EUROMECH 134 : DESIGN AGAINST FAILURE IN SOILS

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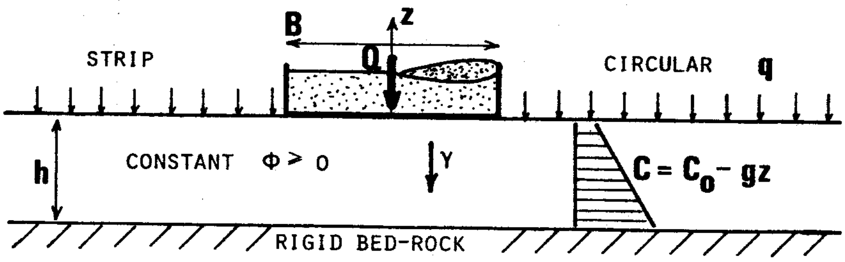
BEARING CAPACITY OF SHALLOW FOUNDATIONS ON NON-HOMOGENEOUS SOILS

This study aims at the determination of the exact value of the theoretical bearing capacity of shallow foundations of the strip - or circular footing types, when the foundation soil is of limited thickness with a cohesion linearly increasing with depth. The notations for the problem are shown on the figure for both types of footings. The contacts between the soil layer and the footing on the one hand and between the soil layer and the underlying rigid bed-rock on the other are assumed to occur with perfect sticking.



The theoretical bearing capacity of the foundation is defined from the point of view of yield design : the foundation soil is characterized by its strength criterion of Tresca's or Coulomb's types, with a constant friction angle φ ($\geqslant 0$) and a cohesion linearly increasing with depth according to the formula $C = C_o - gz$; the bearing capacity is the average pressure under the footing, axially loaded in the present case, corresponding to the maximum value of the load Q for which equilibrium is compatible with the strength criterion. It will be denoted q_u and q_u^o respectively for the strip and circular footings.

Through a mechanical and dimensionnal analysis it is shown that the parameters of the problem can be grouped so that the bearing capacity can be written in the forms :

(1)
$$\begin{cases} q_u = q + (C_o + q\tan\varphi)F_c\left(\dfrac{g+\gamma\tan\varphi}{C_o + q\tan\varphi}B,\ \dfrac{B}{h},\ \varphi\right) \\ q_u^o = q + (C_o + q\tan\varphi)F_c^o\left(\dfrac{g+\gamma\tan\varphi}{C_o + q\tan\varphi}B,\ \dfrac{B}{h},\ \varphi\right) \\ \text{when } (C_o + q\tan\varphi) \neq 0 . \end{cases}$$

(2)
$$\begin{cases} q_u = q + (g + \gamma\tan\varphi)B\,K\,(B/h,\ \varphi) \\ q_u^o = q + (g + \gamma\tan\varphi)B\,K^o\,(B/h,\ \varphi) \\ \text{when } (C_o + q\tan\varphi) = 0. \end{cases}$$

where F_c, F_c^o, K, K^o are scalar functions of the indicated arguments.

These *global* formulae account for all coupling effects to appear in the problem. They require functions F_c, F_c^o, K, K^o to be determined, which has be done, refering to the theorems of yield design, through the use of the theories of plane or axially symmetric limit equilibriums for non-homogeneous soils.

In order to make the practical use of the obtained results easy, it has been found suitable to present them in the form of multi-entry charts for various values of φ in the range (0, 45°). Are given :

(a) the bearing capacity factors to be used in the superposition formula for the strip-footing, as functions of B/h and φ ; this allows to calculate a linear underestimation of the bearing capacity of the strip-footing.

(b) the interaction factor μ_c for the strip-footing as a function of $(g + \gamma \tan \varphi)B/(C_o + q \tan \varphi)$, B/h, φ ; it measures the coupling effect between surface cohesion and surface load on the one hand and cohesion gradient and gravity forces on the other ; *(a)* and *(b)* make it possible to determine the exact value of the bearing capacity for the strip-footing.

(c) the shape factor ν, as a function of $(g + \gamma \tan \varphi)B/(C_o + q \tan \varphi)$, B/h, φ, for passing from a strip-footing to the circular footing with the same width and the same features for the soil layer ;*(a)*, *(b)* and *(c)* allow to determine the exact value of the bearing capacity for the circular footing.

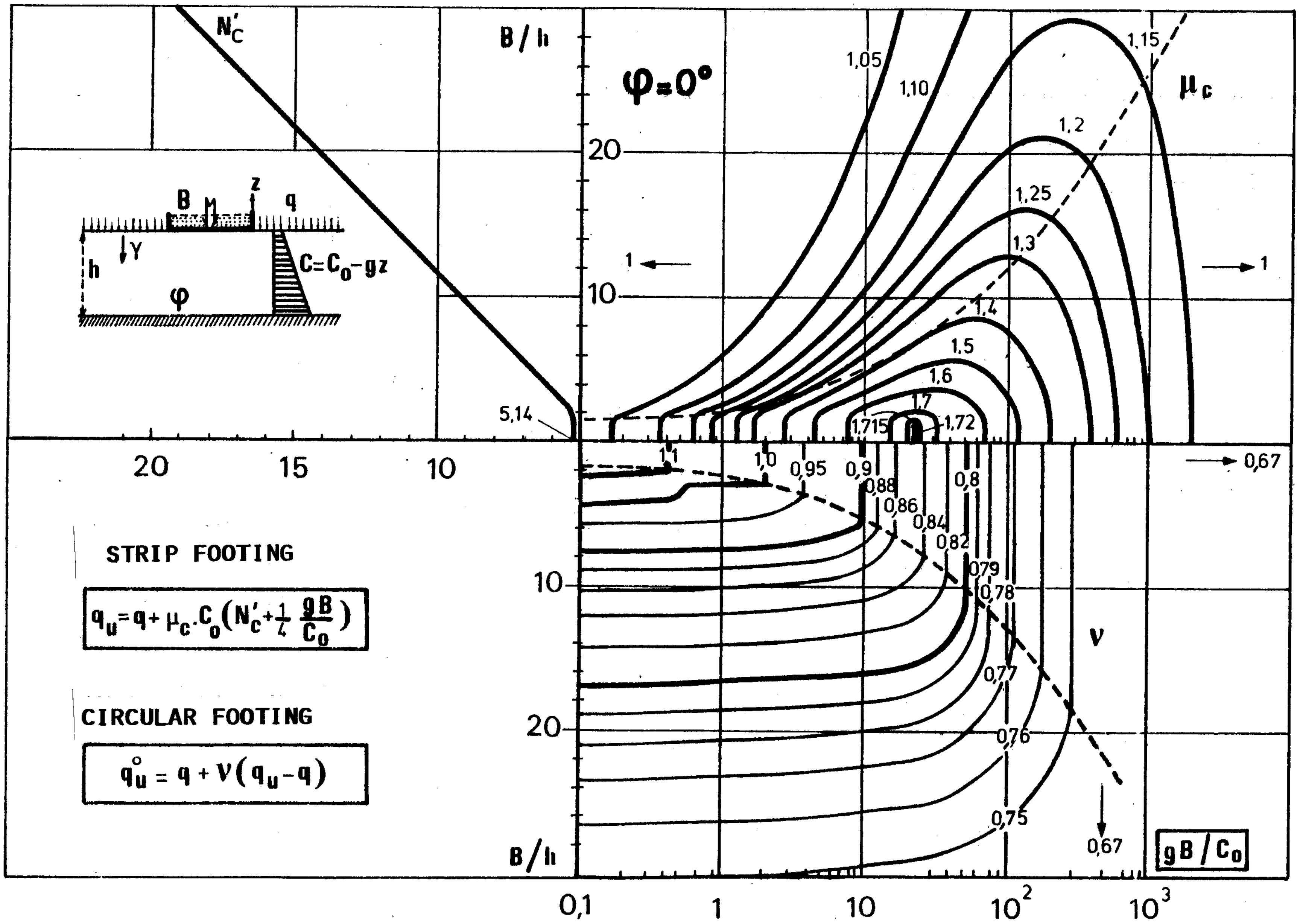
As a comment of these results in a few words :

- the proposed charts make it possible to obtain the exact value of the bearing capacity for those two types of footings both in classical conditions and commonly encountered in practise non-classical ones (e.g. sea soils exhibiting both small friction angle and surface cohesion and important cohesion gradient) ;

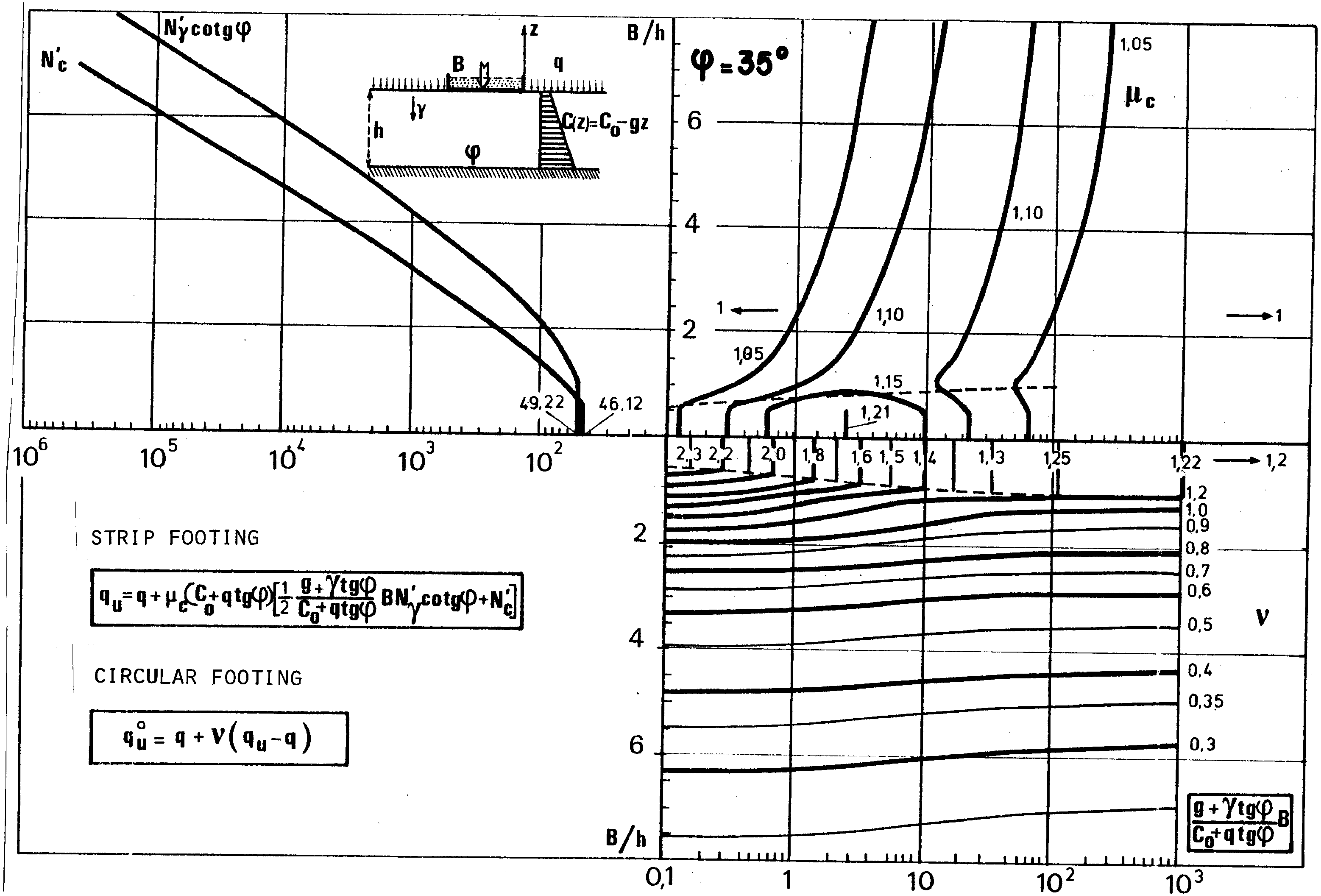
- since strip-and circular footings may be considered as two extreme geometrical patterns for the foundations to be dealt with in practise, it is intended that the adopted presentation be convenient for the engineer since it allows to determine q_u and q_u^o simultaneously and then to interpolate between the two values as felt necessary ;

- it may be noticed that the coupling effect appears to be important for small values of the soil friction angle and of the geometrical ratio B/h (e.g. the maximum value of μ_c is reached when $\varphi = 0$, $B/h < 1.41$ and $gB/C_o = 23$; $\mu_c = 1.72$) ;

- the shape factor is highly dependent on φ; it is also a decreasing function of its two first arguments ; its maximum value is $\nu(0, 0, \varphi)$, the shape factor on the cohesion term for a footing over an unlimited soil layer, which increases with φ and is always greater than 1, whereas $\nu(\infty, 0, \varphi)$, the shape factor on the gravity term, increasing with φ , becomes greater than 1 only when $\varphi > 29°$.



φ=0°
N'c
B/h
μc
ν
gB/Co
5,14
C=Co−gz
STRIP FOOTING
qu=q+μc.Co(N'c+1/4 gB/Co)
CIRCULAR FOOTING
q°u = q + ν(qu−q)
1,05
1,10
1,15
1,2
1,25
1,3
1,4
1,5
1,6
1,7
1,715
1,72
1,1
1,0
0,95
0,9
0,88
0,86
0,84
0,82
0,8
0,79
0,78
0,77
0,76
0,75
0,67
0,1
1
10
10²
10³



STRIP FOOTING

$$q_u = q + \mu_c (C_0 + q\,tg\varphi)\left[\frac{1}{2}\,\frac{g + \gamma\,tg\varphi}{C_0 + q\,tg\varphi}\,B N'_\gamma\,cotg\varphi + N'_c\right]$$

CIRCULAR FOOTING

$$q_u^{\circ} = q + \nu\,(q_u - q)$$

$$\frac{g + \gamma\,tg\varphi}{C_0 + q\,tg\varphi}\,B$$

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