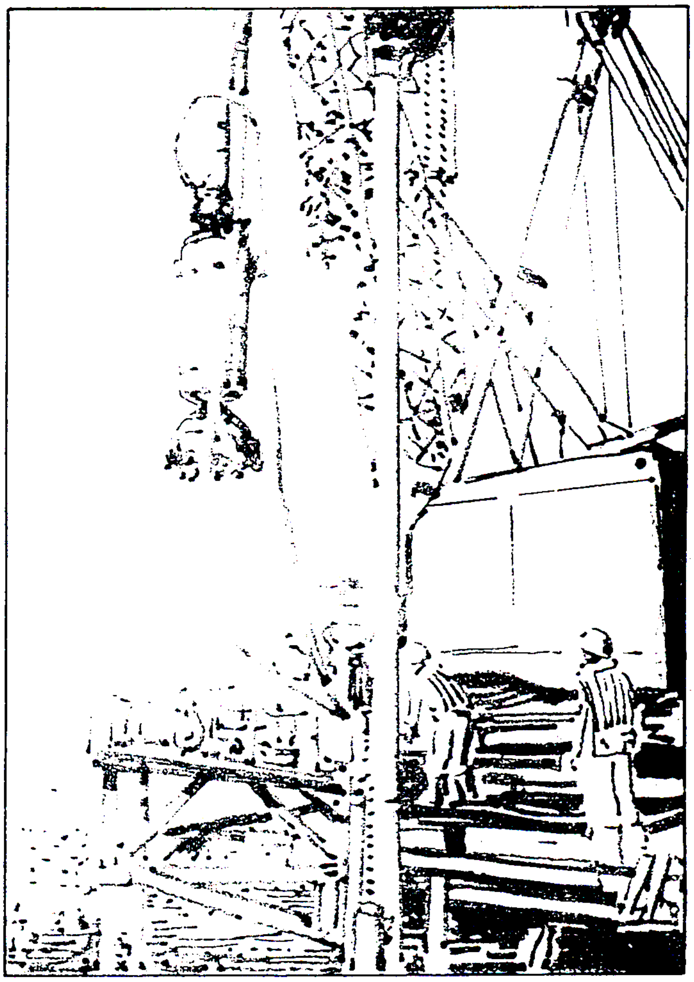
FOUNDATION ENGINEERING



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Bearing capacity of circular shallow foundations

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1. INTRODUCTION

The bearing capacity of shallow foundations of the strip footing type has already been the theme of various studies, among others by the authors of the present work, in cases when the foundation rests on a soil layer overlaying a rigid bedrock, and with a vertical cohesion gradient.

The practical importance of such analyses is quite evident :

- it is common to find a relatively thin foundation soil layer bound with a bedrock which can be considered as being rigid, the soils being either frictional ($\phi \neq 0°$) or purely cohesive;

- a downward vertical cohesion gradient appears in soils with an internal friction angle $\phi \leqslant 25°$, and more often in purely cohesive soils or with ϕ near to 0° such as marine soils, for example.

The present article aims at studying circular shallow foundations, axially loaded and laying upon a non homogeneous soil, in conditions identical to those described above.

Besides being interesting in itself - it is actually possible to find circular shallow foundations of a similar type - this study will moreover provide the pratician dealing with a rectangular shallow foundation, for example, with two quite different elements: first,the bearing capacity of a strip footing and second, the bearing capacity of a circular footing, so as to enable him to evaluate the bearing capacity, he is looking for by means of an appropriate shape factor.

As the reader will see hereunder, we have been concerned mainly with presenting our results in form of charts, for a direct use in common practice. As a matter of fact, these charts are derived directly from those which the authors have chosen to use in their publication on strip footings (MATAR and SALENÇON, 1979). Besides being intrinsecally easy to use, they allow the designing engineer to evaluate, at the same time, the bearing capacities of a strip footing, and of a circular footing of identical width and under identical conditions, which of course, makes interpolations easier for foundations of "intermediate" forms.

2. THE STUDIED PROBLEM

Figure 1 gives the data of the problem : a circular shallow foundation with a diameter B, supposed to be perfectly rough (which means that the soil and the foundation are totally adhesive to one another), is axially loaded by a force Q. The foundation acts upon a soil layer, with a thickness h, whether finite of infinite : this soil layer itself rests with perfect friction on an assumed rigid bedrock.

The strength criterion of the foundation soil is :

- for a purely cohesive soil, with a cohesion C :

  denoting by σ_1, σ_2, σ_3 the principal stresses [1] ordered following $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$

$$\sigma_1 - \sigma_3 \geqslant 2C, \qquad (2.1)$$

  (this is TRESCA's criterion) ;

[1] The stress tensor is noted $\underline{\underline{\sigma}}$. The conventional sign which has been chosen, is the same as for mechanics (tensile stresses positive) so as to make it easier to refer to the formulae that one finds in basic theoretical works about the theroy of limit equilibriums in axial symmetry. This will have no effect at all upon the practical use of the results as shown in the charts, since bearing capacity will, of course, be counted positively, in compression.

- for a soil exhibiting both cohesion and friction, with a cohesion C and a friction angle ϕ :

$$\sigma_1(1+\sin\phi) - \sigma_3(1-\sin\phi) - 2C\cos\phi \leq 0 \qquad (2.2)$$

The foundation soil is supposed to be non homogeneous and to show a vertical cohesion gradient g :

$$g = -dC/dz \qquad (2.3)$$

the Oz axis being upward vertical, and the friction angle supposed to be constant.

The downward vertical gradient will be supposed positive :

$$g \geq 0 \qquad (2.4)$$

It will be seen afterwards that the results are valid as long as the group $(g+\gamma\,tg\,\phi)$ is positive :

$$g + \gamma\,tg\,\phi \geq 0 \qquad (2.5)$$

where γ is the unit weight supposed to be constant, of the foundation soil,

C_o denotes the surface cohesion

q is the uniform lateral surface load.

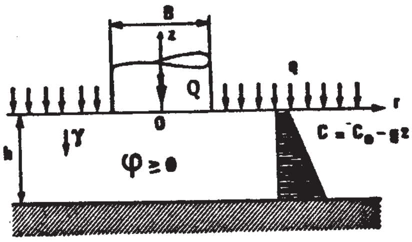

Fig.1 The problem:circular shallow foundation on a non homogeneous soil layer.

3. BEARING CAPACITY OF SHALLOW FOUNDATIONS

3.1 DEFINITION

One knows that the theory of yield design makes it possible to define clearly the ultimate load of a shallow foundation, namely Q^+, characterized by the average pressure then prevailing under this same foundation, that one being called the bearing capacity: Q^+ is the highest load that can applied to the foundation, for which compatibility between the equations of a quasi-static equilibrium and the strength capacity of the soil holds (see e.g. MATAR and SALENÇON, 1979).
Thus, for a shallow foundation of the strip footing type, where Q denotes the load per unit of linear length, the bearing capacity is :

$$q_u = Q^+/B \qquad (3.1)$$

where B is the foundation width.

For the circular shallow foundation described hereabove, (Figure 1), the bearing capacity is :

$$q_u^o = Q^+/(\pi B^2/4) \quad (^1) \qquad (3.2)$$

3.2 RESULTS FOR STRIP FOOTINGS

We recall here some of the results which have been achieved by MATAR and SALENÇON (1979).

The bearing capacity q_u, written in (3.1) is a function of all the parameters defining a problem homologous to that of figure 1.

$$q_u = f(C_o, g, B, h, \gamma, \phi, q) \qquad (3.3)$$

It has been demonstrated, through a mechanical analysis of the problem, based upon the theory of yield design, that the parameters which appear in (3.3) can be put together so that q_u appears under the following form :

$$q_u = q + f_2(C_o + q\,tg\,\phi,\; g+\gamma\,tg\,\phi,\; B,\; h,\; \phi) \qquad (3.4)$$

Dimensional analysis of (3.4) then leads to the formulae :

$$\begin{cases} q_u = q + (C_o + q\,tg\,\phi) \times F_c\left(\dfrac{g+\gamma\,tg\,\phi}{C_o+q\,tg\,\phi}\,B,\; \dfrac{B}{h},\; \phi\right) \\ \text{if} \quad (C_o + q\,tg\,\phi) \neq 0 \end{cases} \qquad (3.5)$$

$$\begin{cases} q_u = q + (g+\gamma\,tg\,\phi)\,B\,K\left(\dfrac{B}{h},\; \phi\right) \\ \text{if} \quad (C_o + q\,tg\,\phi) = 0 \end{cases} \qquad (3.6)$$

where F_c and K represent two scalar functions of the indicated arguments.

Before determining the exact value of q_u, by calculating the values of F_c and K, a lower bound was obtained for the bearing capacity, by using the static approach of the theory of yield design : applying the method of superposition :

$$q_u \geq (q_u)_{superp.} \qquad (3.7)$$

with :

$$\begin{cases} (q_u)_{superp.} = q + (C_o + q\,tg\,\phi)\,N'_c\left(\dfrac{B}{h}, \phi\right) \\ \qquad + \dfrac{1}{2}(\gamma + g\,cotg\,\phi)\,B\,N'_\gamma\left(\dfrac{B}{h}, \phi\right) \\ \text{when} \quad \phi > 0. \end{cases} \qquad (3.8)$$

(1) In order to avoid confusions and to make comparisons easier, the same notations will be used for the values and the magnitudes related to a circular foundation, as for those related to the strip footing of same characteristics, plus the exponent "o".

and

$$\begin{cases} (q_u)_{\text{superp.}} = q + C_o \, N'_c \, (\frac{B}{h}, 0) + \frac{1}{4} \, gB \\ \text{when} \quad \phi = 0. \end{cases} \tag{3.9}$$

Formulae (3.8) and (3.9) yield the exact value for q_u when either $(C_o + q \, tg \, \phi) = 0$, or $(g + \gamma \, tg \, \phi) = 0$. Using them requires the knowledge of the scalar functions : $N'_c \, (\frac{B}{h}, \phi)$ and $N'_\gamma \, (\frac{B}{h}, \phi)$ which have already been determined by MANDEL and SALENÇON (1969 and 1972). Formula (3.9) was proved by MATAR and SALENÇON (1977) using the demonstration which had been given by SALENÇON (1974) for the case of the foundation soil with an unlimited thickness.

Comparing the formulae (3.6) on the one hand, and (3.8) and (3.9) on the other hand, shows that :

$$K \, (\frac{B}{h}, \phi) = \frac{1}{2} \, N'_\gamma \, (\frac{B}{h}, \phi) \, \text{cotg} \, \phi$$
$$\text{for} \quad \phi > 0 \tag{3.10}$$

and

$$K \, (\frac{B}{h}, 0) = \frac{1}{4} \tag{3.11}$$

The numerical calculation of factors F_c was made through the theory of plane limit equilibriums (MATAR (1978), MATAR and SALENÇON (1979)) ; thus the exact value of the bearing capacity q_u is known in any case.

Charts have been given to make practical calculation of q_u easy. These charts are based on the use of factor μ_c defined by :

$$\mu_c \, (\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} \, B, \, \frac{B}{h}, \, \phi) = \frac{q_u - q}{(q_u)_{\text{superp.}} - q} \tag{3.12}$$

which characterizes the underestimation of the bearing capacity due to the superposition method.

Thus, by using triple entry charts, established for values of ϕ = 0°, 4°, 10°, 20°, 25°, 30°, 35°, 40°, 45° making it possible to read or to interpolate the values of the factors N'_c, $N'_\gamma \, \text{cotg} \, \phi$ and μ_c for $\phi > 0$, or the values of the factors N'_c and μ_c for $\phi = 0$, one can calculate the value of q_u by applying the following simple formulae :

$$\begin{cases} q_u = q + \mu_c (C_o + q \, tg \, \phi) \\ \times \left[\frac{1}{2} \, \frac{g + \gamma \, tg\phi}{C_o + q \, tg \, \phi} \, B \, N'_\gamma \, \text{cotg} \, \phi + N'_c \right] \\ \text{if} \quad \phi > 0 \end{cases} \tag{3.13}$$

$$\begin{cases} q_u = q + \mu_c \, C_o \left[\frac{1}{4} \, \frac{gB}{C_o} + N'_c \right] \\ \text{if} \quad \phi = 0. \end{cases} \tag{3.14}$$

Comments about these results have already been made in the various publications quoted before.

3.3 RESULTS FOR CIRCULAR FOUNDATIONS

The same process as has just been described can be followed for circular foundations.

The bearing capacity, which depends on the parameters defining the problem of figure 1, can be written :

$$q_u^o = f^o(C_o, \, g, \, B, \, h, \, \gamma, \, \phi, \, q) \tag{3.15}$$

The parameters can be grouped exactly the same way as in § 3.2, which results in the following formula, homologous to (3.4) :

$$q_u^o = q + f_2^o(C_o + q \, tg \, \phi, \, g + \gamma \, tg \, \phi, \, B, \, h, \, \phi) \tag{3.16}$$

Dimensional analysis yields:

$$\begin{cases} q_u^o = q + (C_o + q \, tg \, \phi) \\ \quad \times \, F_c^o \, (\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} \, B, \, \frac{B}{h}, \, \phi) \\ \text{if} \quad (C_o + q \, tg \, \phi) \neq 0 \end{cases} \tag{3.17}$$

with

$$\begin{cases} q_u^o = q + (g + \gamma \, tg \, \phi) \, B \, K^o \, (\frac{B}{h}, \, \phi) \\ \text{if} \quad (C_o + q \, tg \, \phi) = 0 \end{cases} \tag{3.18}$$

where F_c^o and K^o are scalar functions of the already indicated arguments.

Applying the static approach of yield design proves that a lower bound for the bearing capacity q_u^o is obtained by using the superposition method.

$$q_u^o \geqslant (q_u^o)_{\text{superp.}} \tag{3.19}$$

with

$$\begin{cases} (q_u^o)_{\text{superp.}} = q + (C_o + q \, tg \, \phi) \, N'^o_c \, (\frac{B}{h}, \, \phi) \\ \quad + \frac{1}{2} \, (\gamma + g \, \text{cotg} \, \phi) \, B \, N'^o_\gamma \, (\frac{B}{h}, \, \phi) \\ \text{when} \quad \phi > 0 \end{cases} \tag{3.20}$$

and

$$\begin{cases} (q_u^o)_{\text{superp.}} = q + C_o \, N'^o_c \, (\frac{B}{h}, 0) \\ \quad + K^o \, (\frac{B}{h}, 0) \, gB \\ \text{when} \quad \phi = 0 \end{cases} \tag{3.21}$$

From the very definition of the superposition method, formulae (3.20) and (3.21) yield the exact value of q_u^o when either $(C_o + q\,\mathrm{tg}\,\phi) = 0$, or $(g + \gamma\,\mathrm{tg}\,\phi) = 0$. Using these formulae, requires the knowledge of the factors $N_c'^{o}(\frac{B}{h}, \phi)$, $N_\gamma'^{o}(\frac{B}{h}, \phi)$ and $K^o(\frac{B}{h}, 0)$.

It follows that those factors can be easily determined by calculating the bearing capacity q_u^o of the circular foundation for each of the following three simple cases :

1) Foundation acting on a homogeneous weightless soil layer, without lateral surface load (Figure 2).

2) Foundation acting on a cohesionless, homogeneous, weighted soil layer, without lateral surface load (Figure 3).

3) Foundation acting on a purely cohesive soil layer, $(\phi = 0)$, non homogeneous with a cohesion linearly increasing with depth, and without lateral surface load (Figure 4).

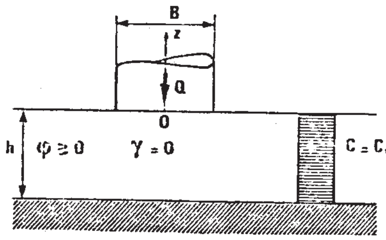

Fig.2 Determination of $N_c'^{o}(\frac{B}{h}, \phi)$.

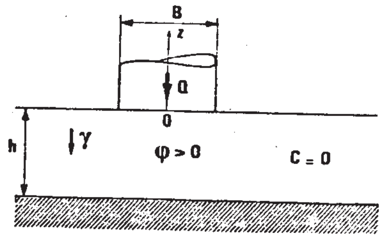

Fig.3 Determination of $N_\gamma'^{o}(\frac{B}{h}, \phi)$.

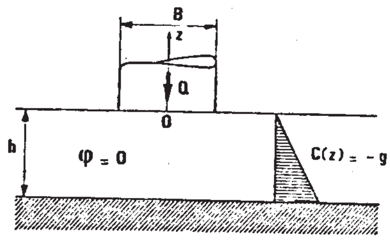

Fig.4 Determination of $K^o(\frac{B}{h}, 0)$.

The first two problems were studied by SALENÇON et al. (1973) through the theory of limit equilibrium in axial symmetry, within the frame of the HAAR-KARMAN hypothesis : the solutions thus obtained [1] made it possible to determine, for some values of ϕ, the factors $N_c'^{o}(\frac{B}{h}, \phi)$ and $N_\gamma'^{o}(\frac{B}{h}, \phi)$.

The comparison between (3.18) and (3.20) shows that, as for the case of strip footings, the following relation holds :

$$K^o(\frac{B}{h}, \phi) = \frac{1}{2} N_\gamma'^{o}(\frac{B}{h}, \phi) \;.\; \mathrm{cotg}\,\phi \qquad \text{when } \phi > 0 \tag{3.22}$$

As for the determination of $K^o(\frac{B}{h}, 0)$, which appears in (3.18) and (3.21), it requires the solution of the third problem stated before. The solution of this problem leads to the following new result :

$$K^o(\frac{B}{h}, 0) = \frac{1}{6} \quad \forall\, B/h \tag{3.23}$$

and to the superposition formula for $\phi = 0$

$$\begin{cases} q_u^o = q + C_o N_c'^{o}(\frac{B}{h}, 0) + \frac{1}{6}\, gB \\ \text{when } \phi = 0 \end{cases} \tag{3.24}$$

The use of the global formula (3.17), which takes into account all coupling effects between the groups of parameters, requires the determination of the factor F_c^o, as a function of these arguments. The exact value of the bearing capacity q_u^o will then be obtained through (3.17).

In order not to make this paper too long, we shall leave aside any discussion related to the method which has been used to determine F_c^o, and to the extensive numerical calculations which have been required to find this value.

In a few words:use is made again of the theory

[1] These solutions are similar to those given by most authors for this type of problem : stress fields obtained by the characteristics method in a zone spreading under the foundation in the soil layer, and emerging at the free surface. To be quite rigorous, and in order to show that this solution really gives the value of $N_c'^{o}$ or $N_\gamma'^{o}$, it would be necessary to complete the stress field in the entire soil layer, respecting the compatibility "equilibrium-bearing capacity", and to build a velocity field associated to this stress field by using the mathematic rule of normality (v.z. theory of yield design); for reason of simplicity this question will be left aside here and afterwards for the determination of K^o and F_c^o.

of limit equilibriums in axial symmetry, with the HAAR-KARMAN hypothesis (which will be justified a posteriori by the fact that it makes it possible to construct a solution). This leads to solving a hyperbolic problem by using the characteristic method. Greater difficulties are encountered than in the case of a strip footing, which are due to the presence of singularities on the axis Oz ($r = 0$). Moreover, the amount of calculations needed to determine the function F_c^o is much greater than that needed to determine F_c in the same value ranges for the various arguments.

4. THE OBTAINED RESULTS

4.1 THE SHAPE FACTOR

A calculation programme has then been devised to perform the calculation of F_c^o in any case.

As a matter of fact, this programme has been designed so as to give results that are easy to use practically for the determination of q_u^o, which is obviously linked to the way of presenting the results.

Two remarks have helped to decide how to present these results :

1) Experience has proved that, in the case of strip footings, triple entry charts are very convenient ;

2) Foundations of the "strip footing" type and circular foundations represent, in a way the two extreme geometrical cases, between which are found the foundations which are being built in actual practice : it would then be interesting to find a means of presenting the results which would make it possible to calculate the bearing capacity in both cases almost simultaneously.

Thus, after having thought of a presentation, similar to the one that has been adopted for strip footings, and which would have used triple entry charts for reading or interpolating the values of the coefficients $N_c'^o$, $N_\gamma'^o \operatorname{cotg}\phi$, and μ_c^o (homologous to μ_c), it was considered more advisable to present the results in axial symmetry, making use of the shape factor ν defined as :

$$\nu\left(\frac{g+\gamma \operatorname{tg}\phi}{C_o + q \operatorname{tg}\phi} B, \frac{B}{h}, \phi\right) = \frac{q_u^o - q}{q_u - q} \qquad (4.1)$$

where q_u^o and q_u represent the bearing capacities under identical conditions of two foundations, of equal width, one of the circular type and the other of the strip footing type.

The calculation programme has been arranged so as to lead to a determination of ν, which That made it necessary to perform the "plane" calculations and the calculations in axial symmetry at the same time ; it should be noted that this proved to be difficult because the calculation procedures are not identical in both cases.

The results are then presented in quadruple entry charts, established for $\phi = 0°, 4°, 10°, 20°, 25°, 30°, 35°, 40°, 45°$,

where N_c', $N_\gamma' \operatorname{cotg}\phi$, μ_c and ν (Figure 5)* can be read or interpolated.

One can thus determine first the bearing capacity for the strip footing, by means of the formulae (3.13) and (3.14) already written, then the bearing capacity of the circular foundation of equal width, by using the formula (4.1), i.e.:

$$\begin{cases} q_u^o = q + \nu\,\mu_c (C_o + q \operatorname{tg}\phi) \left[\dfrac{1}{2}\dfrac{g+\gamma \operatorname{tg}\phi}{C_o + q \operatorname{tg}\phi} B N_\gamma' \operatorname{cotg}\phi + N_c'\right] \\ \text{if } \phi > 0 \end{cases} \qquad (4.2)$$

$$\begin{cases} q_u^o = q + \nu\,\mu_c\, C_o \left[\dfrac{1}{4}\dfrac{gB}{C_o} + N_c'\right] \\ \text{if } \phi = 0 \end{cases} \qquad (4.3)$$

4.2 CHARTS ISO-ν

The adopted presentation thus consists in adding to the charts which had been given before for calculating the bearing capacity of strip footings, new charts to determine the value of the shape factor ν.

This factor, like μ_c, depends on the three arguments :

$$\frac{g+\gamma \operatorname{tg}\phi}{C_o + q \operatorname{tg}\phi} B, \quad \frac{B}{h}, \text{ and } \phi.$$

Therefore, as is shown on figure 5*, charts iso-ν have been drawn for each of the values of ϕ listed hereabove : these charts are constituted by lines of equal value to the factor ν and drawn according to the arguments :

* Figure 5 is related to the charts included in the preceding paper, by MATAR-SALENCON, figure 27 to figure 34.

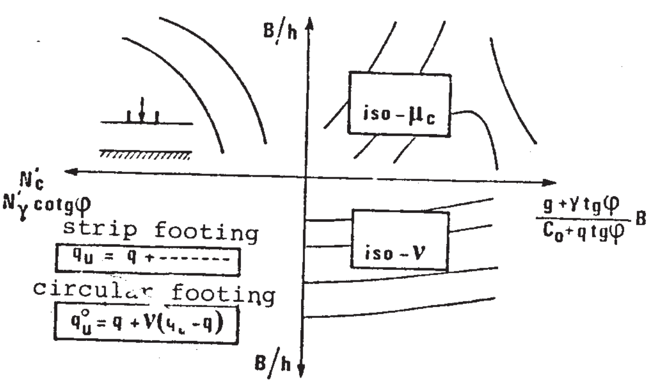

Fig.5 Scheme of the charts for consolidation of the bearing capacity.

$$\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} B, \frac{B}{h}, \phi .$$

4.3 PROPERTIES OF THE SHAPE FACTOR ν

1) ϕ and $\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} B$ being given

ν is a decreasing function of $\frac{B}{h}$.

This means particularly that the shape factor is at its maximum when the foundation acts on a soil layer of unlimited thickness.

2) ϕ and $\frac{B}{h}$ being given

ν is a decreasing function of

$$\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} B$$

3) When $\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} B = 0$, the factor ν reduces to the ratio :

$$\nu \left(0, \frac{B}{h}, \phi\right) = N_c^{\prime o}\left(\frac{B}{h}, \phi\right) / N_c^{\prime}\left(\frac{B}{h}, \phi\right) \quad (4.4)$$

which can be termed as : "shape factor for the cohesion in the superposition formula".

Similarly, for $\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} B \uparrow \infty$, we get :

$$\nu \left(\infty, \frac{B}{h}, \phi\right) = N_\gamma^{\prime o}\left(\frac{B}{h}, \phi\right) / N_\gamma^{\prime}\left(\frac{B}{h}, \phi\right) \quad (^1) \quad (4.5)$$

"shape factor for gravity in the superposition formula".

In particular, for $B/h = 0$, i.e. for a soil layer of unlimited thickness, we have

$$\nu(0, 0, \phi) = N_c^o(\phi) / N_c(\phi) \quad (4.6)$$

and

$$\nu(\infty, 0, \phi) + N_\gamma^o(\phi) / N_\gamma(\phi) \quad (^1) \quad (4.7)$$

The maximum value of the shape factor is thus :

$$\nu(0, 0, \phi) = N_c^o(\phi) / N_c(\phi)$$

4) The shape factor for the cohesion in the case of a soil layer of unlimited thickness which is the maximum value of ν, is an increasing function of ϕ. Its variation is drawn on figure 6. It is always superior to 1.

The shape factor for gravity in the case of soil layer of unlimited thickness is also an increasing function of ϕ. Its variation is drawn on figure 7.

The range of variations of ν with respect to its first two arguments increases with ϕ.

5) For a same value of ϕ, according to the values of the first two arguments, the shape factor can be superior or inferior to 1.

In particular, for a foundation on a soil layer of undefinite thickness, the shape factor is always superior to 1 only when $\phi > 29°$.

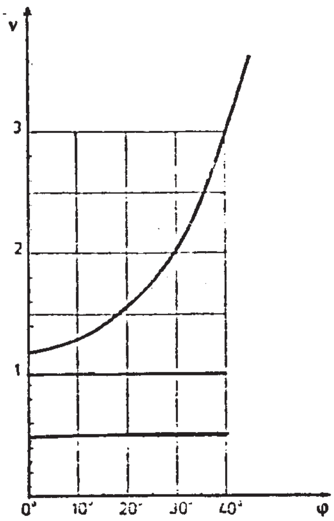

Fig.6 Variation of $\nu(0,0,\phi)$ vs. ϕ.

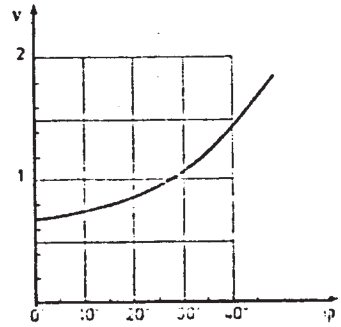

Fig.7 Variation of $\nu(\infty,0,\phi)$ vs. ϕ.

6) It has been shown (see e.g.: MATAR and SALENÇON, 1979), that, in the case of strip footings, there exists a value $\left(\frac{B}{h}\right)_o$, function of ϕ and of $\frac{g + \gamma \, tg \, \phi}{C_o + q \, tg \, \phi} B$ below which the bearing capacity is independent of B/h :

if $\frac{B}{h} < \left(\frac{B}{h}\right)_o$

then q_u is independent of $\frac{B}{h}$.

(^{1}) When $\phi = 0$, this formula still holds as :

$$N_\gamma^{\prime o} \cot g \, \phi / N_\gamma^{\prime} \cot g \, \phi = N_\gamma^{o} \cot g \, \phi / N_\gamma \cot g \, \phi = \frac{1}{6} / \frac{1}{4} = \frac{2}{3}$$

Similarly, for a circular foundation, there exists a value $(\frac{B}{h})^{o}_{o}$ with the same property :

$$\text{if} \quad \frac{B}{h} < (\frac{B}{h})^{o}_{o}$$

then q^{o}_{u} is independent of $\frac{B}{h}$.

Moreover, the following property holds : for any given ϕ , for the same value of the argument $\frac{g + \gamma \operatorname{tg} \phi}{C_o + q \operatorname{tg} \phi} B$, we always have :

$$(\frac{B}{h})_{o} < (\frac{B}{h})^{o}_{o}$$

From which it results that the shape factor ν is independent of B/h as soon as $B/h < (B/h)_o$.

The representative curve of $(B/h)_o$ as a function of $\frac{g + \gamma \operatorname{tg} \phi}{C_o + q \operatorname{tg} \phi} B$, is drawn on the charts iso-μ_c and iso-ν.

5. *CHARTS FOR THE PRACTICAL CALCULATION OF THE BEARING CAPACITY OF SHALLOW FOUNDATIONS*

ϕ = 0°
4°
10°
20°
25°
30°
35°
40°
45°

6. *EXAMPLES OF THE USE OF THE CHARTS*

6.1 INTRODUCTION

To illustrate the use of the above-given charts, we intend to select some of the cases which have been studied in the reference (MATAR and SALENÇON, 1979), with the main objective of comparing strip footings to circular footings.

6.2 THE SITES WHICH WERE STUDIED

6.2.1 Site A

- Soil with internal friction angle : ϕ = 0°, then 4°, 10°
- Foundation width : B = 40 m, then 4 m
- Lateral surface load : q = 0
- Layer thickness : h = 10 m
- Surface cohesion : $C_o = 10^3$ Pa
- Downward vertical cohesion gradient : $g = 2.5 \times 10^3$ N/m³
- Unit weight : $\gamma = 1.6 \times 10^4$ N/m³

(B = 40 m , ϕ = 0°, 4°, 10° : cases A1, A2, A3 ;
B = 4 m , ϕ = 0°, 4°, 10° : cases A4, A5, A6).

6.2.2 Site B

- Soil with internal friction angle : ϕ = 0°, then 4°, 10°
- Foundation width : B = 40 m
- Lateral surface load : q = 0
- Layer thickness : h = ∞
- Surface cohesion : C_o = 0
- Downward vertical cohesion gradient : $g = 0.6 \times 10^3$ N/m³
- Unit weight : $\gamma = 1.6 \times 10^4$ N/m³

6.2.3 Site C

- Soil with internal friction angle : ϕ = 30°
- Foundation width : B = 4 m
- Lateral surface load : $q = 1.8 \times 10^4$ Pa
- Layer thickness : h = ∞
- Surface cohesion : $C_o = 1.6 \times 10^4$ Pa
- Downward vertical cohesion gradient : g = 0
- Unit weight : $\gamma = 1.8 \times 10^4$ N/m³

6.3 TABLE OF RESULTS

The reading of charts for the values of the arguments indicated in columns 2 to 4, makes it possible to determine the factors $N'_\gamma \operatorname{cotg} \phi$, N'_c, μ_c, and ν :

CASES	ϕ	B/h	$\frac{g + \gamma \operatorname{tg} \phi}{C_o + q \operatorname{tg} \phi} B$	$N'_\gamma \operatorname{cotg} \phi$	N'_c	μ_c	ν
A1	0°	4	100	0,5	6,2	1,48	0,77
A2	4°	4	145	1,12	8	1,15	0,76
A3	10°	4	213	2,46	14,5	1,07	0,78
A4	0°	0,4	10	0,5	5,14	1,65	0,9
A5	4°	0,4	14,5	1,12	6,19	1,42	0,87
A6	10°	0,4	21,3	2,46	8,34	1,30	0,87
B1	0°	0	∞	0,5	5,14	1	0,67
B2	4°	0	∞	1,12	6,19	1	0,70
B3	10°	0	∞	2,46	8,34	1	0,74
C	30°	0	1,58	25,55	30,14	1,20	1,57

the bearing capacity being thus:

CASES	(q_u) superp. in 10^4 Pa	q_u^o in 10^4 Pa	q_u in 10^4 Pa
A1	3,12	3,56	4,62
A2	8,96	7,8	10,3
A3	27,38	22,9	29,3
A4	0,76	1,13	1,26
A5	1,43	1,77	2,03
A6	3,45	3,91	4,49
B1	0,60	0,40	0,60
B2	3,85	2,70	3,85
B3	16,45	12,62	16,83
C	133	252	161

6.4 COMMENTS

These results demonstrate how easy it is to use charts for the calculation of the bearing capacity, whether for strip footings or for circular footings.

If we compare one to another the cases A1, A2 and A3 ; A4, A5 and A6 ; B1, B2 and B3, we can notice the stabilizing influence of the internal friction angle : as in the case of strip footings, for a purely cohesive soil, if an internal friction angle can be assumed, even very small (a few degrees), the bearing capacity of a circular footing is considerably increased. This adds to the security of the calculation.

In the case of a purely cohesive soil (site A, cases A1 and A4), it may be noticed that, for both cases, the bearing capacity of a circular footing is lower than that of a strip footing. The wider the foundation, the more noticeable this effect, because of the sense of variation of ν . It is also linked to the presence of the cohesion gradient and to the limited thickness of the soil layer : upon a purely cohesive, homogeneous and unlimited soil, the bearing capacity of a circular footing is superior to that of a strip footing ($\nu = 1.15$).

In the case of a purely cohesive soil, with zero cohesion at the surface, (site B, case B1) which can be assumed as a model for some marine soils, we are now able to calculate the bearing capacity q_u^o for a circular footing. If there is no surface load, this capacity is in the ratio $\nu = 2/3$, with that of a strip footing.

It can be instructive to compare the results that can be obtained with the charts given in chapter 5, for non-standard cases which can be found practically, with the bearing capacity evaluations, which could be obtained by means of empiric rules.

These rules are based on the classical results available in the case of plane problems, that is for strip footings ; they refer to the bearing capacity factors determined in the case of a homogeneous layer : as a matter of fact, it is an application of the superposition method in the plane case, which introduces, instead of a cohesion increasing linearly with depth, a constant cohesion. One will then take either the surface cohesion C_o , either an "average" cohesion to be determined as it has been explained in MATAR and SALENÇON (1979).

We get :

- for site A (case A1) :

  with the surface cohesion :
  $$(q_u)_o = 0.625 \times 10^4 \text{ Pa}$$
  with an average cohesion :
  $$\overline{q_u} = 8.44 \times 10^4 \text{ Pa}$$
  to be compared with :
  $$q_u^o = 3.56 \times 10^4 \text{ Pa}$$

- for site A (case A4) :

  with the surface cohesion :
  $$(q_u)_o = 0.51 \times 10^4 \text{ Pa}$$
  with an average cohesion :
  $$q_u = 2.33 \times 10^4 \text{ Pa}$$
  to be compared with :
  $$q_u^o = 1.13 \times 10^4 \text{ Pa}$$

- for site B (case B1) :

  with the surface cohesion :
  $$(q_u)_o = 0$$
  with an average cohesion :
  $$\overline{q_u} = 4.36 \times 10^4 \text{ Pa}$$
  to be compared to :
  $$q_u^o = 0.40 \times 10^4 \text{ Pa.}$$

One should notice the underestimation introduced by the first method, which can even make any building impossible and the unacceptable overestimation brought in by the latter.

As concerns the example of a soil with friction, which has been studied ($\phi = 30°$), it should be noticed that the bearing capacity of the circular footing is significantly more important than that of the similar strip footing (56.5 %).

This effect, which is evidenced here in the case of a soil of unlimited thickness, appears as long as the thickness of the soil layer remains sufficient, which means as long as the presence of a rigid bedrock influences but little the capacity of the strip footing.

7. CONCLUSION

This study aimed at providing engineers an easy way of calculating bearing capacity of shallow foundations, particularly under conditions of non-homogeneity which occurs in practice and for which the presently available methods often prove insufficient.

This aim has been reached with the calculation of the "shape factor", defined as the ratio of the bearing capacities for two foundations, one of them circular and the other one of the "strip footing" type, of equal width and under identical conditions.

The charts representing this factor have then been drawn for the values of ϕ corresponding to practical needs ($0° < \phi < 45°$). By adding these charts to the charts given previously by the same authors, for a strip footing, one can immediately determine the bearing capacity of circular shallow foundations or of "strip footing" foundations ; this is an answer, whether direct or by interpolation, to the needs of the engineers.

Similarly to the case of strip footings, where a comparison had been given between the results of the theoretical calculations and the results of experiments led on small-scale models, it would also be interesting to study, on a small-scale model, taking into account the results evidenced by the theory, the variation of the shape factor, not only in function of the parameters listed hereabove but also in function of the actual shape of the foundation: for example, square foundation, rectangular foundation, etc...
Finally, attention should be drawn to the fact that the whole analysis has been led assuming perfect adhesion at the interfaces between the soil layer and the rigid base, and between the foundation and the soil layer. Of those two assumptions the latter is likely to be valid in any case, whereas it appears that the former may be, sometimes, questionable, for instance due to the fact that a thin layer of poorer quality might be encountered at the contact with the rigid base. It is very difficult to give more precise ideas about that topic because of the great number of physical parameters that are likely to play a role : the nature of the soil itself, the thickness of the layer, and the level of the applied stresses for instance.

Generally speaking, the strength criterion at the interface between the soil layer and the rigid base is somewhat between the two extreme cases of perfect adhesion on the one hand, and perfect smoothness on the other. Let the bearing capacities in the latter case be denoted q'_u and $q^{o\prime}_u$, it is clear that these may be written :

$$q'_u - q = \lambda\left[\frac{g + \gamma\, tg\,\phi}{C_o + q\, tg\,\phi} B, \frac{B}{h}, \phi\right] (q_u - q)$$

and :

$$q^{o\prime}_u - q = \lambda^o\left[\frac{g + \gamma\, tg\,\phi}{C_o + q\, tg\,\phi} B, \frac{B}{h}, \phi\right] (q^o_u - q)$$

where λ and λ^o are scalar functions of the indicated arguments : λ and $\lambda^o < 1$.

The work done by MANDEL and SALENCON (1972) makes it possible to determine $\lambda\,(0, B/h, \phi)$ for various values of ϕ in the range 0°, 40°. It appears that the reduction of the bearing capacity in the case of a perfectly smooth interface may be important, and that this effect increases with the values of B/h and ϕ. As a matter of fact, since the assumption of a perfectly smooth interface is a very pessimistic one, it appears that the indications derived from the values of $\lambda\,(0, B/h, \phi)$ are sufficient for practical applications. What will be retained is that the results of the charts should be used with great care in the cases of high values of B/h and ϕ, which would correspond to very high values for factors N'_c and $N'_\gamma \cot g\,\phi$ that are practically irrelevant.

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