function of K_1 and K_2. For isotropic soils, $K_1 = K_2 = 1$ in (4.3), the solution is known : $\varphi(K_1,K_2) = \pi + 2$.

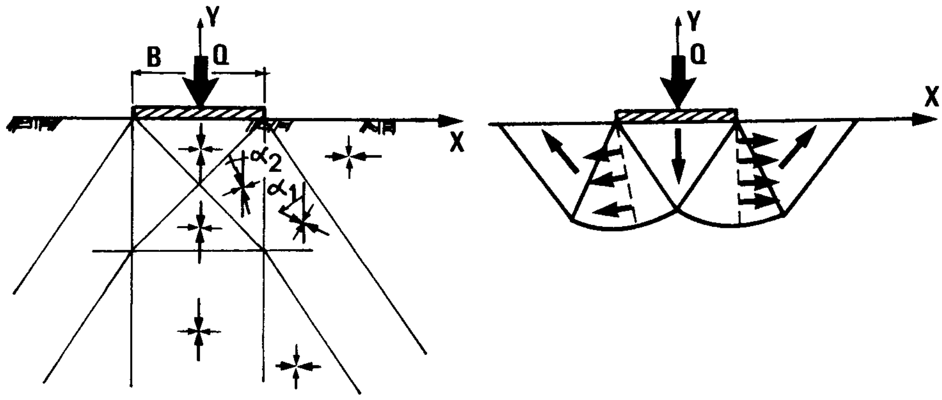

Figure 6 : Bearing capacity of strip-footings on anisotropic cohesive soils : static and kinematic solutions [33].

Results of this analysis are presented on figure 7, for three values of K_2, showing the thin zones delimited by the upper and lower bounds of the correction factor :

$$n(K_1,K_2) = \varphi(K_1,K_2)/(\pi + 2) \quad (5.3)$$

Complete charts are available in [34], and a detailed critical analysis of these results from the practical point of view may be found in [37]. Looking at correction factor (5.3) immediatly gives an estimation of the error committed on the bearing capacity of a footing on an anisotropic soil, if it is calculated assuming the soil to be isotropic from triaxial tests on vertically drilled samples. Although allowing for the usual tolerances in soil mechanics, it appears that the correction may be important either in the conservative or in the non-conservative sense ; moreover, simple empirical rules, as that of a mean cohesion suggested by Meyerhof [38], cannot be considered as sufficient (this one is proved to hold only in the case of Casagrande and Carrillo's soils).

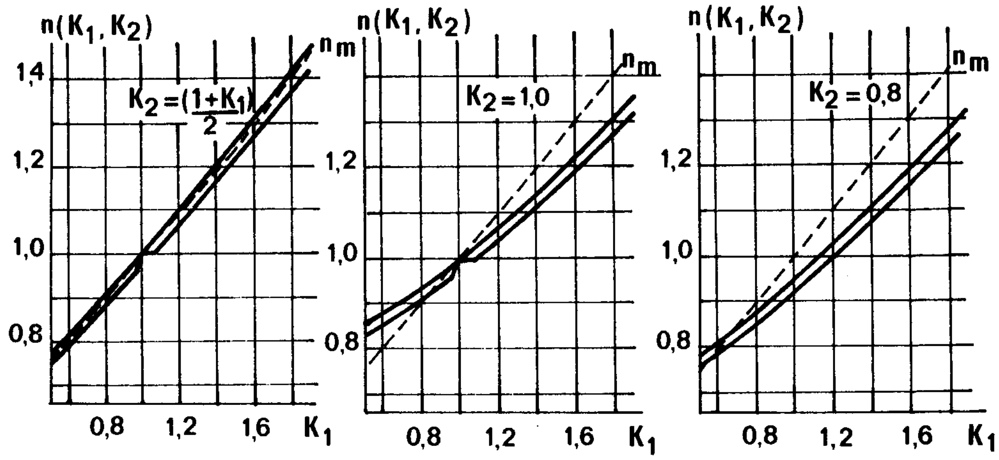

Figure 7 : Bearing capacity of a foundation on a cohesive anisotropic soil [37] ; $K_2 = (1 + K_1)/2$ corresponds to Casagrande-Carrillo's soils; n_m is the empirical result obtained assuming the soil to be isotropic with a cohesion $(C_v + C_h)/2$.

Comparing those results with Davis and Christian's ones shows that both strength-criterion (5.1, 4.3) and Davis and Christian's are convenient for practical applications.

5.4 - Stability of slopes for anisotropic cohesive soils

Let h and β be the geometrical characteristics of the studied slope (figure 8). The constitutive soil with voluminal weight γ, is anisotropic with strength-criterion (5.1, 4.3). Dimensional analysis shows that the stability of this structure is governed by the non-dimensional loading-parameter $N = \gamma h/C_V$; its extreme value is a function of β, K_1, K_2 : $N^*(\beta, K_1, K_2)$.

The construction of static solutions in order to apply the yield-design approach "from inside" has proved difficult and poorly efficient for this kind of problems; therefore they are usually dealt with by means of the yield-design approach "from outside".

Such a stability analysis has been carried out by Tristàn-Lòpez [34], applying Eq. (3.7) with the velocity-fields defined by the rigid body rotation of a block sliding along a circular line. Thus he established the first charts for slope stability evaluation in the case of an anisotropic cohesive soil whose yield-strength is defined by three parameters (i.e. two anisotropy ratios). An example is given at figure 8.

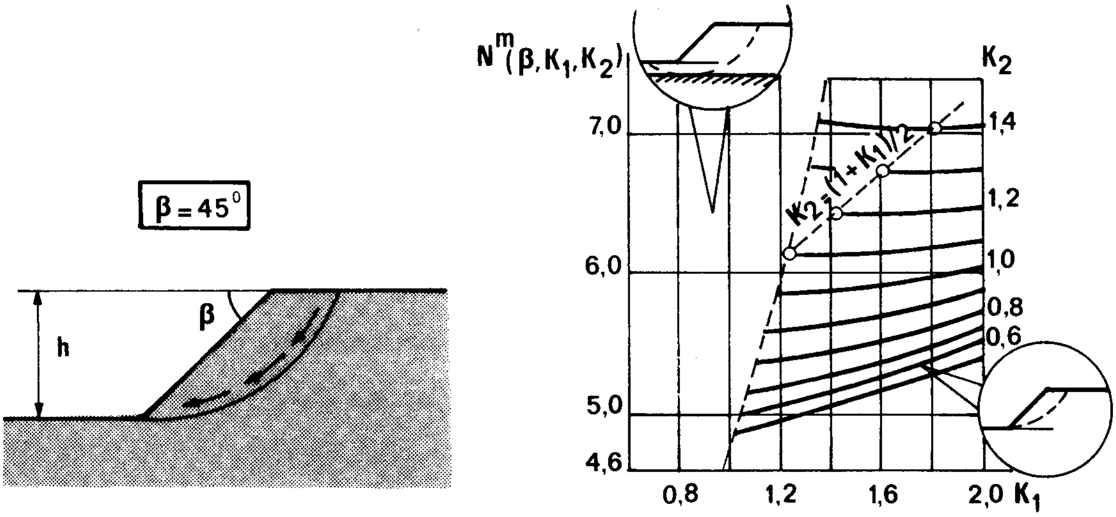

Figure 8 : Slope stability for anisotropic cohesive soil [34] : $\gamma h/C_V \leq N^m(\beta, K_1, K_2)$.

As mentioned before, Bishop's formula reduces to Casagrande and Carrillo's when $K_2 = (1+K_1)/2$; stability analyses were already available in that case (Lo [21], Chen [39]) whose results can be compared with those obtained by Tristàn-Lòpez (figure 9).

This comparison deserves a few comments from the mechanical point of view. As a matter of fact, Lo's and Chen's analyses are performed using the same velocity fields as in Tristàn-Lòpez'; nevertheless, though the strength-criterion is the same for the three authors, the obtained results are different. This discrepancy which originates in the way the approach from outside is realized, emphasizes the importance of a sound mechanical reasoning.Tristàn-Lòpez' analysis relies on

Eq. (3.7) and definition (3.5) of function $\pi(\ )$ to calculate $P(\underline{v})$: therefore it is consistent with the definition of the soil strength-criterion from the mechanical view-point. Lo's analysis is made following the idea mentioned in chap.2: the work of the external forces is compared with the work of the resisting ones ; in order to evaluate the later, the slip-circle in the velocity field is considered as an envelope of failure planes in triaxial tests (figure 9) and the mobilized shear stress on any of these planes (inclined at $\psi = 35°$ on the sample axis) is said to be equal to the corresponding $C(\alpha)$. Since $\psi \neq \pi/4$, this reasoning is indeed not in accordance with the procedure and the interpretation of the triaxial test (uniform cylindrical stress-field around σ_1). The same ideas have been followed by Reddy and Srinivasan, Chen, and Menzies, assuming other values for ψ ($\psi = 0$ in some cases). Due to the values of ψ adopted by the authors on the example of figure 9, the discrepancy between the results obtained that way, and through the mechanical dualization, is slight and can be considered negligible for practical applications ; but the question should not be discarded.

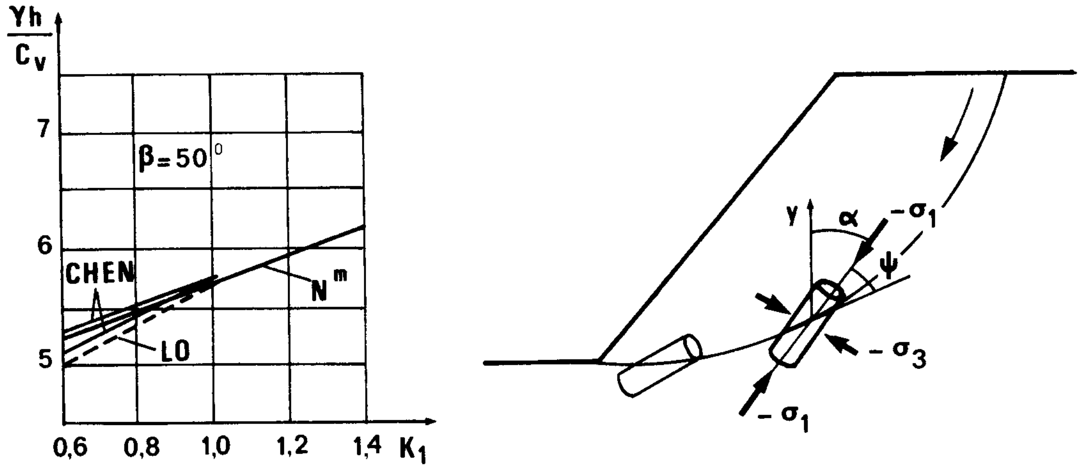

Figure 9 : Casagrande and Carrillo's soil : comparison between Tristán-López', Lo's, and Chen's results.

6 - SOIL REINFORCEMENT AND ANISOTROPY

6.1 - Soil reinforcement

Mechanical soil-reinforcement for building earth-structures has been used since antiquity, but it has enjoyed an extensive development during the late twenty years generated by important technical research. A few examples can illustrate typical techniques :
- spreading of regularly disposed reinforcement in the soil (the principle of reinforced earth),
- substitution of a better soil for a part of the native soil, following a regular mesh (the principle of ballast columns and trenches),
- improvement of the strength-characteristics of the native soil, following a regular mesh (the principle of lime columns),

and it may be noticed that the regularity of the set up reinforcing material, and the existence of preferential orientations, are features common to these processes.

Practical stability analyses for reinforced soil structures (e.g. slope stability, bearing capacity of surface footings) apply two-dimensional yield-design methods adapted from those classical in the case of homogeneous isotropic soils ; therefore any new development on this topic will be welcome. Here, we intend to give a first survey of the methods and results that can be derived from the application of the yield-design theory to the stability analysis of reinforced soil structures.

The characteristic scale of any of the abovementioned reinforcements, the usual civil engineering one, appears macroscopic when compared with that of the soil particles. But it looks like a fabric scale when compared with the scale of the structures themselves. This suggests that it should be possible to analyse the stability of a reinforced-soil structure by referring to a homogeneous "reinforced-soil" material whose definition and use should be clearly specified. From the preferential orientations of the reinforcement one may expect the "reinforced-soil" material to exhibit anisotropic strength properties.

The theoretical developments about homogenization in yield-design and limit analysis, by Suquet [40,41] and de Buhan [42], will not be presented ; the general reasoning and an example of the obtained results will be exposed on a typical problem.

6.2 - Stability of a reinforced soil slope

The considered slope is made of a purely cohesive isotropic native soil with cohesion C_1, reinforced by vertical layers of another purely cohesive anisotropic soil with cohesion $C_2 > C_1$. The stability of this structure will be studied in plane Oxy (figure 10).

Let e_i denote the layer thickness for the material with cohesion C_i $(i=1,2)$; let $e = e_1 + e_2$ and $\lambda_i = e_i/e$ $(i=1,2)$. The contact between the native soil and the reinforcement one is assumed to be perfectly adhesive. As a simplification, both soils will be supposed to have the same voluminal weight γ.

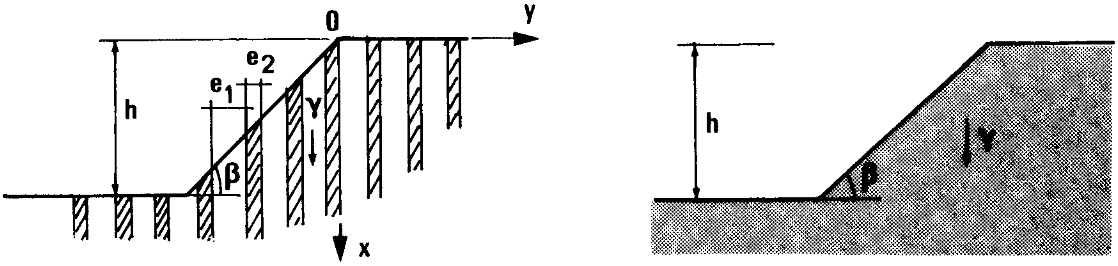

Figure 10 : Stability analysis of a reinforced-soil slope and associated homogeneous structure.

Dimensional analysis applied to the yield-design of that structure shows that the non-dimensional factor $N = \gamma h/C_1$ acts as the loading parameter for the problem ; its extreme value is a function of β, C_2/C_1, λ_2/λ_1, e/h :

$$(6.1) \qquad N = \gamma h/C_1 \quad , \quad N^\star(\beta,\ C_2/C_1,\ \lambda_2/\lambda_1,\ e/h)\ .$$

The value of $N^\star$ is looked for when the thickness of the layers (the scale of the reinforcement) can be considered small with respect to the height of the slope (the scale of the structure), i.e. when $e/h \to 0$. For simplicity we introduce :

$$(6.2) \qquad N_o^\star(\beta,\ C_2/C_1,\ \lambda_2/\lambda_1) = \lim N^\star(\beta,\ C_2/C_1,\ \lambda_2/\lambda_1,\ e/h) \quad \text{as} \quad e/h \to 0.$$

The determination of $N_o^\star$ through the approaches of yield-design proves rather difficult. Due to the two layered materials present in the system, the stress-or velocity-fields to work with soon become sophisticated, except for the velocity-fields defined by the rigid body rotation of a block sliding along a circular line (as in para. 5.4) which remain convenient. These ones are used in the classical methods [44] : thus, applying the kinematic approach with this family of velocity-fields leads to an upper bound N_o^m for $N_o^\star$:

(6.3) $$N_o^m(\beta,\ C_2/C_1,\ \lambda_2/\lambda_1) \geq N_o^\star(\beta,\ C_2/C_1,\ \lambda_2/\lambda_1)\ .$$

It may be proved that N_o^m depends only on β and on the factor $r = [1 + (\lambda_2/\lambda_1)(C_2/C_1)] / (1 + \lambda_2/\lambda_1)$, and writes :

(6.4) $$N_o^m(\beta,\ C_2/C_1,\ \lambda_2/\lambda_1) = r\ N^m(\beta)\ ,$$

where $N^m(\beta)$ is the result obtained for the stability of a cohesive soil slope by the slip circle method. In other words, the slope with the "composite" soil when $e/h \to 0$, is perceived through the slip-circle analysis in just the same way as if it were made of a homogeneous isotropic soil with the cohesion rC_1 i.e. the ponderated mean value of both constitutive soil cohesions.

6.3 - Homogenization and yield-design

Homogenization in yield-design introduces a homogeneous structure associated to the composite soil structure when $e/h \to 0$, which is geometrically identical and is submitted to the same loading process as that one (figure 10). The stability of this associated homogeneous structure is then analysed.

The homogenized material is only defined for this application. A first yield-design problem is solved on the representative elementary volume of the "composite" material : it corresponds to the change of scales and produces the strength criterion to be adopted for the "reinforced-soil" homogenized material from the characteristics C_ν, C_2/C_1, λ_2/λ_1, through the static and kinematic approaches.

In the present state of the theory it is proved that, with this definition of the homogenized material, the stability analysis of the associated homogeneous structure gives an over-estimation of the stability of the composite one (Suquet [41]). A stronger result would indeed better fit the intuitive conjecture made in para. 6.1, but the value of this already available one will be demonstrated through our following example.

6.4 - Strength-criterion of reinforced soil

For the reinforced soil structure describe in para. 6.2, the homogenized material is thus obtained in the form of a purely cohesive soil, transversally isotropic around Oy. For the two-dimensional problems in plane strain parallel to Oy, its strength-criterion writes : let σ_1 and σ_3 be the principal stresses ($\sigma_1 \leq \sigma_3$) and $\alpha = (Oy, \sigma_1)$

(6.5) $$\sigma_3 - \sigma_1 - 2C(\alpha) \leq 0\ ,$$

(6.6) $$C(\alpha) = C_1\ \rho(\alpha,\ C_2/C_1,\ \lambda_2/\lambda_1)\ ,$$

where the non-dimensional factor ρ can be explicitly calculated [42]. Figure 11 shows ρ as a function of α.

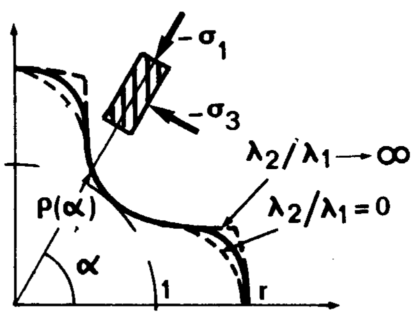

Figure 11 : Polar diagrams for the homogenized soil : $\rho(\alpha) = C(\alpha)/C_1$.

It exhibits the following properties :

$\alpha = \pi/4$: $\rho = 1$, $C_{45} = C_1$ ("weak" soil cohesion)

(6.7) $\left.\begin{array}{l}\alpha = 0\\ \alpha = \pi/2\end{array}\right\}$: $\rho = r$, $C(\alpha) = rC_1$ (ponderated mean value of the soil cohesions)

$0 \leqslant \alpha \leqslant \pi/2$: $\rho(\alpha) = \rho(\pi/2 - \alpha)$ $\quad 1 \leqslant \rho(\alpha) \leqslant r$

6.5 - Stability analysis of the associated homogeneous structure

With the results (6.5, 6.6) the stability analysis of the associated homogeneous slope (figure 10) proves similar to the problem already studied in para. 5.4 for anisotropic natural soils ; here, formula (6.6) is substituted for Bishop's formula in expressing $C(\alpha)$. The corresponding π- functions for plane strain parallel to Oy have been determined [45].

Once again the velocity-fields defined by the rigid-body rotation of a block sliding along a circular line are used in the kinematic approach. An upper bound $N^m_{hom}(\beta, C_2/C_1, \lambda_2/\lambda_1)$ is thus obtained for the extreme value $N^\star_{hom}$ of the loading parameter $\gamma h/C_1$ for the associated homogeneous structure :

(6.8) $$N^m_{hom}(\beta, C_2/C_1, \lambda_2/\lambda_1) \geqslant N^\star_{hom}(\beta, C_2/C_1, \lambda_2/\lambda_1) .$$

Applying Suquet's theorem, the loading parameters being identical for both structures, we can also state that :

(6.9) $$N^m_{hom}(\beta, C_2/C_1, \lambda_2/\lambda_1) \geqslant N^\star_o(\beta, C_2/C_1, \lambda_2/\lambda_1) .$$

Figure 12 presents the obtained results in the case $\beta = 90°$, as functions of parameters λ_2/λ_1 and r.

N^m_{hom} enjoys the following properties whatever β :

- for $r = 1$ (homogeneous soil with cohesion C_1), $N^m(\beta)$ is found again :

(6.10) $$r = 1,\ \forall\ \lambda_2/\lambda_1\ :\ N^m_{hom}(\beta, C_2/C_1, \lambda_2/\lambda_1) = N^m(\beta)\ ;$$

- for $r > 1$, N^m_{hom}/r decreases as r increases when λ_2/λ_1 is fixed

  N^m_{hom}/r increases with λ_2/λ_1 when r is fixed,

  thence, taking Eq. (6.10) into account, we get :

(6.11) $$N^m_{hom}(\beta, C_2/C_1, \lambda_2/\lambda_1) \leqslant r\ N^m(\beta).$$

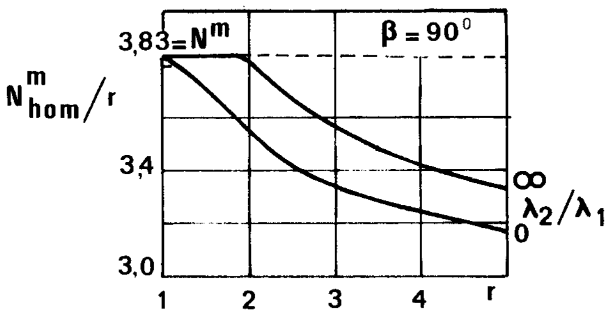

Figure 12 : Stability analysis of a reinforced soil slope : $\gamma h/C_1 \leqslant N^m_{hom}$.

6.6 - Comments : homogenization and anisotropy

Putting together formulae (6.3, 6.4, 6.9 and 6.11), or looking at figure 12, it is easy to compare the results obtained through the application of the slip-circle method : first to the "composite"-soil slope ($e/h \to 0$), then to the associate slope made of a homogenized soil with anisotropic cohesion. It appears then that, working on the same class of simple velocity-fields, and applying rigorously the kinematic approach of yield-design, we get significantly different estimations of the composite-soil slope stability : since it is lower, the value obtained through homogenization shall be retained. A similar conclusion has been drawn by de Buhan [46] when studying the bearing capacity of surface footings, within the same framework.

The property is general and its explication is close to mechanical intuition. It was shown in para. 6.2 that the classical method leads to the same evaluation of the slope stability as if the constitutive soil were homogeneous and isotropic with the cohesion ponderated mean value ; stability analysis after homogenization, thanks to the two successive applications of yield-design approaches it implies, can account for the macroscopic anisotropy of reinforced-soil (and for this one, cohesion $C(\alpha)$ has been proved to be always smaller than the cohesion ponderated mean value).

7 - CONCLUSION

Due to techonological incitements and to progresses in computational means, solid mechanicians have taken increasing interest into the study of anisotropic materials, during the late decades. General fundamental researches have been carried out as well as more specific investigations in many disciplines of applied mechanics.

In soil mechanics, constitutive laws are now formulated, based on modelizations which take anisotropy into account ; numerous papers have been devoted to that subject (a recent general lecture by Hueckel and Nova [9] includes some 250 references !). We have chosen to present here an aspect of the reflexions on soil anisotropy which is directly connected with practical applications : the yield-strength of anisotropic soils from the point of view of the stability analysis of structures.

The necessity of sound mechanical bases to rely on, as well when setting up or interpreting experimental tests, as when devising stability analysis methods, has been evidenced : ambiguities, which we can remain unaware of in the case of isotropy, may lead to inconsistency when the material is anisotropic. The theory of representation of isotropic tensorial functions and that of yield-design proved highly valuable for that purpose. From the practical view point the obtained results are relevant.

Apart from the anisotropy of natural soils, the macroscopic anisotropy of mechanically reinforced soils can also be taken into account. Applying the homogenization process to yield-design makes it possible to formulate efficient methods for the stability analysis and the design of reinforced-soil structures. New developments should appear in this field during the next few years, which are more likely to be of an applied character, in order to simulate the real reinforcement techniques better.

REFERENCES :

[1] Coulomb, C.A., Essai sur une application des règles de Maximis et Minimis à quelques problèmes de statique relatifs à l'architecture, mémoire présenté à l'Académie Royale des Sciences (Paris, 1773).

[2] Chandler, R.J., Two early contribution to the study of slopes, *Géotechnique*, 31, 4, (1981), 553-554.

[3] Morton, J., *The natural history of Northamptonshire* (London, 1712).

[4] Desrues, J., *La localisation de la déformation dans les matériaux granulaires*, th. Dr Sc., Univ. Grenoble, (1984).

[5] Habib, P., Les surfaces de glissement en mécanique des sols, *Revue Française de Géotechnique*, 27, (1984), 7-21.

[6] Boehler, J.P., Lois de comportement anisotrope des milieux continus, *J. de Mécanique*, 17, 2, (1978), 153-190.

[7] Wang, C.C., A new representation theorem for isotropic functions, Part. I & II, *Arch. Rat. Mech. An.*, 36, (1970), 166-223, Corrigendum, 43, (1971), 392-395.

[8] Salençon, J., *Calcul à la rupture et analyse limite*, Presses de l'E.N.P.C., Paris, (1983).

[9] Hueckel, T., Nova, R., Anisotropic failure criteria for geotechnical materials, recent trends, *Int. Coll. CNRS, "Failure criteria of structured media"*, Villard-de-Lans, (1983).

[10] Ung Sen, Y., *Etude de l'influence des paramètres d'essais sur la mesure de la résistance au cisaillement des argiles raides*, th. Dr Ing., E.N.P.C., Paris, (1975).

[11] Casagrande, A., et Carrillo, N., Shear failure of anisotropic materials, *J. Boston Soc. Civil Eng.*, 31, 4, (1944), 74-87.

[12] Allirot, D., Boehler, J.P., Evolution de l'anisotropie d'une roche sous compression cyclique isotrope, *Bul. Ac. Pol. Sc.*, série Sciences et Tech., 24, 9, (1976), 405-409.

[13] Goguel, J., La prise en compte de l'anisotropie dans la mécanique des déformations tectoniques, *Bul. Soc. Géol.*, *Fr.*, 18, 6, (1976), 1489-1495.

[14] La Rochelle, P., & Marsal, R.J., Slope stability, Gen. Rep. 11, *Proc. 10th I.C.S.M.F.E.*, Stockholm, (1981), 141-162.

[15] Broms, B.B., et Ratnam, M.V., Shear strength of an anisotropically consolidated clay, *Journal of the Soil Mech. and Found. div.*, Proc. A.S.C.E., 92, SM6, (nov. 1963), 1-26.

[16] Broms, B.B., et Casbarian, A.O., Effects of rotation of the principal stress axes and of the intermediate principal stress on the shear strength, *Proc. 6th I.C.S.M.F.E.*, Montreal, (1965), 1, 179-183.

[17] Saada, A.S., et Zamani, K.K., The mechanical behavior of cross anisotropic clays, *Proc. 7th I.C.S.M.F.E.*, Mexico, (1969), 1, 351-359.

[18] Saada, A.S., Discussion of "Anisotropy in heavily overconsolidated Kaolin", *Jnl Geotech. Eng. Div.*, A.S.C.E., 102, GT 7, (1976), 823-824.

[19] Boehler, J.P., Raclin, J., Critères de résistance des composites verre-résine sous états de contrainte complexes, *C.R. JNC 3*, Paris, (1982), 309-319.

[20] Duncan, J.M., et Seed, H.B., Anisotropy and stress reorientation in clays, *Jnl Soil Mech. and Found. Div.*, Proc. A.S.C.E., 92, SM6, (sept. 1966), 21-50.

[21] Lo, K.Y., Stability of slopes in anisotropic soils, *Jnl Soil Mech. and Found. Div.*, Proc. A.S.C.E., 91, SM4, (july 1965), 85-106.

[22] Duncan, J.M., et Seed, H.B., Strength variations along failure surfaces in clay, *Jnl Soil Mech. and Found. Div.*, Proc. A.S.C.E., 92, SM6, (nov. 1966), 81-104.

[23] Bishop, A.W., The strength of soils as engineering materials, Géotechnique, 16, 2, (1966), 89-130.

[24] Lo, K.Y., et Milligan, V., Shear strength properties of two stratified clays, *Jnl Soil Mech. and Found. Div.*, Proc. A.S.C.E., 93, SM1, (jan. 1967), 1-15.

[25] Caquot, A., Conférence à l'Académie des Sciences d'Italie, 1953, (quoted in Caquot, A., & Kérisel, J., *Traité de mécanique des sols*, Gauthier-Villars, Paris, (1966).

[26] Goldenblat, I.I., *Some problems of mechanics of deformable media*, Noordhoff, Groningen (1962).

[27] Sobotka, Z., The fundamental relations of theory of plasticity and a new concept of the plastic potential for anisotropic materials, Stavebnicky Casopis, S.A.V. 14, (1966), 377-380.

[28] Boehler, J.P., et Sawczuk, A., Equilibre limite des sols anisotropes, *J. de Mécanique*, 9, 1, (1970), 5-33.

[29] Nova, R., Sacchi, G., A model of the stress-strain relationship of orthotropic geological media, *J. Meca. Th. Appl.*, 1, 6, (1982), 927-949.

[30] Nova, R., Sacchi, G., A generalized failure condition for orthotropic solids, *Proc. Euromech 115, Coll. CNRS "Comportement mécanique des solides anisotropes"*, Villard-de-Lans, (1979).

[31] Boehler, J.P., *Contributions théoriques et expérimentales à l'étude des milieux plastiques anisotropes*, Th. Dr Sc., Grenoble, (1975).

[32] Pastor, J., Turgeman, S., Approches numériques des charges limites pour un matériau orthotrope de révolution en déformation plane, *J. Méca. Th. Appl.*, 2, 3, (1983), 393-416.

[33] Salençon, J., et Tristán-López, A., Force portante des semelles filantes sur sols cohérents anisotropes homogènes, *C.R. Ac. Sc. Paris*, 292, II, (1981), 1097-1102.

[34] Tristán-López, A., *Stabilité d'ouvrages en sols anisotropes*, Th. Dr -Ing. E.N.P.C., Paris, (1981).

[35] Davis, E.H., et Christian, J.T., Bearing capacity of anisotropic cohesive soils, *Jnl of the Soil Mech. and Found. Div.*, Proc. A.S.C.E., 97, SM5, (may 1971), 753-769.

[36] Hill, R., *The mathematical theory of Plasticity*, Clarendon Press, Oxford, (1950).

[37] Salençon, J., Tristán-López, A., Calcul à la rupture en mécanique des sols : cas des sols cohérents anisotropes, *Ann. I.T.B.T.P.*, 413, (1983), 53-83.

[38] Meyerhof, G., Discussion of stability of slopes in anisotropic soils, *J. Soil Mech. and Found. Div.*, Proc. A.S.C.E., 91, SM6, (1965), 132.

[39] Chen, W.F., *Limit analysis and soil plasticity*, Elsevier Sc. Publ. Cy., Amsterdam, (1975).

[40] Suquet, P., *Plasticité et homogénéisation*, Th. Dr Sc. Univ. Pierre et Marie Curie, Paris, (1982).

[41] Suquet, P., Analyse limite et homogénéisation, *C.R. Ac. Sc. Paris*, 296, II, (1983), 1355-1358.

[42] de Buhan, P., Homogénéisation en calcul à la rupture, *C.R. Ac. Sc. Paris*, 296, II, (1983), 933-936.

[43] de Buhan, P., Salençon, J., Analyse de stabilité d'ouvrages en sols renforcés, to appear in *Proc. 13rd I.C.S.M.F.E.*, San Francisco, (1985).

[44] Lime Column Method., Linden-Alimak, Sweden, (1980).

[45] Zghal, A., *Stabilité d'ouvrages en matériaux composites*, Mémoire de D.E.A., Laboratoire de Mécanique des Solides, Palaiseau, (1983), E.N.I.T., (Tunis).

[46] de Buhan, P., Détermination de la capacité portante d'une fondation sur sol renforcé par une méthode d'homogénéisation, to appear in *Proc. Coll. Int. "Renforcement des sols et des roches"*, Paris, 1984.

(1) and that the orientation of the stress-tensor should appear only through the inclination of the major principal pressure towards the orthotropy axis.