92

PROCEEDINGS OF THE CNRS INTERNATIONAL COLLOQUIUM NO 351
VILLARD-DE-LANS / FRANCE / 21-24 JUNE 1983

Failure Criteria of Structured Media

Edited by
J.P. BOEHLER
University Joseph Fourier, Grenoble, France

OFFPRINT

A.A. BALKEMA / ROTTERDAM / BROOKFIELD / 1993

Failure Criteria of Structured Media, Boehler (ed.) © 1993 Balkema, Rotterdam, ISBN 90 6191 179 6

Determination of a macroscopic yield criterion for a multi-layered material

P. de Buhan & J. Salençon
Ecole Polytechnique, Palaiseau, France

ABSTRACT : A macroscopic yield criterion for a multi-layered material is defined as the solution of an asymptotic yield design problem on representative elementary volumes of this material. In the case of the two-dimensional multi-layered medium with Tresca constituents, a purely cohesive anisotropic criterion is obtained. It is applied to the stability analysis of a multi-layered vertical cut.

RESUME : On définit le critère de rupture à l'échelle macroscopique d'un matériau multicouche à partir de la solution d'un problème asymptotique de calcul à la rupture sur des volumes élémentaires représentatifs de ce matériau. On établit alors dans l'hypothèse du matériau multicouche bidimensionnel à constituants de Tresca, que le critère obtenu est du type cohérent anisotrope. Un tel critère est utilisé pour analyser la stabilité d'un talus vertical multicouche.

This paper is based on the theory of yield design (Salençon, 1983), which allows the determination of the potentially safe loads of a structure, taking into account its geometry, loading conditions, and with the knowledge of a yield criterion in each point of the constitutive material, as sole information concerning its mechanical behaviour. The problem will be studied within the framework of three-dimensional continuum mechanics in sections 1 and 2, and of two-dimensional continuum mechanics in sections 3 to 6.

1 A FIRST APPROACH TO THE CONCEPT OF MACROSCOPIC YIELD CRITERION

A multi-layered vertical cut with a depth h (fig.1) is taken as an example. The two homogeneous constitutive materials are denoted by i (i = 1,2), any quantity relative to them being characterized by the corresponding subscript. The layer thickness is e_i and we introduce :

$$\lambda_i = e_i/e, \quad \text{where} \quad e = e_1 + e_2 . \qquad (1)$$

The strength capacities of each material are defined through a convex domain G_i determining the set of allowable stress tensors $\underset{\approx}{\sigma}$.

Interfaces between layers will be assumed in a first step to be perfectly adhesive.

The specific weights of the two materials are assumed to be equal to γ . γ is the only loading parameter for the problem.

The stability of such a structure is determined by finding the extreme value γ^+ defined as follows :

$$0 \leq \gamma \leq \gamma^+ \Longleftrightarrow \left\{ \begin{array}{l} \exists \ \underset{\approx}{\sigma} \text{ statically admissible} \\ \quad \text{(S.A.) with } \gamma \ , \\ \quad \text{such that :} \\ \underset{\approx}{\sigma}(\underline{x}) \in G_i \ (i = 1 \text{ or } 2), \forall \ \underline{x} \end{array} \right. \qquad (2)$$

As many difficulties arise when trying to solve this problem directly (due to the complexity of the stress fields $\underset{\approx}{\sigma}$ in the static approach, and of the velocity fields $\underline{v}$ in the kinematic approach), it seems reasonable to look for the solution of a homogenized problem, obtained through the assimilation of the multi-layered material to a homogeneous one, in every point of which a macroscopic yield criterion with a domain G_H will be defined. This latter problem is solved by determining the extreme value γ_H^+, thus defined :

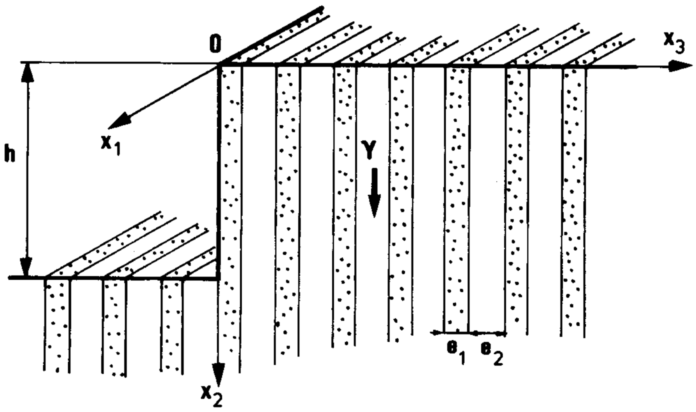

Fig. 1 : Multi-layered vertical cut.

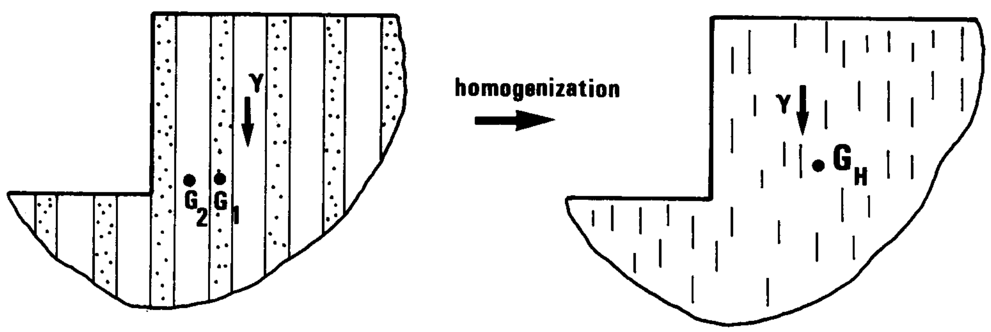

Fig. 2 : A definition of the equivalent homogenized problem.

$$0 \leqslant \gamma \leqslant \gamma_H^+ \Longleftrightarrow \begin{cases} \exists\ \underline{\sigma} \text{ S.A. with } \gamma \\ \underline{\sigma}(\underline{x}) \in G_H\ ,\ \forall\ \underline{x} \end{cases} \qquad (3)$$

(see fig.2)

Two questions will be successively considered :

a) How to define and calculate such a macroscopic yield criterion (sections 2 to 4) ?
b) To what extent does the solution of the homogenized problem (3) approximate that of the initial problem (2), thus justifying the homogenization process (section 5) ?

2 DEFINITION OF THE MACROSCOPIC YIELD CRITERION

2.1 Representative elementary volumes of the multi-layered material

A fixed geometrical domain Ω of unit volume with a boundary $\partial\Omega$ is considered in space $\mathbb{R}^3$ (with co-ordinate system $Ox_1x_2x_3$) A representative elementary volume (Le Nizhery, 1976) of multi-layered material is then defined as any volume of this material occupying Ω, such that :

- the direction of the layers is parallel to Ox_1x_2,
- and $e = \varepsilon . L$, where ε is a small dimensionless parameter and L a characteristic length of Ω (for instance the smallest diameter of Ω normal to the layer direction : fig.3).

Such a volume will be denoted by Ω_ε.

2.2 The case of the homogeneous material : $G_1 = G_2 = G$

The multi-layered material can then be considered as homogeneous with regard to its strength capacities, so that the de-

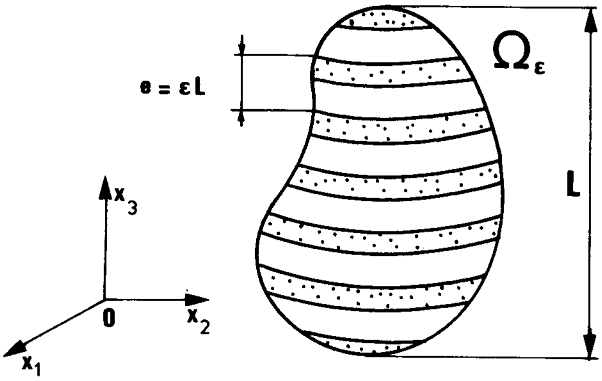

Fig.3 : Representative elementary volume of multi-layered material.

pendence on ε of any representative elementary volume of this material can be omitted : $\Omega_\varepsilon = \Omega$.

Generally speaking, the notion of stress solicitation on the boundary $\partial\Omega$ of Ω, denoted by S, can be defined as the datum of the stress vector $\underline{T}$ acting upon Ω in each point $\underline{x}$ of $\partial\Omega$:

$$S = \{\underline{T}(\underline{x}) \ , \ \underline{x} \in \partial\Omega\} \ ,$$

under the conditions :

$$\int_{\partial\Omega} \underline{T}(\underline{x})\,ds = 0, \quad \text{and} \quad \int_{\partial\Omega} \underline{x} \wedge \underline{T}(\underline{x})\,ds = 0 \ .$$

Likewise, the work of S in any velocity field $\underline{v}$ can be defined as the integral :

$$P(S, \underline{v}) = \int_{\partial\Omega} \underline{T}(\underline{x})\underline{v}(\underline{x})\,ds \ .$$

We suppose then that Ω is subjected to a uniform stress solicitation, that is :

$$\forall \ \underline{x} \in \partial\Omega \qquad \underline{T}(\underline{x}) = \underline{\Sigma} \cdot \underline{n}(\underline{x}) \ ,$$

where $\underline{\Sigma}$ is a constant symmetric tensor and $\underline{n}(\underline{x})$ the normal unit vector at the point $\underline{x}$ of $\partial\Omega$. Such a solicitation is denoted by $S_{\underline{\Sigma}}$.

Two properties can be then established as follows :

Property 1 : Ω is subjected to a 6 parameter loading process.

Indeed, it can be written :

$\forall \ \underline{\sigma}$ statically admissible with $S_{\underline{\Sigma}}$

$\forall \ \underline{v}$ kinematically admissible,

$$P(S_{\underline{\Sigma}},\underline{v}) = \int_{\partial\Omega} \underline{n}(\underline{x}) \cdot \underline{\sigma}(\underline{x}) \cdot \underline{v}(\underline{x})\,ds = \underline{\Sigma}(\underline{\sigma}) : \langle \underline{d} \rangle (\underline{v})$$

where

$$\langle \underline{d} \rangle (\underline{v}) = \frac{1}{2} \int_{\partial\Omega} (\underline{n} \otimes \underline{v} + \underline{v} \otimes \underline{n})\,ds \ .$$

The set of tensors $\underline{\Sigma}$ is a six-dimensional vectorial space, made identical to its dual space (strain rate tensors), and for which the double contracted tensorial product (denoted by :) is a scalar product. Then :

$$P(S_{\underline{\Sigma}},\underline{v}) = \underline{Q} \cdot \dot{\underline{q}} \quad , \quad \text{where} \quad \underline{Q}\,, \dot{\underline{q}} \in \mathbb{R}^6 .$$

Property 2 : K, domain of the potentially safe loadings $\underline{\Sigma}$ is identical with the yield domain G :

$$K = \{\underline{\Sigma} \mid \exists \ \underline{\sigma} \ \text{S.A.} \ \underline{\Sigma}, \ \underline{\sigma}(\underline{x}) \in G, \ \forall \ \underline{x} \in \Omega\} \equiv G$$

Then the yield criterion for a homogeneous material (local datum) appears as the solution of a yield design problem on any volume of this material.

2.3 General case ($G_1 \neq G_2$). Definition of G_H

These results induce us to propose a definition of the yield criterion of the multilayered material on the macroscopic scale, that is to say on the scale of the representative elementary volumes of this material. Some definitions must be adopted first :

2.3.1 Let $\{\Omega_{\varepsilon_n}\}$, $n \in N$ be a sequence of representative elementary volumes such that :

$$\lim_{n\to\infty} \varepsilon_n = 0 \ .$$

Such a sequence will be called asymptotically homogeneous.

Let us define Ω_n as $\Omega_n = \Omega_{\varepsilon_n}$. The part of Ω_n occupied by the constituent i will be denoted by Ω_n^i, and $G_n(\underline{x})$ will be the convex domain describing the strength capacities at the point $\underset{\sim}{x}$ in Ω_n :

$$G_n(\underset{\sim}{x}) = G_i \quad \forall \ \underset{\sim}{x} \in \Omega_n^i .$$

2.3.2 Likewise, a sequence of stress solicitations $\{S_n\}$, $n \in N$

$$S_n = \{\underset{\sim}{T}_n(\underset{\sim}{x}), \ \underset{\sim}{x} \in \partial\Omega\}$$

will be called asymptotically uniform, if there exists a tensor $\underset{\sim}{\Sigma}$ such that :

$$\lim_{n\to\infty} P(S_n, \underset{\sim}{v}) = P(S_{\underset{\sim}{\Sigma}}, \underset{\sim}{v}) \quad , \forall \ \underset{\sim}{v} \ \text{K.A.}$$

This will be written conventionally :

$$\{S_n\} \to S_{\underset{\sim}{\Sigma}} .$$

2.3.3 The following definition of the macroscopic yield domain $G_H (\subset \mathbb{R}^6)$ is then adopted :

$\underset{\sim}{\Sigma}$ belongs to G_H, if and only if, given any asymptotically homogeneous sequence $\{\Omega_n\}$, there exists a sequence $\{S_n\}$ such that $\{S_n\} \to S_{\underset{\sim}{\Sigma}}$, and checking : (4)

S_n acting on Ω_n is potentially safe (P.S.) for every n ;

this means that it is possible to exhibit a stress field $\underset{\sim}{\sigma}_n$ statically admissible with S_n and respecting the yield criterion everywhere :

$$\underset{\sim}{\sigma}_n(\underset{\sim}{x}) \in G_n(\underset{\sim}{x}) \qquad \forall \ \underset{\sim}{x} \ \in \Omega_n .$$

The diagram below clearly shows the consistency of the proposed definition, by making a comparison with the homogeneous case :

Homogeneous Material $G_1 = G_2 = G$	Multi-Layered Material $G_1 \neq G_2$
$G = \{\underset{\sim}{\Sigma} \mid S_{\underset{\sim}{\Sigma}} \ . \ \text{P.S.}\}$	$G_H = \left\{ \underset{\sim}{\Sigma} \mid \begin{array}{l} \forall \{\Omega_n\} \text{ asymp. homogeneous} \\ \exists \{S_n\} \to S_{\underset{\sim}{\Sigma}} \\ \forall n \ S_n \ \text{P.S.} \end{array} \right\}$
Property	Definition

3 INDEPENDENCE OF THE DEFINITION IN THE CASE OF THE TWO DIMENSIONAL MULTI-LAYERED MATERIAL

Definition (4) a priori involves the geometry of the representative elementary volumes. For our reasoning to be fully justified, the independence of the criterion so-obtained with respect to the choice of Ω has then to be proved.

This independence property can be expressed in a very simple way, when the multi-layered material can be described within the framework of two-dimensional continuum mechanics ; this is, for instance, the case of "plane strain" yield design problems in a direction parallel to the symmetry axis of the multi-layered material. Each convex $G_i (\subset \mathbb{R}^3)$ describes then the "plane-strain" strength capacities of the constituent i .

The following sections of this paper are devoted to problems studied within the framework of two-dimensional continuum mechanics.

3.1 The two-dimensional multi-layered material being studied in the plane Oxy (fig.4), it can be shown (de Buhan, 1982) that G_H can be defined as follows :

$$\underset{\sim}{\Sigma} \in G_H \Longleftrightarrow \left\{ \begin{array}{l} \exists \ (\underset{\sim}{\sigma}_1, \underset{\sim}{\sigma}_2) \text{ such that} \\ \quad \underset{\sim}{\Sigma} = \lambda_1 \underset{\sim}{\sigma}_1 + \lambda_2 \underset{\sim}{\sigma}_2 \\ \text{satisfying } \ \underset{\sim}{\sigma}_i \in G_i \\ \quad (i = 1, 2) \\ \text{and} \\ \sigma_1^{xy} = \sigma_2^{xy} \quad \text{(components in} \\ \qquad\qquad\quad \text{co-ordinate} \\ \sigma_1^{yy} = \sigma_2^{yy} \quad \text{system Oxy)} \end{array} \right. \qquad (5)$$

where λ_1 and λ_2 are defined by (1).

The major interest of definition (5) lies in the fact that it makes the practical determination of the macroscopic yield criterion much easier.

Note : when the material is homogeneous ($G_1 = G_2 = G$), definition (5) leads to $G_H = G$.

3.2 "Dual" definition of G_H :

It can be proved that :

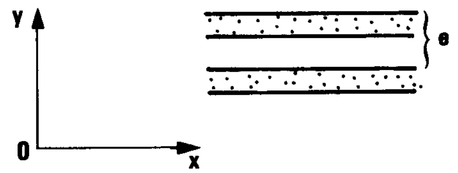

Figure 4 : Two-dimensional multi-layered material.

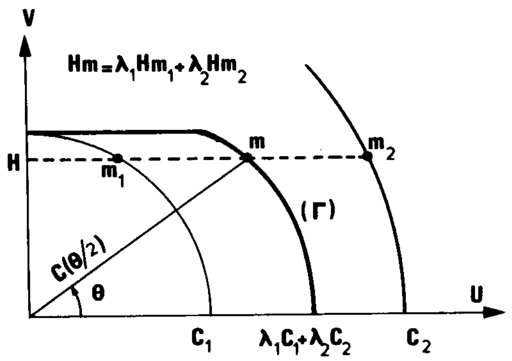

Figure 5a : Representation of the yield domain in the deviatoric plane.

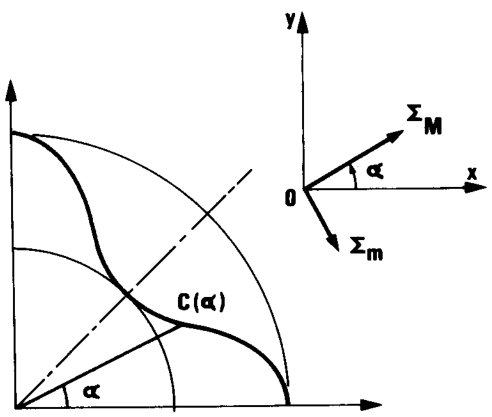

Figure 5b : Polar diagram for cohesion.

$$G_H = \bigcap_{\underset{\sim}{D} \in \mathbb{R}^3} \left\{ \underset{\sim}{\Sigma} \;;\; \underset{\sim}{\Sigma} : \underset{\sim}{D} \leqslant \Pi_{G_H}(\underset{\sim}{D}) \right\} \qquad (6)$$

where $\Pi_{G_H}(\underset{\sim}{D})$ (defined as Sup $\{\underset{\sim}{\Sigma} : \underset{\sim}{D}, \underset{\sim}{\Sigma} \in G_H\}$) is equal to :

$$\Pi_{G_H}(\underset{\sim}{D}) = \underset{(\underset{\sim}{d}_1, \underset{\sim}{d}_2)}{\mathrm{Min}} \left\{ \lambda_1 \Pi_{G_1}(\underset{\sim}{d}_1) + \lambda_2 \Pi_{G_2}(\underset{\sim}{d}_2) \;, \; \lambda_1 \underset{\sim}{d}_1 + \lambda_2 \underset{\sim}{d}_2 = \underset{\sim}{D} \right\}$$

under the condition : $d_1^{xx} = d_2^{xx}$ (strain rates along Ox).

4 APPLICATION TO THE CASE OF PURELY COHESIVE ISOTROPIC CONSTITUENTS

4.1 The two constituents are now supposed to be Tresca materials :

$$\underset{\sim}{\sigma} \in G_i \Longleftrightarrow \sigma_M - \sigma_m - 2C_i \leqslant 0 \qquad (7)$$

$(i = 1,2)$

where σ_M (resp. σ_m) is the major (resp. minor) principal stress of the tensor $\underset{\sim}{\sigma}$ and C_i the cohesion of constituent i.

It can be shown (de Buhan, 1982) from (5) and (7) that the macroscopic yield criterion can be expressed as follows :

$$\underset{\sim}{\Sigma} \in G_H \Longleftrightarrow \Sigma_M - \Sigma_m - 2C(\alpha) \leqslant 0 \;, \qquad (8)$$

where Σ_M and Σ_m are the major and minor principal stresses of Σ, and α the angle made by the major principal stress with Ox. Thus, the multi-layered material with purely cohesive isotropic constituents behaves, as regards its yield criterion, like a homogeneous, but anisotropic cohesive material.

Formula (8) is clearly of the same form as the yield criteria proposed by many authors who have dealt with two dimensional plane strain problems for naturally anisotropic purely cohesive materials such as soils (Casagrande & Carillo, 1944, Bishop, 1966, Salençon & Tristan Lopez, 1980).

4.2 Geometrical representation of the criterion

As criteria (7) and (8) are independent of the hydrostatic component of stress $\underset{\sim}{\Sigma}$, they can be represented in the two-dimensional space of deviator stresses with co-ordinate system :

$$U = (\Sigma^{yy} - \Sigma^{xx})/2 \quad , \quad V = \Sigma^{xy} \; .$$

The boundary line of each domain G_i in the plane (U,V) is a circle of radius C_i, centered at the origin, and that of G_H a curve (Γ) obtained through a very simple geometrical construction (fig.5a). The corresponding equation in polar co-ordinates is :

$$\rho(\theta) = C(\theta/2) \; .$$

The polar cohesion diagram $C(\alpha)$ can be

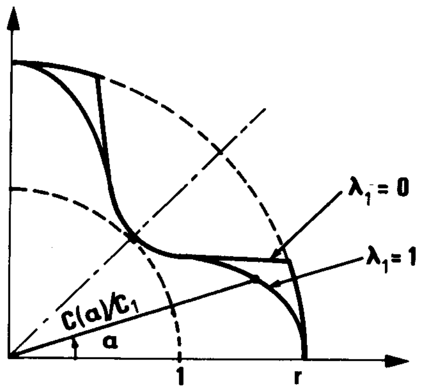

Figure 6 : Variation of the polar diagram as a function of λ_1.

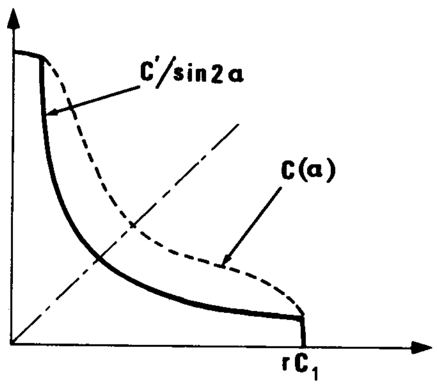

Figure 7 : Polar diagram for cohesion in the case of interfaces with cohesion C'.

easily deduced, showing the symmetry properties (figure 5b) :

$$C(\alpha) = C(-\alpha) \tag{9}$$

and especially $C(\pi/2 - \alpha) = C(\alpha)$.

Two dimensionless parameters can conveniently be introduced :

$$k = C_2/C_1 \quad \text{(suppose that } k \geqslant 1)$$

and $r = (\lambda_1 C_1 + \lambda_2 C_2)/C_1 = \lambda_1 + k\lambda_2$

$= C(\alpha = 0)/C(\alpha = \pi/4)$.

The evolution of the dimensionless polar diagram $C(\alpha)/C_1$, as a function of λ_1, with r remaining constant, is shown on fig.6. Two particular cases have to be underlined :

- $\lambda_1 = 0$: it represents layers of constituent 2 (with cohesion rC_1) seperated by interfaces with cohesion C_1
- $\lambda_1 \to 1$ (whence $k \gg 1$) : it corresponds to constituent 1 with inclusions of very thin layers ($\lambda_1 \ll 1$) of constituent 2 ($C_2 \gg C_1$). (This configuration might schematize reinforced earth).

4.3 Macroscopic yield domain in the case of yield conditions at the interface

We now assume that the interfaces between the layers are no longer perfectly adhesive and that a yield criterion governs the contact. Such a condition can be described by a scalar convex function g defined by :

$$g(\underset{\sim}{T}) \leqslant 0 \Longleftrightarrow \left\{ \begin{array}{l} \text{the stress vector } \underset{\sim}{T} \\ \text{acting on the interface} \\ \text{respects the criterion.} \end{array} \right.$$

It can be easily shown that the corresponding macroscopic yield domain G_H^{int} becomes :

$$G_H^{int} = G_H \cap \{\underset{\sim}{\Sigma} \mid g(\Sigma^{xy}, \Sigma^{yy}) \leqslant 0\} \tag{10}$$

In a general way, the macroscopic yield criterion is then no longer of the anisotropic purely cohesive type (even in the case of isotropic constituents) except in the case of an interface yield condition defined by :

$$g(\Sigma^{xy}, \Sigma^{yy}) = |\Sigma^{xy}| - C' \ .$$

Formula (8) becomes then :

$$\underset{\sim}{\Sigma} \in G_H^{int} \Longleftrightarrow \Sigma_M - \Sigma_m \leqslant 2 \operatorname{Min}\left(C(\alpha), \frac{C'}{\sin 2\alpha}\right) \tag{11}$$

(fig.7).

4.4 "Failure" modes of the elementary volume of multi-layered material

We shall speak of the "failure of the elementary volume" of multi-layered material, when the "macroscopic" solicitation $\underset{\sim}{\Sigma}$ is located on the boundary of G_H. Let then $\underset{\sim}{\sigma}_1$ and $\underset{\sim}{\sigma}_2$ be the "microscopic" stress tensors corresponding to definition (5) of $\underset{\sim}{\Sigma}$, and $\underset{\sim}{d}_1$, $\underset{\sim}{d}_2$ the "associated" strain rate tensors defined by :

$$\underset{\sim}{\sigma}_i : \underset{\sim}{d}_i = \Pi_{G_i}(\underset{\sim}{d}_i) \quad (i = 1,2) \tag{12}$$

Two cases have to be distinguished according to the orientation α of the major principal stress (but due to relations (9), it is possible to consider only $\alpha \in [0, \pi/4[$.

• $0 \leq \alpha \leq \alpha_o$ where $\sin 2\alpha_o = 1/r$ (fig.8a). Tensors $\underset{\sim}{\sigma}_i$ are both located on the boundaries of the domains G_i. The criterion is reached in both constituents, and "failure" occurs by deformation of both materials ($\underset{\sim}{d}_i \neq 0$, $i = 1,2$).

(Actually the term "failure" is misused, and should rigorously be used only in the case of perfectly plastic materials obeying the principle of maximum plastic work. Relation (12) defines then the corresponding plastic flow rule).

• $\alpha_o < \alpha \leq \frac{\pi}{4}$ (fig. 8b) . Only the stress state in constituent 1, which is the one with the weaker cohesion C_1, is on the yield surface. "Failure" occurs then through pure shearing strain of constituent 1 : the principal directions of $\underset{\sim}{d}_1$ make 45° angles with the vertical direction, and $\underset{\sim}{d}_2 = 0$.

5 APPLICATION TO THE STABILITY ANALYSIS OF A MULTI-LAYERED VERTICAL CUT (Zghal, 1983)

Let us come back to the vertical cut example (section 1). Assuming now that the constituents obey Tresca's criterion, the problem can be solved as a "plane strain" problem in the plane Ox_2x_3, and the previously established results (sections 3 and 4) are then directly applicable here.

A dimensional analysis of the initial problem (2) shows that γ^+ is necessarily written as :

$$\gamma^+ = K^+(\lambda_1,\ r,\ e/h).C_1/h$$

where

$$K^+ = K^+(\lambda_1,\ r,\ e/h) = \frac{\gamma^+ h}{C_1}$$

is a dimensionless factor characterizing the stability of the vertical cut.

Likewise, for the homogenized problem (3) :

$$K^+ = \frac{\gamma_H^+ \cdot h}{C_1} = K_H^+(\lambda_1,\ r) \ .$$

The justification of the homogenization process achieved by substituting problem (3) for problem (2), involves the equality between K_H^+ and the limit value of K^+ when the thickness of the layers tends to zero (with h remaining constant), to be proved :

$$K_H^+ = \lim_{e/h \to 0} K^+ (= K_o^+) \ .$$

The aim of this paper is only to compare

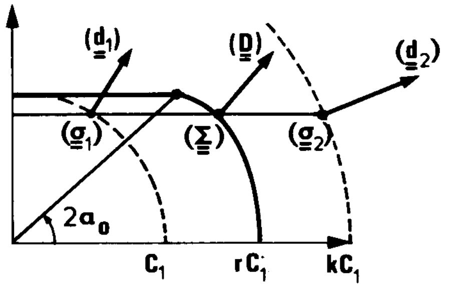

Figure 8a : $0 \leq \alpha \leq \alpha_o$

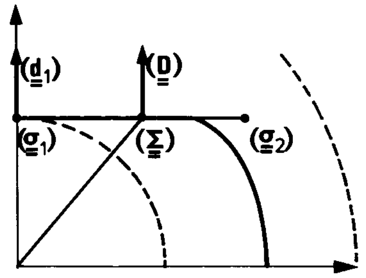

Figure 8b : $\alpha_o < \alpha < \frac{\pi}{4}$.

the upperbound values of K_o^+ and K_H^+ (respectively denoted by K_o^m and K_H^m) obtained from kinematic approaches using rigid block mechanisms, with circular failure lines as shown on fig.9. The analysis leads to the following results :

direct calculation

$$K_o^+ \leq \min_{\theta_1,\theta_2} F(\theta_1,\theta_2\ ;\lambda_1,\ r) = K_o^m \approx 3.83\ r$$

"homogenized" calculation

$$K_H^+ \leq \min_{\theta_1,\theta_2} G(\theta_1,\theta_2\ ;\lambda_1,\ r) = K_H^m \ .$$

The variations of K_o^m/r and K_H^m/r as functions of r , are represented by curves displayed on fig. 10.

In the case where $r = 1$ (homogeneous vertical cut), Taylor's classical result is thus obtained :

$$K_o^m(r = 1) = K_H^m(r = 1) \approx 3.83.$$

Moreover, the upperbound value K_o^m ($\approx 3.83\ r$) is identical to the one which would have been obtained by performing the same kinematic approach on a homogeneous vertical cut with isotropic cohesion $rC_1 = \lambda_1 C_1 + \lambda_2 C_2$.

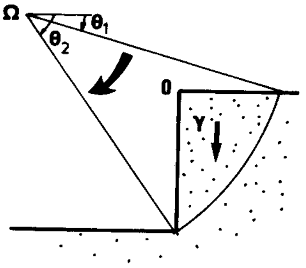

Figure 9 : Rigid block mechanism with circular failure line.

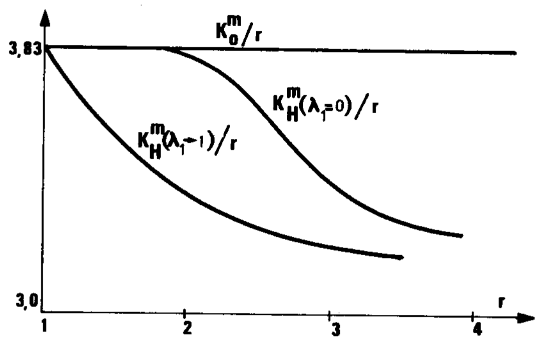

Figure 10 : Variations of K_o^m/r and K_H^m/r as functions of λ_1.

6 COMMENTS

6.1 Definitions (5) and (6) of G_H coincide in the case of the two dimensional multi-layered material, with those assumed by Suquet concerning periodically structured materials (i.e. $G(\underset{\sim}{x})$ a periodical function of $\underset{\sim}{x}$.

Besides, the approach adopted in this paper can be easily extended as a whole to this latter type of material. The case of the two dimensional multi-layered material being noticeable because of the very simple calculation of the macroscopic yield criterion.

6.2 The following inequality, deriving from a result by Suquet can be proved :

$$K_o^+ \leqslant K_H^+ .$$

Thence, taking into account the inequality :

$$K_H^m \leqslant K_o^m \quad \text{(figure 10)},$$

it follows :

$$K_o^+ \leqslant K_H^+ \leqslant K_H^m \leqslant K_o^m .$$

These inequalities emphasize the great interest of the homogenized calculation, which, by using the same failure mechanism as for the direct calculation, leads to a better result. It is likely that more sophisticated mechanisms would have been necessary to achieve the same result through direct calculation.

6.3 Lowerbound values for K_o^+ and K_H^+ can also be achieved through static approach. It is then necessary touse heavier methods (finite element methods : Pastor, 1983), in order to obtain significant results.

6.4 The homogenization method within the framework of the yield design theory can be applied to other types of structures and problems : for instance the indentation of a multi-layered half-space could be studied.

REFERENCES

BISHOP, A.W., 1966, The strength of soils as engineering materials. Géotechnique, vol.16, n°2, pp. 89-130.

de BUHAN, P., 1982, Définition et calcul d'un critère de résistance macroscopique pour un matériau composite multicouche. Mémoire de DEA, ENPC, Paris VI.

CASAGRANDE, A. & CARILLO, N., 1944, Shear failure of anisotropic materials. Journal of the Boston Society of Civil Engineers, vol. 31, n°4, pp. 74-87.

LE NIZHERY, D., 1976, Comportement plastique des matériaux composites multicouches Thèse de Docteur Ingénieur, Paris VI.

PASTOR, J., 1983, Applications de la théorie de l'analyse limite aux milieux isotropes et orthotropes de révolution. Thèse d'Etat, Grenoble.

SALENÇON, J., 1983, Calcul à la rupture et analyse limite. Presses de l'ENPC, Paris.

SALENÇON, J. & TRISTAN LOPEZ, A., 1980, Analyse de stabilité des talus en sols cohérents anisotropes, CRAS, t.290, série B, pp. 493-496.

SUQUET, P., 1982, Plasticité et homogénéisation. Thèse d'Etat, Paris VI.

SUQUET, P. , 1983, Analyse limite et homogénéisation, CRAS, t. 286, série II, pp. 1355-1358.

TAYLOR, D.W., 1960, Fundamentals of soil mechanics, John Wiley.

ZGHAL, A., 1983, Stabilité des ouvrages en matériaux composites. Mémoire de DEA, LMS, Palaiseau, ENIT Tunis.