FOUNDATION ENGINEERING

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Volume 1

Soil properties -
Foundation design and construction

Foundation Engineering

This book is a review of the French modern practice in soil studies ; it deals with building foundation design and construction.

More specifically, it takes into account the know-how acquired and the research and development results obtained during the past few years. It deals first with the determination of soil properties and puts the emphasis on in situ tests, among which the pressuremeter test, because of their common use today in France. The second part of the book is devoted to shallow foundations, including a general solution to the bearing capacity problem ; it presents analytical methods for calculating stability and settlement on the basis of in situ test data. Comparisons between predicted and measured settlement of completed structures are also made.

Finally deep foundations are dealt with, beginning with the theoretical approach to pile performance under vertical and lateral loads. This is followed by a discussion of methods for bearing capacity calculation derived from in situ tests and the static loading test. This book also describes construction techniques especially for bored piles and details instrumentation for deep foundation and methods of improving the defects in the structure.

Bearing capacity of strip footings*

M. Matar, J. Salençon

ABSTRACT

The paper is concerned with the bearing capacity of a strip-footing resting on a soil layer of limited or unlimited thickness, the cohesion of which increases linearly with depth.

The bearing capacity is defined through the theory of yield-design. A linear formula is given extending the superposition method in order to take the cohesion gradient into account and yielding a conservative value for the bearing capacity.

A global analysis of the problem is then performed. It leads to the determination of the exact value of the bearing capacity. This enables to appreciate how the value of the bearing capacity is underestimated through the use of the superposition method.

Multi-entry charts are proposed which makes it possible to calculate the exact value of the bearing capacity easily.

The interest of these results is attested by various examples of calculations of bearing capacities. For instance let us mention that, for a soil with a cohesion gradient, methods such that "taking only the surface cohesion into account" or "use of an average value of the cohesion" may prove to be either too much conservative, up to the point of making construction impossible, or on the contrary very risky and even dangerous.

1. INTRODUCTION

Surface foundation designing is usually
based on two criteria :
settlement and stability.

- The settlement criterion is used to ob-
  tain the admissible load for the foun-
  dation with the condition that a certain
  settlement, absolute or differential,
  does not exceed a value considered accep-
  table for the good holding of the rele-
  vant work. The load-settlement ratio,
  required for the use of this sizing cal-
  culation criterion, is obtained either
  by calculations that for the most part
  use a model having an isotropic linear
  elastic behavior or by formulae brought
  into relation with the field tests.

- As for the second criterion, it is the
  foundation "stability" that concerns
  us here and the admissible load is limi-
  ted for this latter to a fraction of
  the bearing capacity. This concept of
  bearing capacity is derived from an ana-
  lysis of the foundation "at failure"
  based on the criterion's datum charac-
  terizing the foundation's soil "strength".

In the following we will be more especial-
ly concerned with the bearing capacity
aspects as it relates to "strip-footing"
type foundations, axially loaded. However,
certain of the inferences used will be
of a more general nature and can be trans-
posed without any difficulty to other
types of foundations and to other types
of loads.

Various commonly used formulae enable the
bearing capacity of strip-footing to be
ascertained from those parameters that
characterize soil strength which are inte-
gral parts of the classic criteria of
COULOMB (C, ϕ) or of TRESCA (C) for unli-
mited and homogeneous foundation soils (1).

Based upon this bearing capacity estimate,
the admissible load for a foundation is
limited by introducing a safety factor
which is to take the following aspects
into consideration (fig. 1).

   1. Modelizing, which is that which con-
verts the practical problem
" design a real foundation to ensure the
proper working of the relevant work"
into schematic problem wherein
" the foundation is assumed to be of the
strip-footing type, infinitely rigid, sub-
jected to a deterministic uniaxial load
resting on an unlimited homogeneous soil
layer"
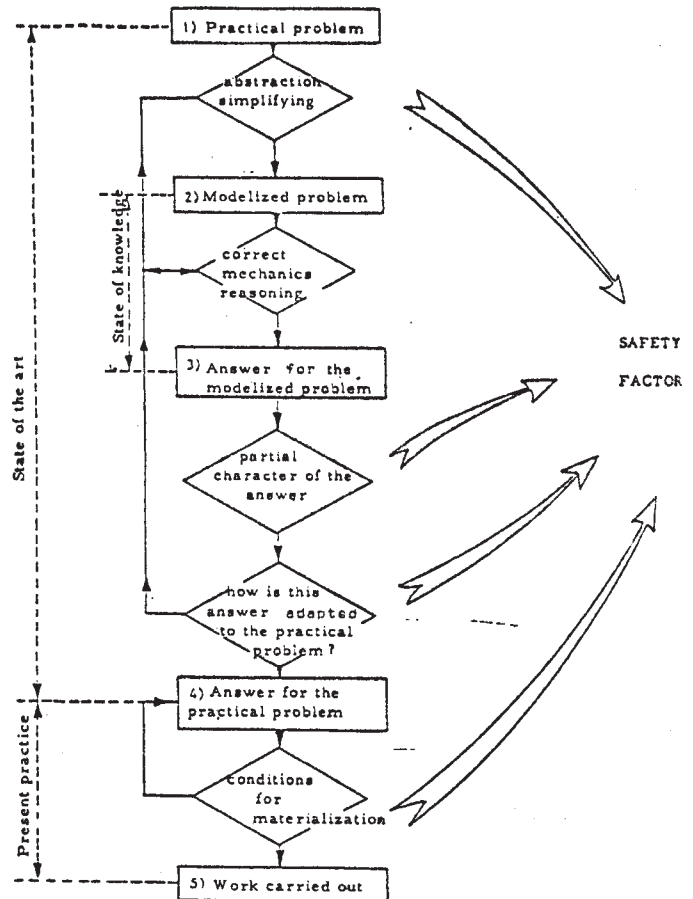
1. Design a foundation to ensure the
   proper working order of the work.

2. Strip-footing, rigid ; soil yield crite-
   rion ; deterministic load mode.

3. Multi-entry charts to compute bearing
   capacity

4. Foundation sizing.

5. Foundation laid (site, various condi-
   tions contractors...).

Fig.1 Diagram of reasoning processes related
to bearing capacity of superficial foundation

   2. The fact the solution obtained from
the modelized problem is more or less li-
mited as it accounts for only one aspect
of soil behavior, i. e. strength.

   3. The adaptation of this partial solu-
tion from the modelized problem to the
question raised by the concrete practical
problem.

Let us note in passing that the second
point brings one back to a purely mecha-
nics problem to whith can be applied ri-
gorous arguments enabling one to give
exact values to the results obtained.

The purpose of the work described here is to advance the state of knowledge concerning the above modelized problem, both for the classic case of an unlimited homogeneous foundation soil and for the case of a soil layer of limited thickness that has cohesion that varies with depth. This means that for this problem, results, of which the significance in mechanics is more exact than those actually possible when they exist, are obtained. For soil mechanics this depends on the theory of yield design (cf. e. g. SALENÇON, 1978) and on the new calculations made by MATAR (1978).

In addition, a comparison is made with the laboratory results on scale model of TOURNIER (1972) and of TOURNIER and MILOVIC (1977) in an attempt to calibrate, vis-à-vis this modelized problem, the partial character of the obtained results, that is, the significance, vis-à-vis this bearing capacity problem thusly calculated. The theory, in fact, causes the bearing capacity to appear as the "extreme" load for the foundation and it is advisable to situate it, by experimentation, in relation to the loads that bring about the ruin of the foundation. It is thereby possible to appreciate, among other factors, the importance of the form assumed for the yield criterion for the soil, of the pre-failure distorsions, etc.

The paper will deal only with the modelized problem and is as follows.

Chapter 2 is devoted to defining the concept of bearing capacity while showing that the problem posed is a typical yield design problem and by underscoring that the traditionally used superposition method is a direct application thereof.

In chapter 3, the case of a foundation on a soil layer of limited thickness and having a cohesion varying linearly with depth is analyzed and it is further shown that it is possible to apply the superposition method to it yielding, by means of previously known results, a non-trivial lower bound approximation of the bearing capacity.

Chapter 4 offers a global analysis of the problem, leading to the use of new computations that supply the value of the bearing capacity taking the effects of coupling between all the problem's parameters into consideration.

Then in chapter 5 these results are given in multi-entry chart form so as to be easily useable.

Chapter 6 gives various examples of their use, revealing, under certain conditions, the importance of those coupling effects neglected by the ordinary formulae for classic cases and by showing, for the non-classic case of a soin layer of varying cohesion, the drawbacks, and even dangers, that might be entailed by the use of cer-

tain empirical rules which, moreover, are frequently rather impractical.

In chapter 7, the comparison with the tested scale models is considered.

For certain aspects of our discussion we will be intentionally brief, notably concerning the developed computing programs and digital computations carried out. Should the reader desire further information he can, for the most part, find this in MATAR (1978).

2. THE CONCEPT OF BEARING CAPACITY

2.1. Classic formulae

As already mentioned, numerous authors have proposed formulae to determine the bearing capacity of superficial foundations of the strip-footing type resting on an unlimited homogeneous soil. In these formulae, the soil is characterized by its yield criterion (cf. SALENÇON and HALPHEN, 1979). With σ and τ designating the normal and tangential components of the stress vector T acting on the facet with and inward normal n (fig. 2), is used :

for fully cohesive soils (short term behavior), the TRESCA criterion :

$$(2.1) \qquad |\tau| \leqslant C , \forall \ \underline{n}$$

for other soils, the COULOMB criterion :

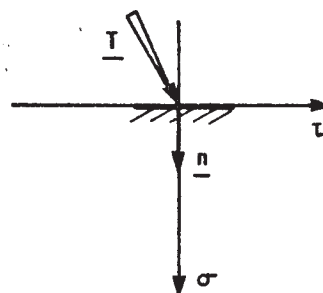$$(2.2) \qquad |\tau| < C + \sigma \ tg \ \varphi, \ \forall \ \underline{n} \qquad (\varphi \neq 0).$$

Fig.2 Conventional stress signs

The formulae proposed to determine the bearing capacity of a strip-footing, with a width B, an axially loaded force Q (per unit running length) acting on a soil of a unit weight γ, with a uniform lateral surface load q (fig. 3), are based on the superposition method, the conservativeness of which is acknowledged and which results in the breaking down of the bearing capacity into three terms in the form of (TERZAGHI, 1943) :

$$(2.3) \qquad (q_u)_{superp.} = \frac{1}{2} \gamma B N_\gamma (\varphi) + C N_c (\varphi) + q N_q (\varphi)$$
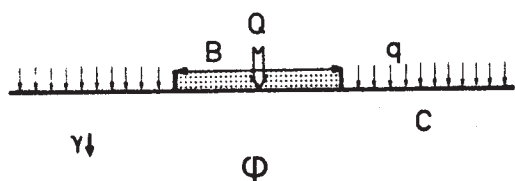
Fig.3 Strip-footing on homogeneous
      unlimited soil

In this fashion, computation of the bea-
ring capacity requires, as results from
equation 2.3, that the scalar coefficients
$N_\gamma(\phi)$, $N_c(\phi)$ and $N_q(\phi)$, entering into the
surface, cohesion and surface load terms,
be known. The values of the coefficients
$N_c(\phi)$ and $N_q(\phi)$ are now practically the
same for all the authors, however, diffe-
rences continue as concerns $N_\gamma(\phi)$.

2.2. Definition of bearing capacity

It is obvious that before starting any dis-
cussion concerning the validity of a for-
mula or concerning the values of the coef-
ficients to be used, the very notion of
bearing capacity needs be defined. Further-
more, it will be seen that once this defi-
nition found and stated, the difficulties
relating to the interpretation of the so-
lutions and of the values proposed by the
various authors for $N_\gamma(\phi)$ will disappear
by themselves.

Recalling what has been said about this
subject and from equation 2.3, it is seen
that the bearing capacity of a foundation
depends, as far as soil behavior is con-
cerned, only on the parameters C and ϕ
which characterize the soil's "strength".
This is an indicator of the logical proce-
dure followed when dealing with bearing
capacity. Based on the only knowledge of
the strength capacities of the soil ele-
ment (from the macroscopie viewpoint of
continuous medium mechanics), information
concerning the loads that the studied
superficial foundation can support will be
sought out. The considered load is
composed of the surface load q and the soil's
weight (unit weight) -stabilizing elements-
and the actual load applied to the foun-
dation.

This procedure was followed by all the au-
thors for the problems related to founda-
tion bearing capacity. It is the same type
of ressoning which, ever since COULOMB
(1773), has also been followed concerning
slope stability and ground thrust and pres-
sure. It is typical of yield design reaso-
ning.

The soil's strength capacities at a point $\underline{x}$
are characterized by a criterion, imposed
condition at the stress state $\underline{\underline{\sigma}}(\underline{x})$ at this
point so as to be able to be supported
there by the material element and which is
defined by a scalar function f of $\underline{\underline{\sigma}}(\underline{x})$ (1).

(2.4) $\underline{\underline{\sigma}}(x)$ supported by the ground element

$$<=> f(\underline{x}, \underline{\underline{\sigma}}(\underline{x})) \leqslant 0$$

From these data, the maximum load that the
work being studied may support is sought
arter. This is obviously done without it
being possible to state, unless supplemen-
tary information on the behavior is forth-
coming, that the work will actually support
this maximum load.

This leads to the definition of the bearing
capacity. In fact, by situating oneself
within the initial geometry of the problem
and assuming it to be invariable, it is clear
that in order that the foundation support
a load Q, there needs be a stress field $\underline{\underline{\sigma}}$
which equilibrates the load made up of
(Q, γ, q) while the strength criterion be-
ing satisfied everywhere (soil and inter-
face). It turns out this condition, needed
so as to be capable of being supported by
the foundation, limits the Q forces to a
straight line segment (O, Q^+) (fig. 4).

Fig.4 Forces likely to be supported
      by the foundation within a
      given geometry

Thus :

(2.5) $\begin{cases} \text{F supported by the foundation (in} \\ \quad \text{initial geometry)} \\ \exists \underline{\underline{\sigma}} : \underline{\underline{\sigma}} \text{ equilibrates (Q, } \gamma, \text{ q)} \text{ , and} \\ f(\underline{x};\underline{\underline{\sigma}}(\underline{x})) \leqslant 0 \, \forall \, \underline{x} \end{cases}$

$$Q \in (O, Q^+)$$

It is important to analyze the signifi-
cance of this result.

- It is obviously necessary that the pro-
  blem's geometry be given and invariable,
  as the reasoning, from the point of
  view of behavior, only refers to the
  "strength characteristics", that is, to
  the soil yield criterion.

- From these data, the stated result which
  is based on the convexity of the criteria
  (2.1) and (2.2) is the strongest that can
  be obtained.

(1) Tensors are underlined in a number equal
to their order. We will often interchange
the field $\underline{\underline{\sigma}}$ and its local value $\underline{\underline{\sigma}}(\underline{x})$, but
this will cause no ambiguity.

As a result, it is quite natural to define the bearing capacity of the foundation by the average value :

(2.6) $q_u = Q^+/B$

2.3. Static approach ; superposition method

It immediately follows from this definition of the bearing capacity that the bringing forth of a stress field $\underline{\sigma}$, statically admissible and satisfying everywhere the soil strength criterion (by lower bound approximation) for the bearing capacity (this is the static theorem of yield design).

For instance, taking the three load cases illustrated in figure 5 and assuming that the following stress fields are available :

a) $\underline{\sigma}_\gamma$, statically admissible and satisfying the strength criterion of the material element everywhere for the weighted, non-cohesive and unloaded medium (fig. 5a) corresponding to the average pressure under the foundation : $1/2\gamma BN_\gamma(\phi)$.

b) $\underline{\sigma}_c$, statically admissible and satisfying the strength criterion of the material element everywhere for the cohesive, weightless and unloaded medium (fig. 5b), corresponding to the average pressure under the foundation : $CN_c(\phi)$.

c) $\underline{\sigma}_q$, statically admissible and satisfying the strength criterion of the material element everywhere for the loaded, weightless and non-cohesive medium (fig. 5c), corresponding to the average pressure under the foundation : $qN_q(\phi)$.
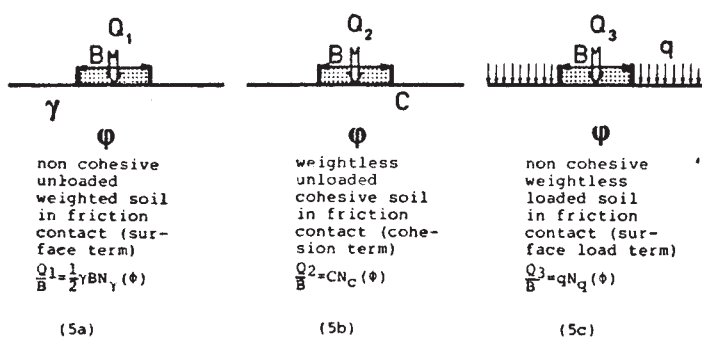

Fig.5 Superposition method

By means of the MOHR representation for instance, it is possible to immediately verify that the stress field (1) :

$$\underline{\sigma} = \underline{\sigma}_\gamma + \underline{\sigma}_c + \underline{\sigma}_q$$

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(1) This result is based on the fact that the domain of the stress states that can be supported by the COULOMB criterion is a convex cone.

found by adding at each point of the medium the stresses produced by $\underline{\sigma}_\gamma$, $\underline{\sigma}_c$ and σ_q, is statically admissible and everywhere satifies the strength criterion for a weighted cohesive soil, with a surface load (fig. 3).
Consequently, by using the above theorem for the problem given in figure 3, we obtain :

(2.7) $q_u \geqslant (q_u)_{superp.} = \frac{1}{2}\gamma BN_\gamma(\phi) + CN_c(\phi) + qN_q(\phi)$

which shows the conservative character of the superposition method.

2.4. Kinematic method - Complete solutions by yield design

It is further demonstrated that the upper bounds of q_u can be easily approximated by kinematics. This result is obtained by dualizing mathematically the equilibrium equations and this by using the principle of virtual work and the support function derived from the strength criterion.

We thus have two approaches which belong to the general formulation of the approaches from the inside and outside in yield design. While these are analogous to the well known limit analysis theorems they are in nowise related to the theory of Plasticity. An example of their detailed application to the analysis of the stability of earth works is to be found in COUSSY and SALENÇON (1979).

The combination of these two approaches, through the construction of complete solutions -from the yield design viewpoint-, make known the exact value of q_u.

2.5. Consequences

Once this precise definition of the concept of bearing capacity, corresponding exactly to the guiding lines followed by the authors working in this field, is established the interpretation of the various methods proposed for the computation of the coefficients' values, that were given in equation 2.3, presents no difficulty. Determining to which type of approach each of the reasonings used is related will suffice. A detailed analysis of this matter is given in MATAR (1978) from which is derived that $N_c(\phi)$ and $N_q(\phi)$ are given by the equations issuing from the PRANDTL solution as completed by SHIELD (1954) ::

(2.8) $\begin{cases} N_c(\phi) = cotg\phi[e^{\pi tg\phi} . tg^2(\frac{\pi}{4}+\frac{\phi}{2})-1] \\ \\ N_q(\phi) = e^{\pi tg\phi} . tg^2(\frac{\pi}{4}+\frac{\phi}{2}) \end{cases}$

As for $N_\gamma(\phi)$, its value is obtained from the LUNDGREN and MORTENSEN solution (1953) as completed by DAVIS and BOOKER (1971).

2.6. Remarks

We have seen that the only possible definition for a bearing capacity q_u using an approach based solely on a knowledge of the foundation's soil strength capacities is that of an <u>extreme</u> load as used in yield design : q_u is the heaviest load <u>likely</u> to be supported by the foundation under given conditions. This is clearly a partial answer to the given (modelized) problem, if for no other resson than that nothing lets us state that any load inferior to q_u will always be actually supported by the foundation, independantly of such factors as loading path, etc. Moreover, the level of distortions before failure and their possible future role are not without importance. This is what the comparison with the model tests, discussed in chapter 7, will attempt to evaluate.

3. BEARING CAPACITY OF A STRIP-FOOTING ON NON-HOMOGENEOUS SOIL - SUPERPOSITION METHOD

3.1. Previously studied cases of non-homogeneity

The most classic foundation studied is obviously that of a strip-footing on unlimited homogeneous soil. The relatively small number of parameters needed to describe the problem in terms of bearing capacity effectively diminishes the volume of calculations to be made when using the superposition method and enables are to easily present the results : tables or curves for $N_\gamma(\phi)$, $N_c(\phi)$ and $N_q(\phi)$.

The case of a strip-footing on homogeneous soil of limited thickness resting on a rigid base (fig. 6) has been studied too by MANDEL and SALENÇON (1969 and 1972). These authors used the superposition method based on the general theory § 2.2. and by constructing complete solutions for the three basic problems that correspond in structure to those in figure 5.
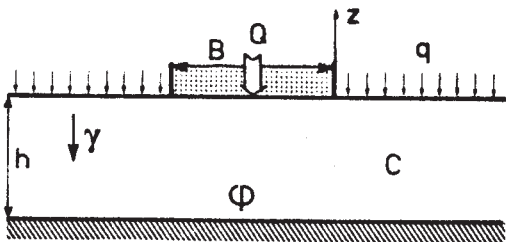

Fig.6 Strip-footing on homogeneous soil of limited thickness

The dimensional analysis shows that the bearing capacities for the three basic problems are, using the notation of figure 6, expressed as :

$$(3.1)\begin{cases} a) & q_u = \frac{1}{2}\gamma B N'_\gamma\left(\frac{B}{h}, \phi\right) \\ b) & q_u = C N'_c\left(\frac{B}{h}, \phi\right) \\ c) & q_u = q N'_q\left(\frac{B}{h}, \phi\right) \end{cases}$$

Thereby, the bearing capacity for the general problem as per figure 6 is underestimated, using the superposition method in the following:

$$(3.2)\quad q_u \geqslant (q_u)_{superp.} =$$

$$\frac{1}{2}\gamma B N'_\gamma\left(\frac{B}{h}, \phi\right) + C N'_c\left(\frac{B}{h}, \phi\right) + q N'_q\left(\frac{B}{h}, \phi\right)$$

The authors studied various cases of interface friction between the soil layer and its rigid base. As for the foundation, it is assumed to be perfectly rough. In that which follows, solely will be considered the case where there is perfect adhesion between the soil layer and the rigid base.

The values of the coefficients of equation 3.2 are given in the references noted. They are graphically represented in figures 10 and 11. Between $N'_c(B/h, \phi)$ and $N'_q(B/h, \phi)$ the relation is : $N'_q(B/h, \phi) = N'_c(B/h, \phi) \, tg\phi + 1$.

Note that for each one of the basic problems, a critical value for the ratio B/h exists, that is, $(B/h)_\gamma$ and $(B/h)_c = (B/h)_q$, functions of ϕ such that :

$$(3.3)\begin{cases} \frac{B}{h} \leqslant \left(\frac{B}{h}\right)_\gamma & <=> \quad N'_\gamma\left(\frac{B}{h}, \phi\right) = N_\gamma(\phi) \\ \frac{B}{h} \leqslant \left(\frac{B}{h}\right)_c & <=> \quad N'_c\left(\frac{B}{h}, \phi\right) = N_c(\phi) \\ & \qquad\quad N'_q\left(\frac{B}{h}, \phi\right) = N_q(\phi) \end{cases}$$

This means that when the soil layer is sufficiently thick, the rigid base's influence is no longer significant on the bearing capacity of the foundation vanishes.

Yet another type of non-homogeneity has been studied : an unlimited foundation soil the cohesion of which linearly increases with depth. This type of non-homogeneity is encountered in current practice where $\phi < 25°$, and notably where $\phi = 0°$ or nearing $0°$, such as the sea soils that were used as examples by DUNCAN and BUCHIGNANI (1973) and LE TIRANT (1976), (fig. 7).
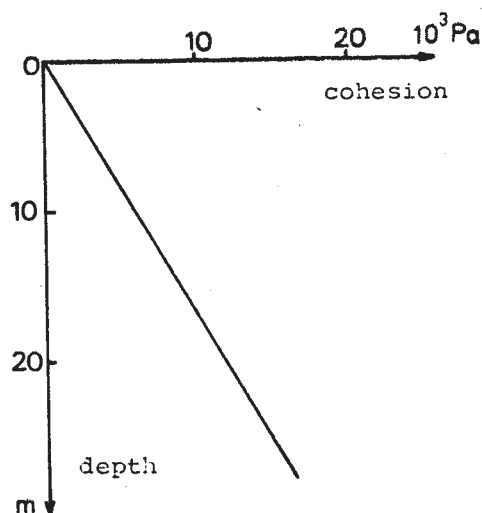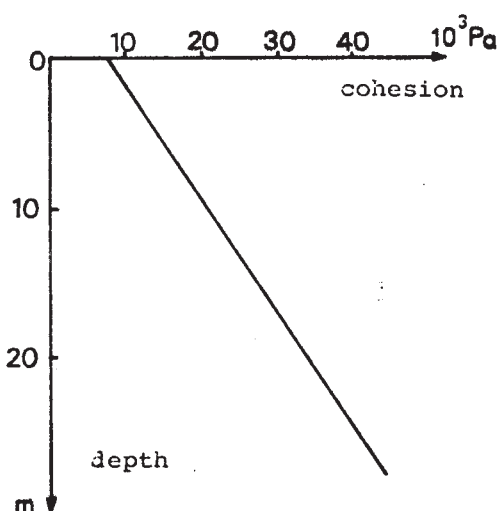
Fig.7 Examples of sea soil shear strength

The analysis of this problem rendered it possible to handle it by means of the superposition method and a global calculation taking all the coupling effects into account (SALENÇON, FLORENTIN and GABRIEL, 1976). As with the preceding ones, these are particular instances of the results of this study and the details are not needed here.

Lastly, MATAR and SALENÇON (1977) recently published results concerning a purely cohesive soil with cohesion linearly varying with depth.

The works mentioned above are those directly related, through the method used, to the present study. Other authors have worked on these problems, attacking them through different methods. Those interested may refer to OBIN (1972) and to GIROUD, TRAN-VO-NHIEM and OBIN (1973) for the bibligraphical references ; the interpretation of the solutions should be done in reference to the theorems §§ 2.3 and 2.4.

3.2. The problem studied (problem P)

Let us now consider the bearing capacity of a strip-footing, width B, axially loaded, resting on a soil layer of limited thickness h, unit weight γ, constant friction angle γ and with a cohesion that linearly increases with depth according to the law :

(3.4) $C(z) = C_O - gz$

C_O is surface cohesion and $g = - dC/dz$ (>0) is the cohesion gradient with depth ; C_O and g are constants.

The soil layer rests on a rigid base with a lateral surface load q (fig. 8).
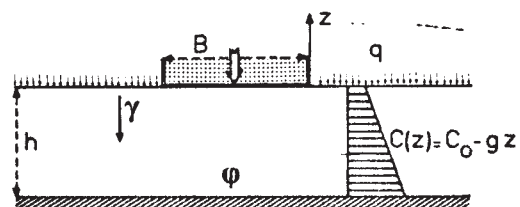

Fig.8 Problem P ; weighted soil, variable cohesion

It is assumed in the following that we are dealing with a rough strip-footing and a rough rigid base. By rough footing (resp. base) we mean that the adhesion between footing and soil is total, i. e. any failure occuring occurs in the soil and not in the soil-footing (resp. soil-base) interface.

The problem thus stated (1), the bearing capacity of the footing is a priori a function of the parameters C_O, g, B, h, γ, ϕ, q :

(3.5) $q_u = f(C_O, g, B, h, \gamma, \phi, q)$

3.3. Soil in friction contact, use of superposition method

Through a mechanics analysis of the problem, it will be seen that for the case of soil in friction contact ($\phi \neq 0$) a lower bound for the bearing capacity can be obtained from equation 3.2, i. e. by using the superposition method without necessitating new calculations.

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1. It is evident that this problem contains, as particular instances all those previously mentioned.

The following theorem is demonstrated :

"P-P[1] equivalence"

For a soil with $\phi \neq 0$, the bearing capacity q_u for problem P is yielded by the equation :

(3.6) $q_u = q_u{}^1 + q$

of the bearing capacity $q_u{}^1$ corresponding to problem P[1] geometrically identical to the preceding one defined as : homogeneous soil of same friction angle ϕ, unit weight $\gamma^1 = \gamma + g \cot g\ \phi$ and cohesion $C^1 = C_o + q\ tg\ \phi$.

This property is analogous to those stated by SALENÇON, BARBIER and BEAUBAT (1973) and by SALENÇON, FLORENTIN and GABRIEL (1976).

Figure 9 details the correspondence between the two problems.

Thus, the lower bound for the capacity obtained through by applying the superposition method to problem P[1] follows in an abvious manner by means of equation 3.2 :

(3.7) $q_u \geqslant (q_u{}^1)$ superp.

$+ q = \frac{1}{2}(\gamma + g\ \cot g\ \phi)\ BN'_\gamma(\frac{B}{h},\ \phi)$

$+ (C_o + g\ tg\ \phi)\ N'_c(\frac{B}{h},\ \phi) + q$

$= (q_u)_{superp.}$
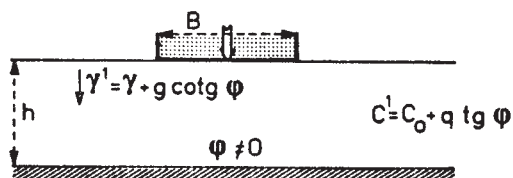
Problem P[1]
Weighted soil, modified unit weight, modified constant cohesion
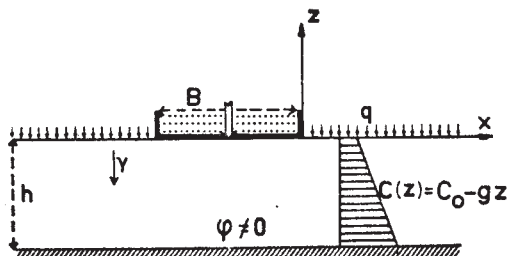
Problem P
Weighted soil
variable cohesion

Figure 9

Problems P[1] and P ; correspondance between stress fields : $\underline{\sigma}^1 = \underline{\sigma} - (gz\ \cot g\phi + q)\ \underline{1}$; correspondance between bearing capacities : $q_u{}^1 = q_u - q$

It is there by seen that a simple mechanics analysis of the given problem yields, without requiring new calculations, a nontrivial lower bound for the bearing capacity, taking into account the cohesion gradient's effect in the case of a frictional soil layer friction.

3.4. Purely cohesive soil

Due to the presence in the above P-P[1] correspondance equations of the factor $\cot g\ \phi$, it is evident that the above method cannot be directly used with a purely cohesive soil. A specific analysis must thus be made.

The following two properties are demonstrated (cf. MATAR and SALENÇON, 1977) :

a) If $C_o = 0$ the footing's bearing capacity is :

(3.8) $q_u = \frac{1}{4}\ gB + q \rightarrow \forall\ \frac{B}{h}$;

b) For the general case of a purely cohesive soil ($\phi = 0$) the superposition of the bearing capacity corresponding to a constant cohesion equal to C_o and to the surface load q (1) and of that due to the cohesion gradient yields a conservative value for the bearing capacity in problem P :

(3.9) $q_u \geqslant (q_u)_{superp.} = C_o N'_c(\frac{B}{h},\ 0) + \frac{1}{4}\ gB + q$

Equation 3.9 thus yields a lower bound for the footing's bearing capacity where $\phi = 0$, taking into account the cohesion gradient's effect ; this is a superposition formula. In addition, this formula is homologous with equation 3.7 in that the continuity between these two equations a $\phi \rightarrow 0$ corresponds to the property : $N'_\gamma(B/h,\ \phi) \sim \phi/2$ when $\phi \rightarrow 0$, $\forall B/h$.

3.5. Remarks

The "P-P[1] equivalence" theorem stated in § 3.3 shows for a material frictional:

an equivalence between the cohesion C and the surface load q effects : this is the so-called "corresponding states" theorem;

and an equivalence between the vertical cohesion gradient and the unit weight effects : from the point of view of bearing capacity, the vertical cohesion gradient is equivalent to an additional unit weight equal to g cotg ϕ. In the next chapter we will make use of this but in the so to say "opposite direction".

_____

(1 For $\phi = 0$, the surface load q has a purely additive effect.

Here these two equivalence brought about a grouping of terms from which it was possible to make use of equation 3.2. Figures 10 and 11 indicate the values of the coefficients $N'_\gamma(B/h, \phi)$ and $N'_c(B/h, \phi)$ to be used in 3.7 as well $N'_c(B/h, 0)$ for 3.9.
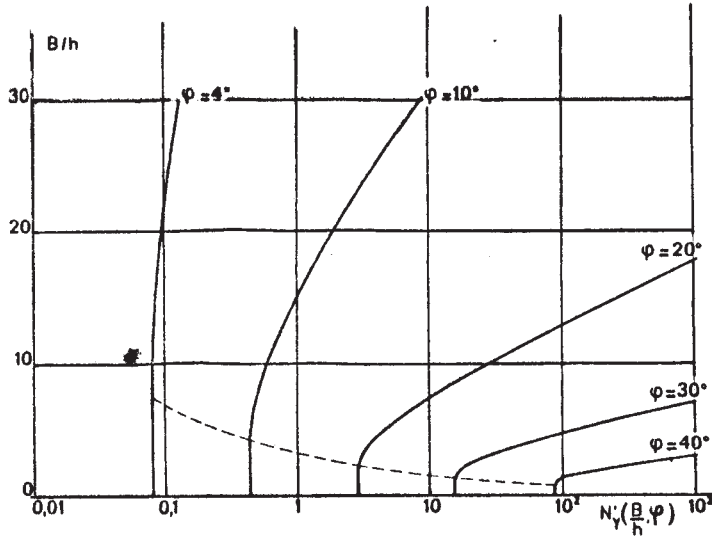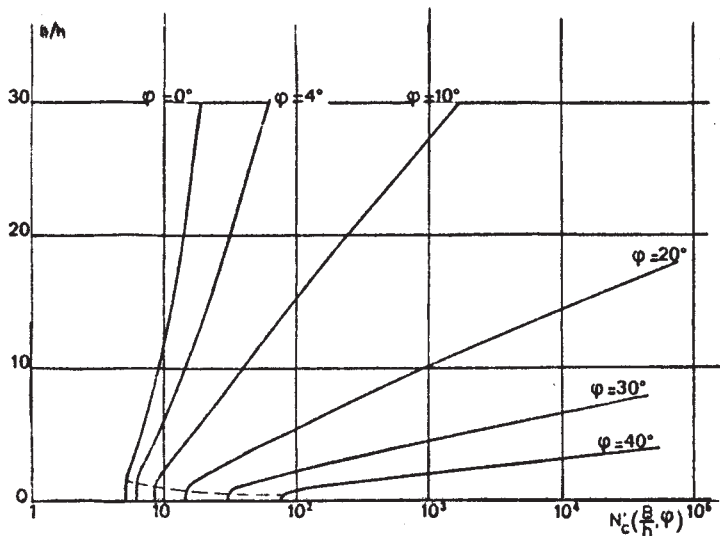
Fig.10 Values of $N'_\gamma(\frac{B}{h}, \phi)$

Fig.11 Values of $N'_c(\frac{B}{h}, \phi)$

As was stated, the superposition method yields, when applied to problem P^1, the lower bound (eq. 3.7). It may be tempting, in accordance with the general theorem in § 2.3, to directly apply this method to problem P. However, as MATAR (1978) explained, such an application would necessitate new computations which would be in fact identical to those made in the following chapter for the global calculation of the bearing capacity for this very same problem P. This application of the superposition method to problem P would thus lack any real interest.

4. BEARING CAPACITY OF A STRIP-FOOTING ON NON-HOMOGENEOUS SOIL - GLOBAL CALCULATION

### 4.1. Position of the problem, P-P2 equivalence

For the following analysis, it is convenient to take-up in the opposite direction the ressoning that was developed in § 3.3 concerning the "P-P1 equivalence". Thusly, the following theorem is demonstrated :
"P-P2 equivalence"

The determination of the bearing capacity q_u^2 for problem P brings us through the equation :

(4.1) $q_u = q_u^2 + q$

to the determination of the bearing capacity q_u for problem P^2 geometrically identical to the preceding one and defined as : weightless soil, friction angle ϕ, variable cohesion $C^2(z) = (C_o+q \, tg \, \phi) - (g \pm \gamma tg \phi)z$, without surface load.

Figure 12 details the correspondance between these two problems while table 1 summarizes the positions of P, P^1 and P^2.
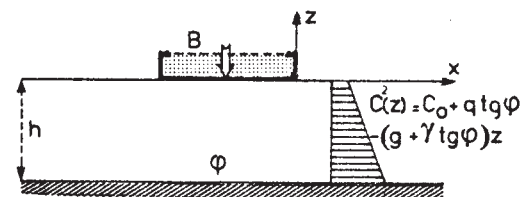
Problem P^2

weightless soil
modified variable
cohesion
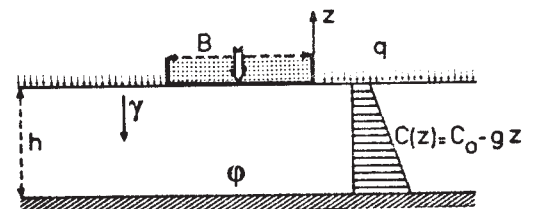
Problem P

weighted soil, variable cohesion

Figure 12

Problems P^2 and P ; correspondance between stress fields : $\underline{\underline{\sigma}}^2 = \underline{\underline{\sigma}} + (\gamma z - q) \underline{1}$; correspondance between bearing capacities : $q_u^2 = q_u - q$.

Table I

Position of problems P, p1 and p2

PROBLEM	P $\phi > 0$	P¹ $\phi > 0$	P² $\phi > 0$
figure	9 and 12	9	12
unit weight	γ	$\gamma^1 = \gamma + g\,\mathrm{cotg}\,\phi$	0
cohesion gradient	g	0	$g^2 = g + \gamma\,\mathrm{tg}\,\phi$
surface cohesion	C_o	$C_o + q\,\mathrm{tg}\,\phi$	$C_o + q\,\mathrm{tg}\,\phi$
lateral surface load	q	0	0
bearing capacity	q_u	$q_u^1 = q_u - q$	$q_u^2 = q_u - q$
stress field	$\underline{\sigma}$	$\underline{\sigma}^1 = \underline{\sigma} - (gz\,\mathrm{cotg}\,\phi + q)\underline{1}$	$\underline{\sigma}^2 = \underline{\sigma} + (\gamma z - q)\underline{1}$

Note that in this expression ϕ is without restriction. Indeed the demonstration remains valid for $\phi = 0$. Thus it is possible to group together under the same formalism, the cases of purely cohesive soil and of frictional soil [1].

From this equivalence theorem is immediately derived that the bearing capacity q_u depends on the parameters defining problem P, indicated in equation 3.5, only through the mediation of the groups defining problem P^2. In more precise terms, considering equation 4.1 and with the notation of 3.5 we obtain :

$$q_u = q + f(C_o + q\,\mathrm{tg}\,\phi,\ g + \gamma\,\mathrm{tg}\,\phi, B, h, O, \phi, O)$$

As a result of the <u>dimensional analysis,</u> it is seen that the bearing capacity q_u for problem P necessarily takes the following forms :

(4.2)
$$\begin{cases} q_u = q + (C_o + q\,\mathrm{tg}\,\phi)\ F_C\left(\dfrac{g + \gamma\,\mathrm{tg}\,\phi}{C_o + g\,\mathrm{tg}\,\phi}B,\ \dfrac{B}{h},\ \phi\right) \\ \text{when } (C_o + q\,\mathrm{tg}\,\phi) \neq 0 \end{cases}$$

(4.3)
$$\begin{cases} q_u = q + (g + \gamma\,\mathrm{tg}\,\phi)\ B.K\left(\dfrac{B}{h},\ \phi\right) \\ \text{when } (C_o + q\,\mathrm{tg}\,\phi) = 0 \end{cases}$$

where F_C and K are scalar functions of the indicated arguments.

These equations have an obvious interest : reducing the number of parameters on which q_u really depends, first by mechanics analysis then by dimensional analysis obviously considerably reducing the volume of calculations which are needed for the complete determination of q_u. It is also seen that the presentation will be greatly facilitated.

4.2. Determining the function $K(B/h, \phi)$

In equation 4.3 corresponding to the case where $(C_o + q\,\mathrm{tg}\,\phi) = 0$ the determination of the function $K(B/h, \phi)$ reduces to the calculation of q_u^2 for problem P^2 in the case where the surface cohesion $C^2(z)$ is zero : $C^2(0) = 0$.

Let us first assume $\phi \neq 0$; q_u^2 is then equal to q_u^1 for the homologous problem P^1 (table I), i. e. where the surface cohesion is zero. This problem is thus one of the three basic problems expressed in § 3.1 for the strip-footing on a homogeneous soil layer. It is in fact, problem "a" and by referring to equation 3.1 we obtain :

$$q_u = \frac{1}{2}\ (\gamma + g\,\mathrm{cotg}\,\phi)\ B\ N'_\gamma\left(\frac{B}{h},\ \phi\right)$$

---

(1). This proves that it is the factor $N'_q = N'_\gamma$ $\mathrm{cotg}\,\phi$ which is significative and not N'_γ in itself (the same for $N'_C = (N'_q - 1)\,\mathrm{cotg}\,\phi$ and not $N'_q - 1$). This will be apparent from the multi-entry charts for practica computations given in chapter 5.

From this is derived :

$$(4.4) \begin{cases} K(\frac{B}{h}, \phi) = \frac{1}{2} N'_\gamma (\frac{B}{h}, \phi) \cot g \phi \\ \text{when } \phi \neq 0 \end{cases}$$

When $\phi \neq 0$, the problem P^2 to investigate is that of the strip-footing on a purely cohesive soil of limited thickness, surface cohesion is zero and cohesion linearly increasing with depth in accordance with the gradient $g^2 = g + \gamma tg\phi$. This is that problem which was mentioned in § 3.4 and the bearing capacity there is given by equation 3.8, thus :

$$q_u = \frac{1}{4} (g + \gamma tg\phi) B = \frac{1}{4} gB$$

From this is derived :

$$(4.5) \quad K(\frac{B}{h}, 0) = \frac{1}{4}$$

The function $K(B/h, \phi)$ is thereby determined for $\phi \geq 0$ without needing to make any new calculation.

4.3. Determining the function

$$F_C (\frac{g + \gamma tg\phi}{C_o + qtg\phi} B, B/h, \phi)$$

To determine the function F_C a complete solution for the problem P^2 is constructed, where $(C_o + qtg) \neq 0$. To this end, the theory of plane limit equilibrium for non-homogeneous soils (cf. e. g. OLSZAK, RYCHLEWSKI and URBANOWSKI, 1962 or SALENÇON, 1974 for the equations relevant to this problem) is used.

We limit ourselves here to giving only some brief indications concerning the process followed and the shape of the characteristic line networks obtained. Details are available in MATAR, 1978, and notably concerning the extensions of the velocity and stress fields.

Construction of the solution for a limited thickness soil layer is derived from the solution of the semi-infinite soil case $(h = \infty)$ in a fashion similar to the one described by MANDEL and SALENÇON (1969 and 1972) and by MATAR and SALENÇON (1977). This was done with the hypothesis $(g + \gamma tg \phi) \geq 0$.

. Unlimited soil (h = 0)

Figure 13 shows a network of characteristics such as constructed by BERTHET, HAYOT and SALENÇON (1972) for a purely cohesive soil $(\phi = 0)$ and by SALENÇON, BARBIER and BAUBAT (1973) for a frictional soil $(\phi \neq 0)$.

This network is made from :

1. a RANKINE field in non-homogeneous medium OAB

2. a PRANDTL fan in non-homogeneous medium, center at O, aperture $3\pi/4+\phi/2$: OBC.

3. Network construction is then continued by the (α) characteristics issued from OC and the (β) characteristics tangent to O'O as a consequence of the foundation being perfectly rough. The network of characteristics has to be symetric with respect to the foundation axis and is thereby limited by the characteristic SCBA (α) and ST (β) which intersect on the axis of symetry at a point S in such a manner that the main direction of the stresses corresponding to the maximum compression be vertical. This condition determines the length of OA, and of the other network dimensions, which were a priori onknown.

Thus constructed (fig. 13) the network descends into the soil to a depth h_o, proportional to B and depending on :

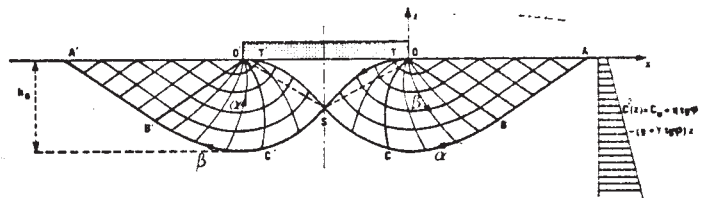$$\frac{g + \gamma tg\phi}{C_o + qtg\phi} B \text{ and } \phi.$$

Figure 13

Fig.13 Foundation on soil of unlimited thickness ; network of characteristics

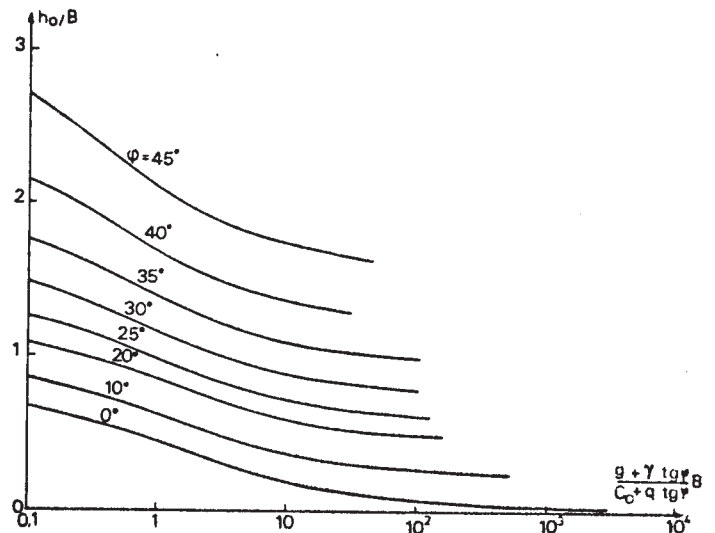Figure 14 represente h_o/B as a function of these variables :

Fig.14 Values of h_o/B

If $(g + \gamma tg\phi) = 0$, then $B/h_o = B/h_c$ corresponding to a weightless cohesive homogeneous soil ; if $(C_o + q tg \phi) = 0$, then $B/h_o = (B/h)_\gamma$, cf. equation 3.3. Thus, h_o/B is a decreasing function of $(g + \gamma tg \phi)B/(C_o + qtg \phi)$ at a given ϕ. This corresponds to the fact that at a given surface cohesion C_o and surface load q, the soil is disturbed so much the shallower

as it becomes more rapidly more resistant, that is, as the gradient $g^2 = (g + \gamma \, tg \, \phi)$ is larger.

From the solution so constructed, it is possible to determine :

$$F_c \left(\frac{g + \gamma tg \, \phi}{C_o + g \, tg \, \phi} \, B, \, O, \, \phi \right)$$

Soil layer $h \geqslant h_o$ thick

For a soil layer of thickness limited to $h > h_o$, the construction of the network of characteristics of figure 13 can be used without any change (fig. 15).
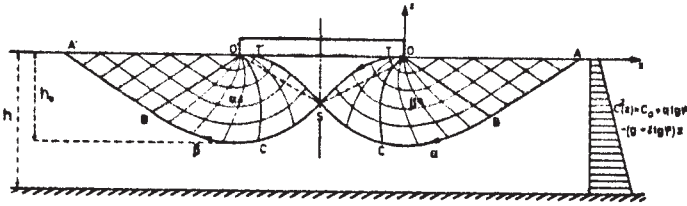

Fig.15 Foundation on a layer $h \geqslant h_o$ thick

In conclusion :

with $B, \dfrac{g + \gamma tg \, \phi}{C_o + q \, tg \, \phi} \, B$ and ϕ given, if $h \geqslant h_o$

the rigid base does not influence the bearing capacity of the strip-footing.

In other words :

$$(4.6) \begin{cases} F_c \left(\dfrac{g + \gamma tg \, \phi}{C_o + q \, tg \phi} \, B, \, \dfrac{B}{h}, \, \phi \right) = \\ = F_c \left(\dfrac{g + \gamma tg \, \phi}{C_o + q \, tg \, \phi}, \, O, \, \phi \right) \\ \text{if } O \leqslant \dfrac{B}{h} \leqslant \dfrac{B}{h_o} \end{cases}$$

Soil layer $h < h_o$ thick.

It is obvious that the network of characteristics of figure 15 cannot be used for a soil layer having a thickness h less than h_o. In this case, the construction of the solution, symetric to the foundation axis, is made as shown on figure 16.
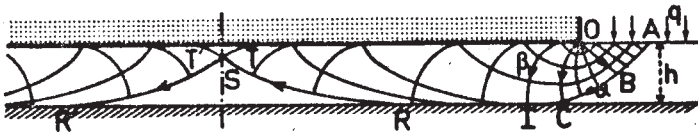

Fig.16 Foundation on a layer $h \leqslant h_o$ thick ;
network of characteristics

1. OAB is a RANKINE field in non-homogeneous medium

2. OBC is a PRANDTL fan in non-homogeneous medium, center at O aperture $3\pi/4 + \phi/2$.

3. Construction of the network of characteristics beyond these fields is continued from OC and CI based on the conditions at the limits.

The network is limited by the characteristics SR and ST which intersect on the foundation axis as in the case of an unlimited soil and it is completed by symetry.

It remains to compute, for the various solutions, the bearing capacity q_u2. This is the integral of the vertical stress (corresponding to the constructed solution) beneath the foundation divided by its width :

$$(4.7) \quad q_u2 = \frac{1}{B} \int_{-B}^{O} \sigma_{zz} \, (x, \, O \quad dx)$$

Thus, to calculate q_u^a it suffices to integrate the known stresses on STO and to take the vertical component of it which represents the value $q_u2.B/2$.

The iso-B/h curves on figures 17, 18 and 19 represent the values of :

$$(4.8) \quad F_c \left(\frac{g + \gamma tg \, \phi}{C_o + q \, tg \, \phi} \, B, \, \frac{B}{h}, \, \phi \right) = \frac{q_u - q}{C_o + q \, tg \, \phi}$$

$$= \frac{q_u2}{C_o + q \, tg \, \phi}$$

(see the figures next page).

4.4. Comparison with the superposition method

Equations 2.4 and 4.3 in conjunction with the knowledge of the functions $K(B/h, \, \phi)$

and $F_c \left(\dfrac{g + \gamma tg\phi}{C_o + qtg\phi} \, B, B/h, \, \phi \right)$ obtained from

equations 4.4 and 4.5 and from curves such as those in figures 17 to 19 for the various values of ϕ, enable one to calculate, for problem P, the exact value of the bearing capacity, i. e. q_u.

This exact value, result of the global calculation, accounts for the coupling effects possible between the various parameters defining the problem. The mechanics and dimensional analyses made in § 4.1 by parameter grouping and the non-dimensional factors that they bring forth, clearly indicate which coupling effects are likely to occur.

From equation 4.2, it is seen that the coupling effect on $(q_u - q)$ is to be sought out between $(C_o + qtg\phi)$ and $(g + \gamma tg\phi)B$.

Having the exact value of q_u, it is important to compare it with its lower bound approximation obtained by the superposition method which does not take the above effects into account : equations 3.7 and 3.9. Note that insofar as equation 4.2 has as a particular instance the classic case of strip-footing on unlimited soil, the proposed comparison enables the conservative character of the superposition method, as used in current practice, to be calculated.
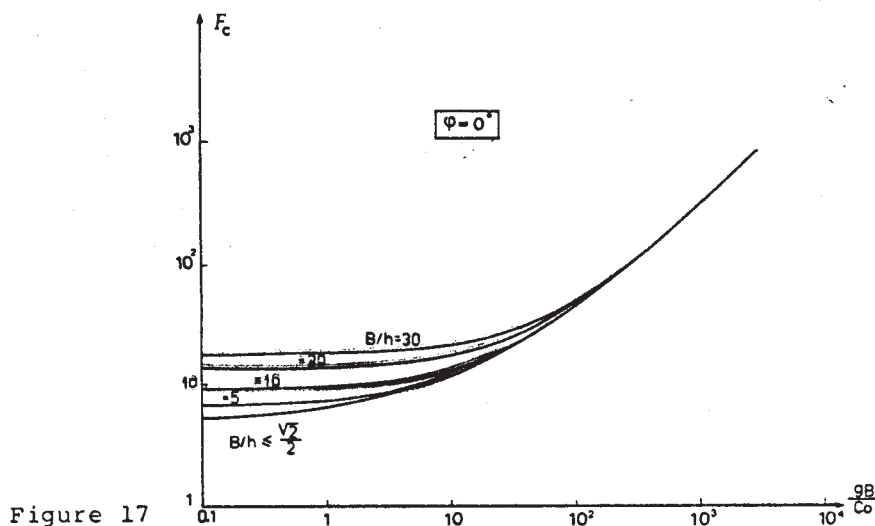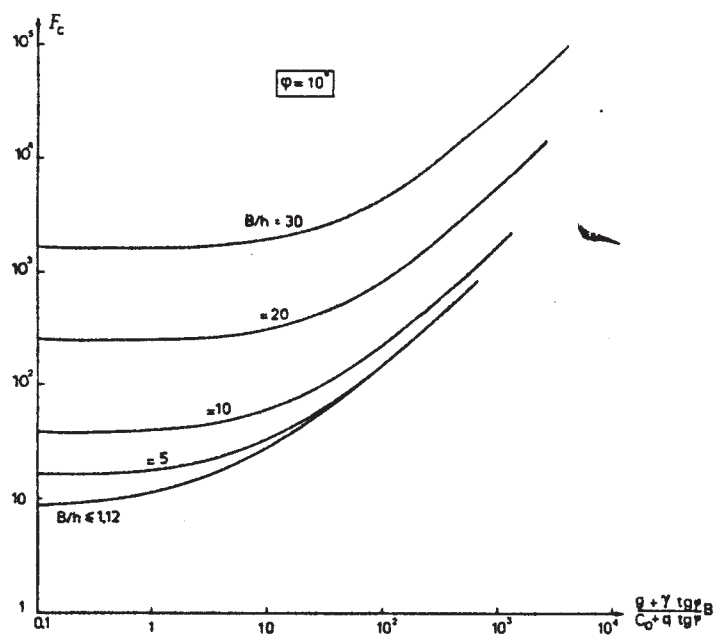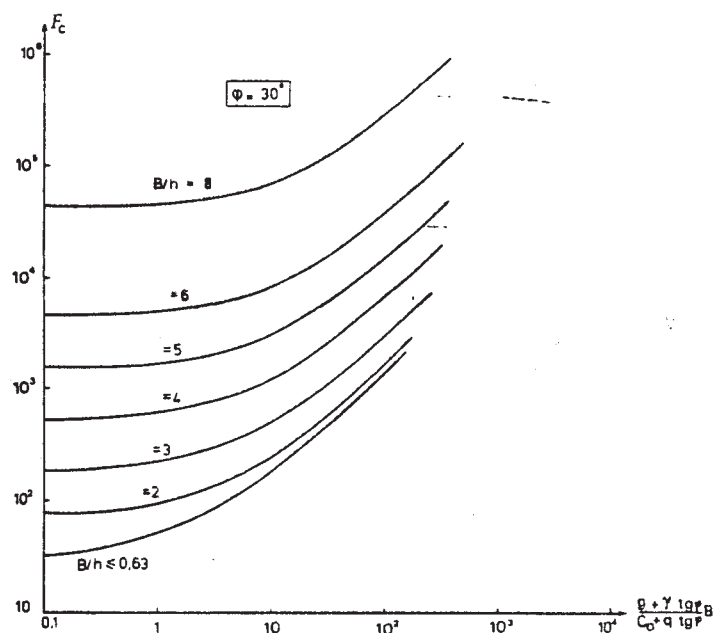
Figure 17

Figure 18

Figure 19

Values of F_c $\phi = 0°,\ 10°,\ 30°$

To make this comparison, we introduce the coefficient μ_c, function of $(\dfrac{g+\gamma tg\phi}{C_0+q\ tg\phi}\ B,$ $B/h,\ \phi)$ defined as

$$(4.9) \qquad \mu_c = \frac{q_u - q}{(q_u)_{superp} - q}$$

This coefficient is analogous to the one used by LUNDGREN and MORTENSEN (1953).

It represents, when q=0 , the ratio between q_u and $(q_u)_{superp}$.

When $q \neq 0$, the introduction of this coefficient is justified, in addition to its suitability for representing the results, by the fact that when making foundation design calculations the safety factor bears not on q_u but rather on (q_u-q) (cf. LEONARDS, 1968, etc.).

By referring to equations 3.7, 3.9 and 4.2, we obtain for μ_c :

$$(4.10)\ \begin{cases} \mu_c\ (\dfrac{g\ +\ \gamma tg\phi}{C_0\ +\ q\ tg\phi}\ \ B,\ \dfrac{B}{h},\ \phi) \\[2em] = \dfrac{F_c\ (\dfrac{g\ +\ \gamma tg\phi}{C_0\ +\ q\ tg\phi}\ \ B,\ \dfrac{B}{h},\ \phi)}{\dfrac{1}{2}\dfrac{g\ +\ \gamma tg\phi}{C_0\ +\ q\ tg\phi}\ BN'_\gamma(\dfrac{B}{h},\ \phi)cotg\phi\ +\ N'_c(\dfrac{B}{h},\ \phi)} \end{cases}$$

when $(C_0 + q\ tg\ \phi) \neq 0$

by taking $N'(B/h,\ \phi)\ cotg\phi = 1/2$ when $\phi = 0$.

Thus :

$$(4.11) \quad \mu_c \geqslant 1 \quad \forall \; \frac{g + \gamma tg\phi}{C_0 + q\, tg\phi} \; B, \; \frac{B}{h}, \qquad \phi \geqslant 0$$

On the other hand, an upper bound can be obtained for μ_c by noting that q_u^2 is necessarily less, in accordance with the static theorem of § 2.3, than the bearing capacity of a footing on a homogeneous layer with a cohesion $C_0+qtg\phi+(g+\gamma tg\phi)h$ which, for problem P^2, is the maximum cohesion in the layer. Whence :

$$1 \leqslant \mu_c (\frac{g+\gamma tg\phi}{C_0+qtg\phi} \; B, \; \frac{B}{h}, \; \phi)$$

$$\leqslant \frac{1 + \frac{g+\gamma tg\phi}{C_0+qtg\phi} \; B . \; (\frac{B}{h})^{-1}}{\frac{1}{2} \frac{g+\gamma tg\phi}{C_0+qtg\phi} \; B \; \frac{N'\gamma (B/h, \; \phi)}{N'_C (B/h, \; \phi)} \; cotg\phi \; + \; 1}$$

Let us now indicate the properties of μ_c, demonstrated in MATAR (1978) :

$$(4.12) \quad \mu_c (0, \; \frac{B}{h}, \; \phi) = 1 \qquad \forall \; \frac{B}{h}, \forall \phi \geqslant 0$$

$$(4.13) \quad \mu_c (\infty, \; \frac{B}{h}, \; \phi) = 1 \qquad \forall \; \frac{B}{h}, \forall \phi \geqslant 0$$

$$(4.14) \quad \mu_c (\frac{g+\gamma tg\phi}{C_0+qtg\phi} \; B, \; \infty, \; \phi) = 1 \qquad \forall \; \frac{g+\gamma tg\phi}{C_0+qtg\phi} B, \forall \phi \geqslant 0$$

As indicated in the preceding § and in figure 14, $(B/h)_0$ is an increasing function of $\frac{g+\gamma tg\phi}{C_0+qtg\phi} \; B$:

$$(\frac{B}{h})_C \leqslant (\frac{B}{h})_0 \leqslant (\frac{B}{h})_\gamma$$

Thus, with equation 3.3, it is immediately seen that :

$$(4.15) \begin{cases} \text{If } (\frac{B}{h}) \leqslant (\frac{B}{h})_C \\ \\ \text{Then } \mu_c \text{ is independent of } \frac{B}{h}, \; \forall \; \phi > 0. \end{cases}$$

On the other hand, μ_c depends on B/h, $\forall \phi \geqslant 0$ as soon as B/h is greater than $(B/h)_C$.

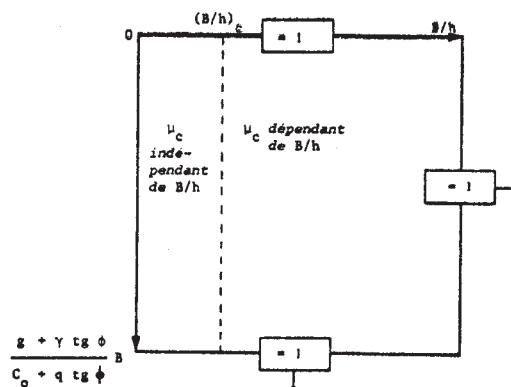These properties, used to plot the multi-entry charts in chapter 5, are summarized in figure 20.

Fig.20 Properties of μ_C

In figure 21, 22 and 23, μ_c is represented, as a function of its first argument, by "iso-B/h" curves with ϕ constant. In figure 24 to 26, it is represented as a function of its first argument by "iso-ϕ" curves with B/h constant when the values of B/h (1) are small. The obtained curves call for the following remarks :
for figure 21 to 23, it is seen that :

- the maximum value μ_c^{max} of μ_c, corresponding to the curve peaks, is a decreasing function of B/h

- the bearing capacity underestimation due to the superposition method may be <u>large</u>, especially <u>when the value of B/h is small</u>.

For figures 24 to 26 :

- the maximum value μ_c^{max} of μ_c, corresponding to the curve peaks, is a decreasing function of ϕ

- the bearing capacity underestimation due to the superposition method may be <u>large</u>, especially <u>when the value of ϕ is small</u>.

For a purely cohesive soil ($\phi=0$), μ_c reaches a maximum value equal to 1.72, when $gB/C_0 = 22$.

It follows from these results that bearing capacity underestimation resulting from the superposition method may be <u>large for values of B/h and ϕ that are small</u> and that it reaches up to 40 % of the exact value of q_u.

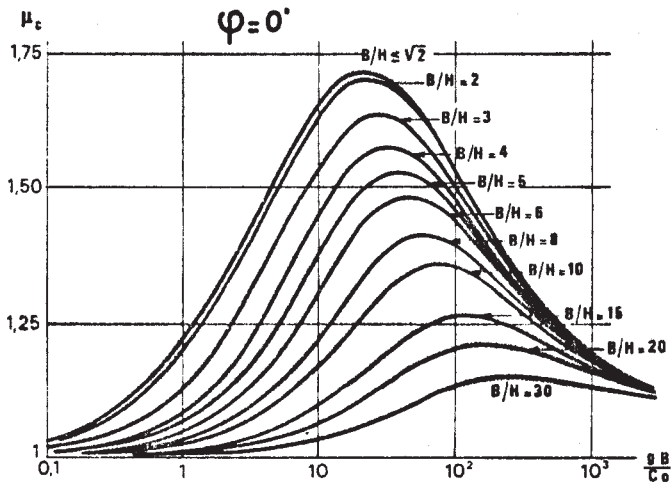(1) The case B/h=0, corresponding to unlimited soil, has been given by SALENÇON, FLORENTIN and GABRIEL (1976).
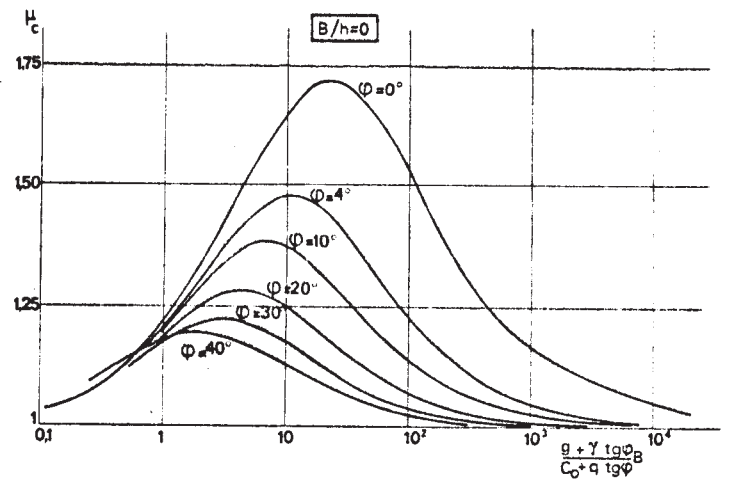
Fig.21 Values of μ_C , $\phi = 0°$
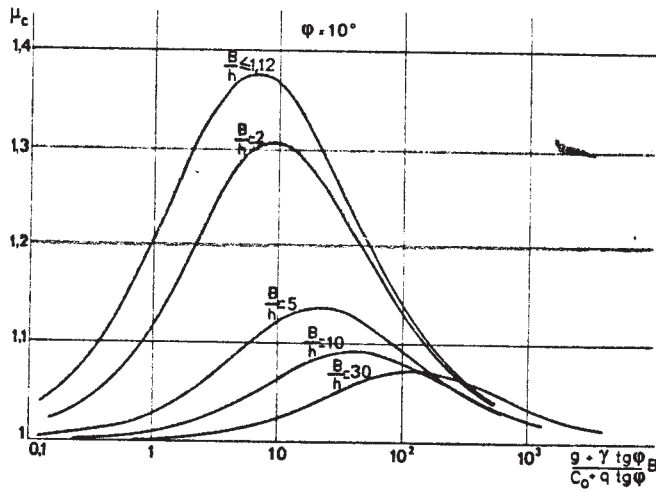

Fig.24 Values of μ_C, B/h = 0

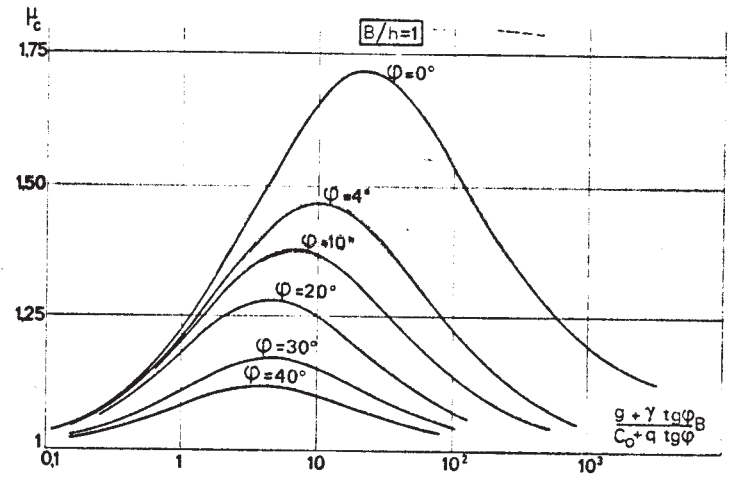Fig.22 Values of μ_C, $\phi = 10°$

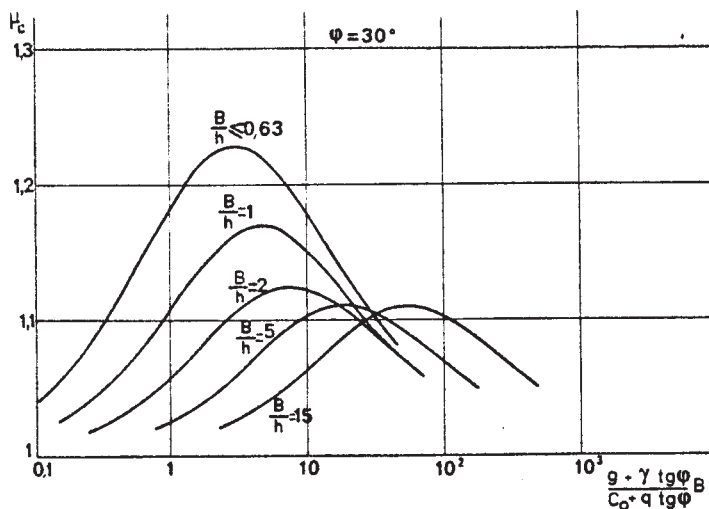Fig.25 Values of μ_C, B/h = 1

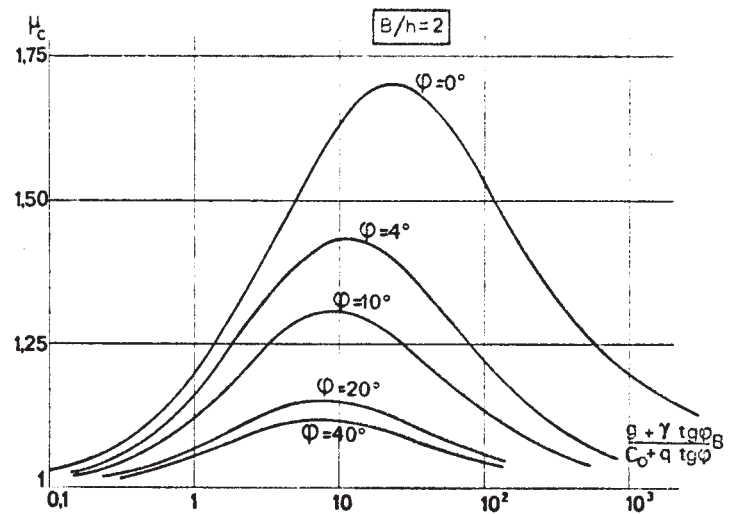Fig.23 Values of μ_C, $\phi = 30°$

Fig.26 Values of μ_C, B/h = 2

5. MULTI-ENTRY CHARTS FOR THE COMPUTATION OF THE BEARING CAPACITY OF STRIP-FOOTINGS

5.1. Presentation

Due to the high numerical values attained by the bearing capacity q_u, it has to be represented on graphs whose F_C axis is expressed on a logarithmic scale such as given in figures 17 to 19. This type of representation is rather clumsy in actual practice.

As the use of the superposition method is usually rather simple, we propose to base the computation of the bearing capacity on this method by making use of correction factors obtained from the definition of μ_C given by equation 4.9.

To this effect, with ϕ constant, are given "iso-μ_C" curves in the

$(\dfrac{q+\gamma tg\phi}{C_0+qtg\phi} B, \dfrac{B}{h})$ plane already introduced in figure 20.

Taking this plane into account, curves were plotted giving $N'_C(B/h, \phi)$ and $N'_\gamma(B/h, \phi)$ cotgϕ as functions of B/h.

These multi-entry charts, where the values of the coefficients μ_C, N'_C, N'_γ cotgϕ can be read or interpolated, are easily used for the actual computation of the

bearing capacity by applying the following formulae derived from equations 3.7, 3.9 and 4.9 :

For a COULOMB soil ($\phi \neq 0$)

(5.1) $q_u = q + \mu_C(C_0 + q\ tg\ \phi)$

$$[\frac{1}{2} \frac{g + \gamma tg\phi}{C_0 + q\ tg\phi} B\ N'_\gamma\ cotg\phi + N'_C]$$

For a purely cohesive soil ($\phi = 0$)

(5.2) $q_u = q + \mu_C\ C_0(N'_C + \frac{1}{4} \frac{gB}{C_0})$

"Iso-μ_C" multi-entry charts are given below corresponding to the case where $\phi = 0°$, $4°$, $10°$, $20°$, $25°$, $30°$, $35°$, $40°$ and $45°$.

5.2. Iso-μ_C" multi-entry chart for $\phi = 0°$, $4°$, $10°$, $20°$, $25°$, $30°$, $35°$, $40°$, $45°$

Nota :

The original charts for strip footings, are dealing with curves located on the upper part of the figures 27 to 34. The curves located on the lower part are refering to computation of circular shallow foundations. Detailed comments for use are given in the next paper by SALENÇON and MATAR.
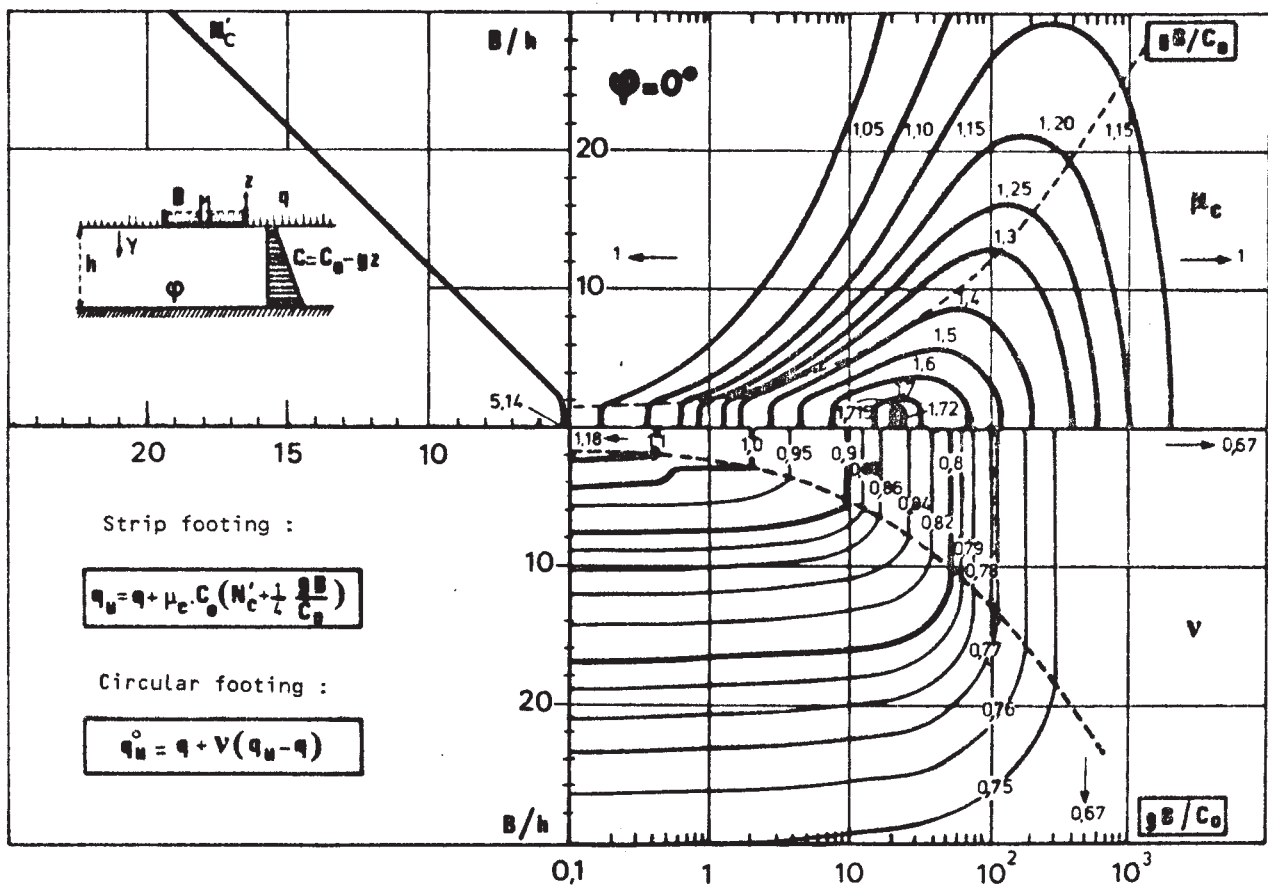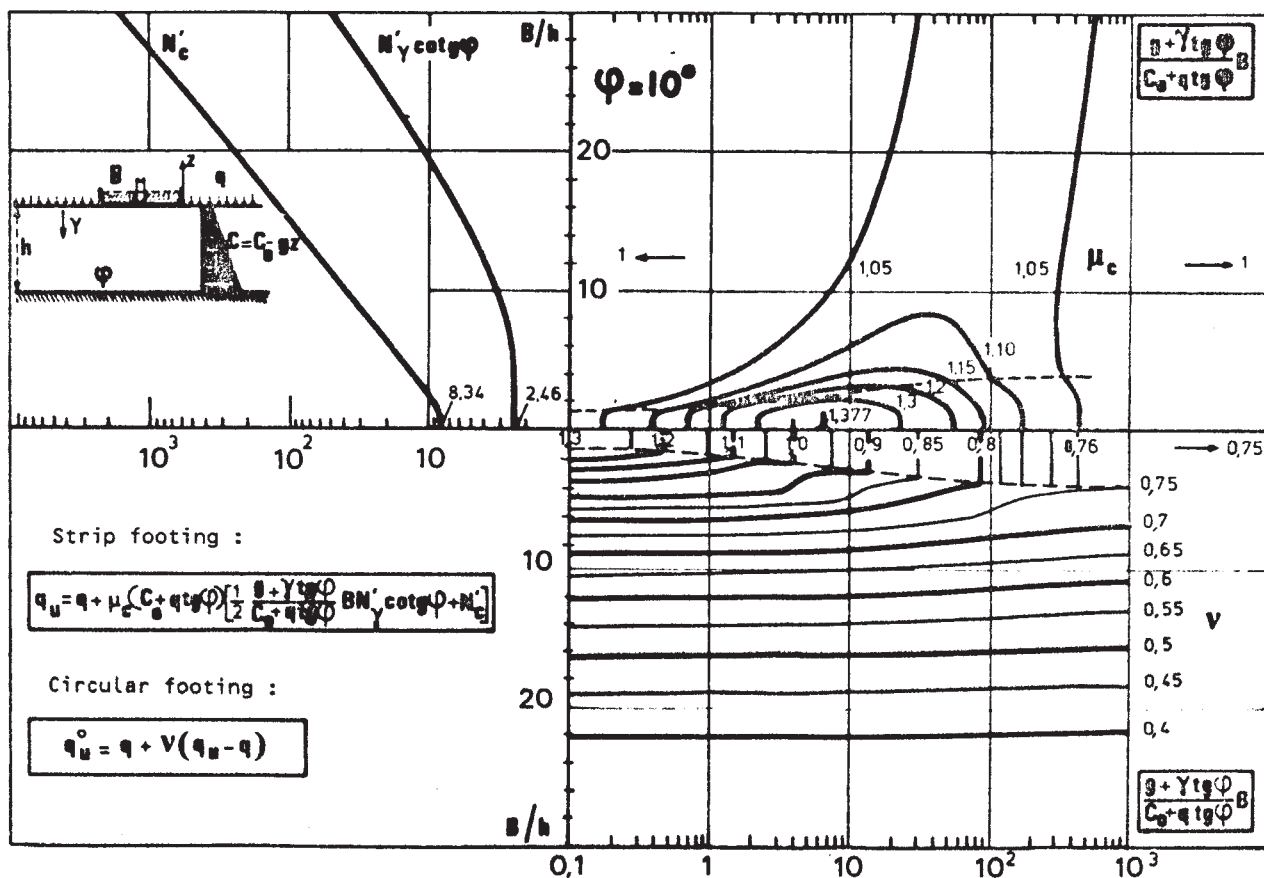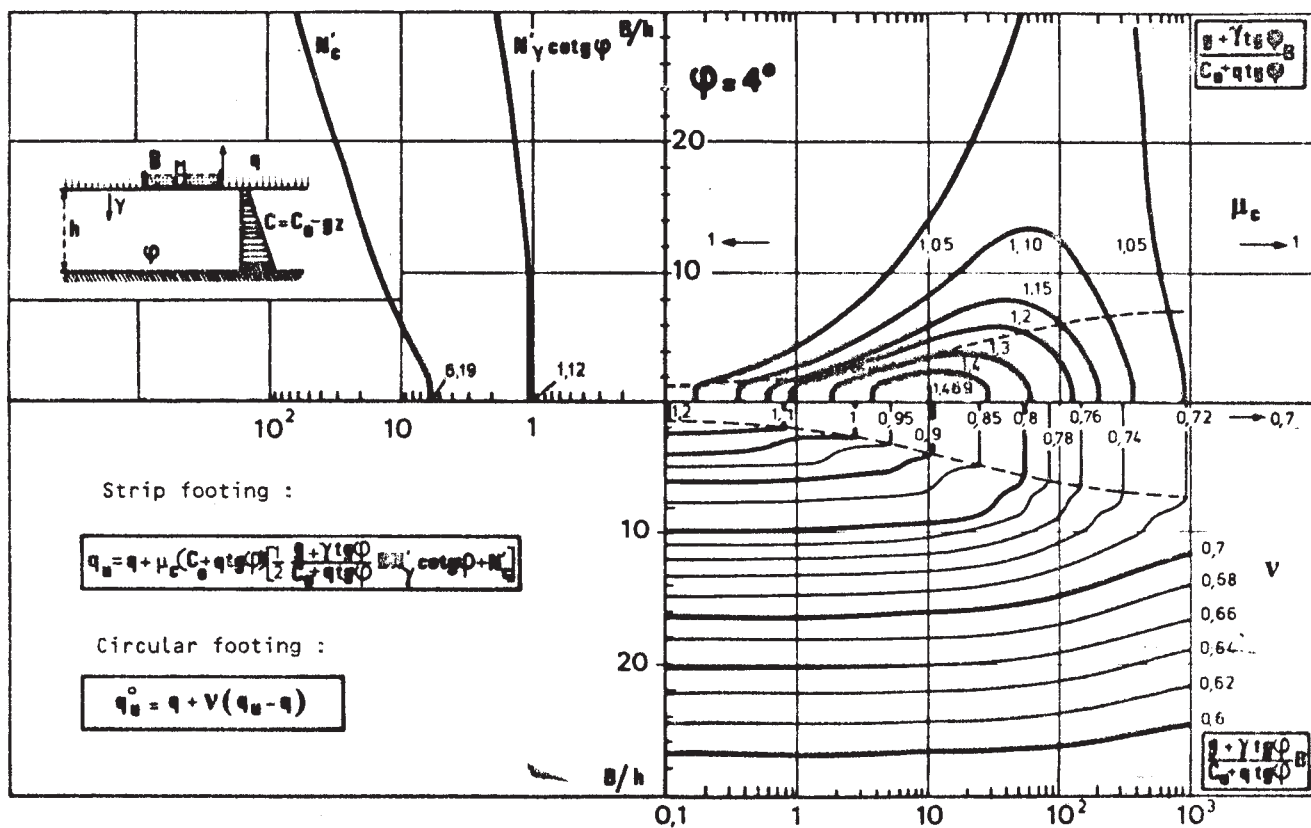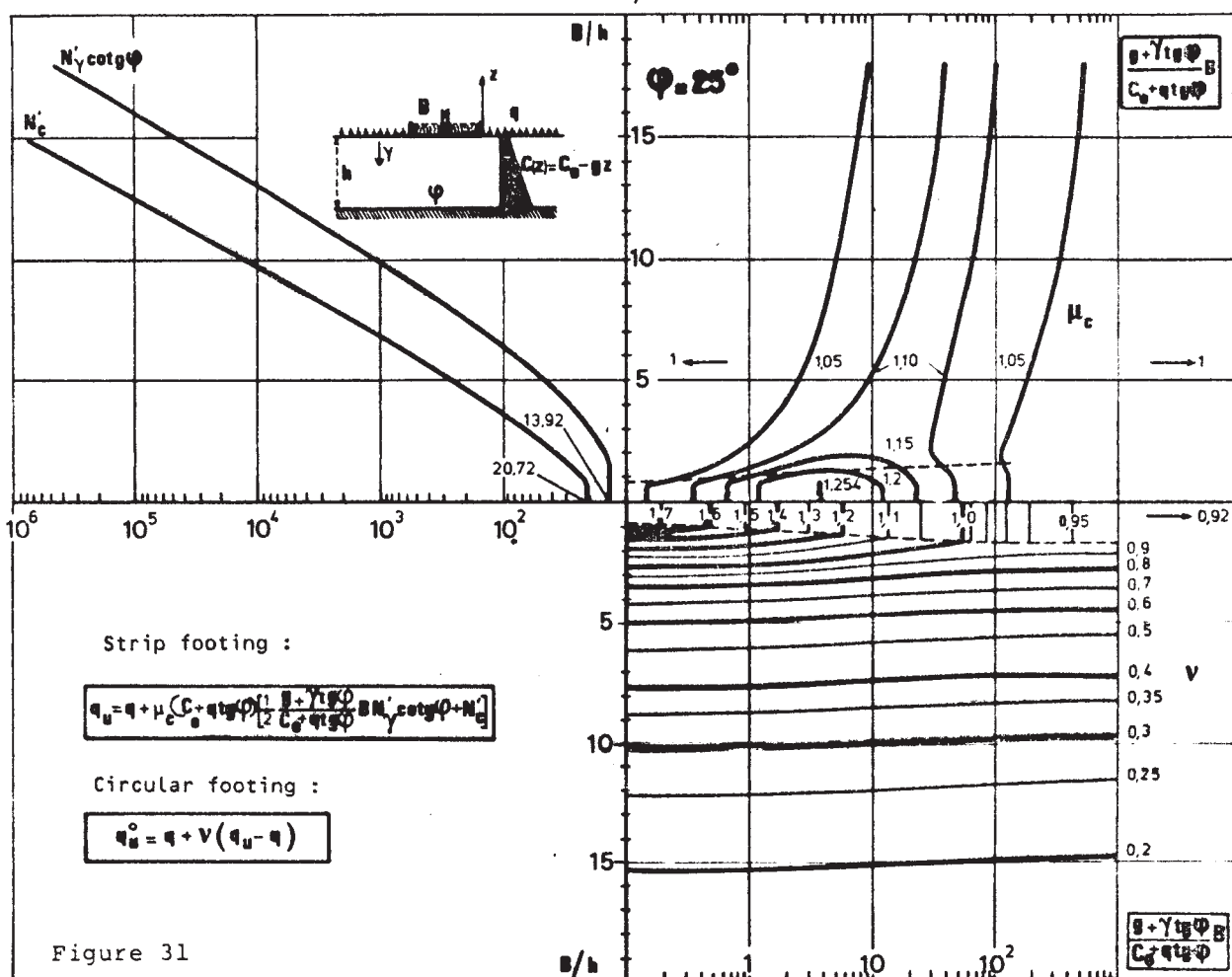

Figure 27

Figure 28

Figure 29

Figure 30

Strip footing :

$$q_u = q + \mu_c \left(C_o + q\,tg\varphi \right) \left[\frac{1}{2} \frac{g + \gamma\,tg\varphi}{C_o + q\,tg\varphi} B\,N'_\gamma\,cotg\varphi + N'_c \right]$$

Circular footing :

$$q_u^o = q + \nu \left(q_u - q \right)$$

$\varphi = 20°$

Figure 31

Strip footing :

$$q_u = q + \mu_c \left(C_o + q\,tg\varphi \right) \left[\frac{1}{2} \frac{g + \gamma\,tg\varphi}{C_o + q\,tg\varphi} B\,N'_\gamma\,cotg\varphi + N'_c \right]$$

Circular footing :

$$q_u^o = q + \nu \left(q_u - q \right)$$

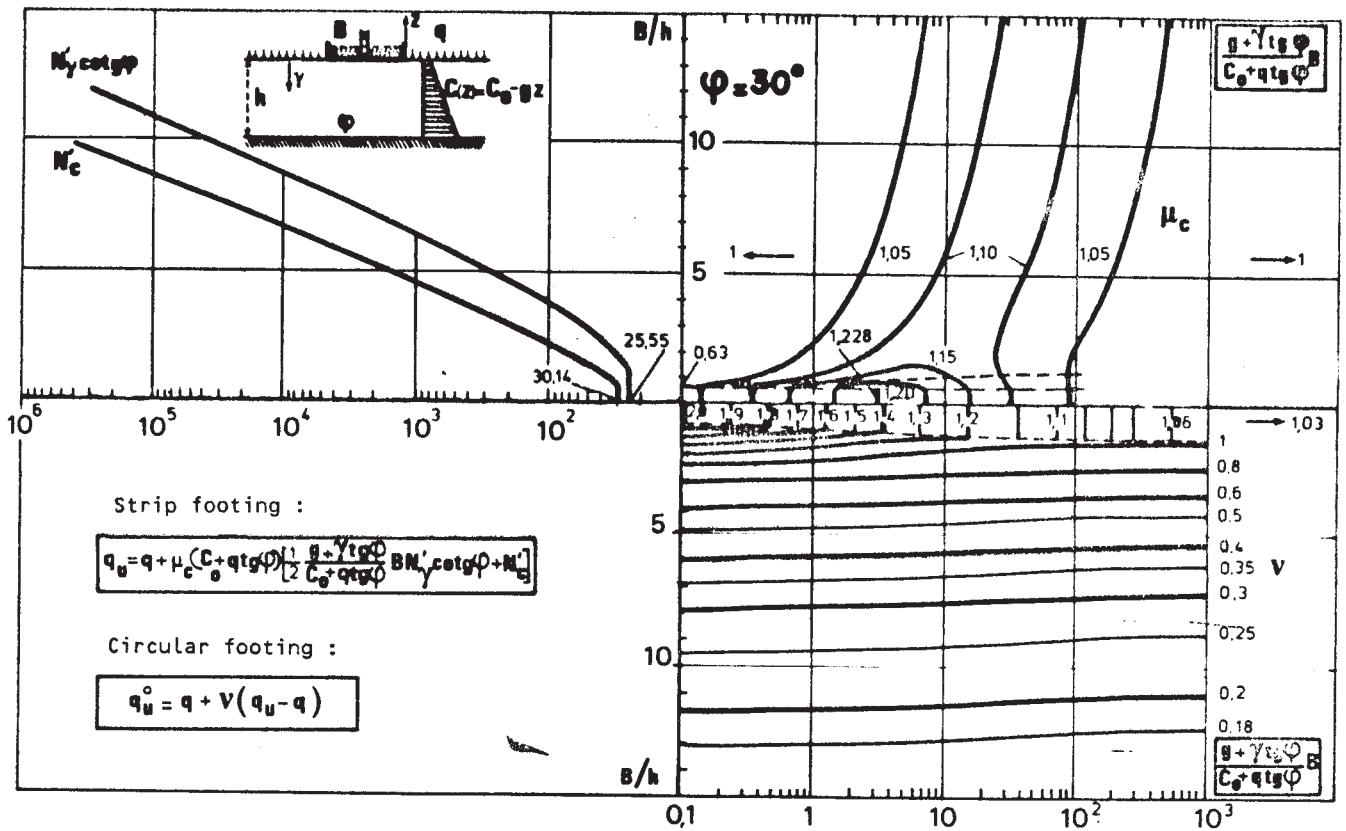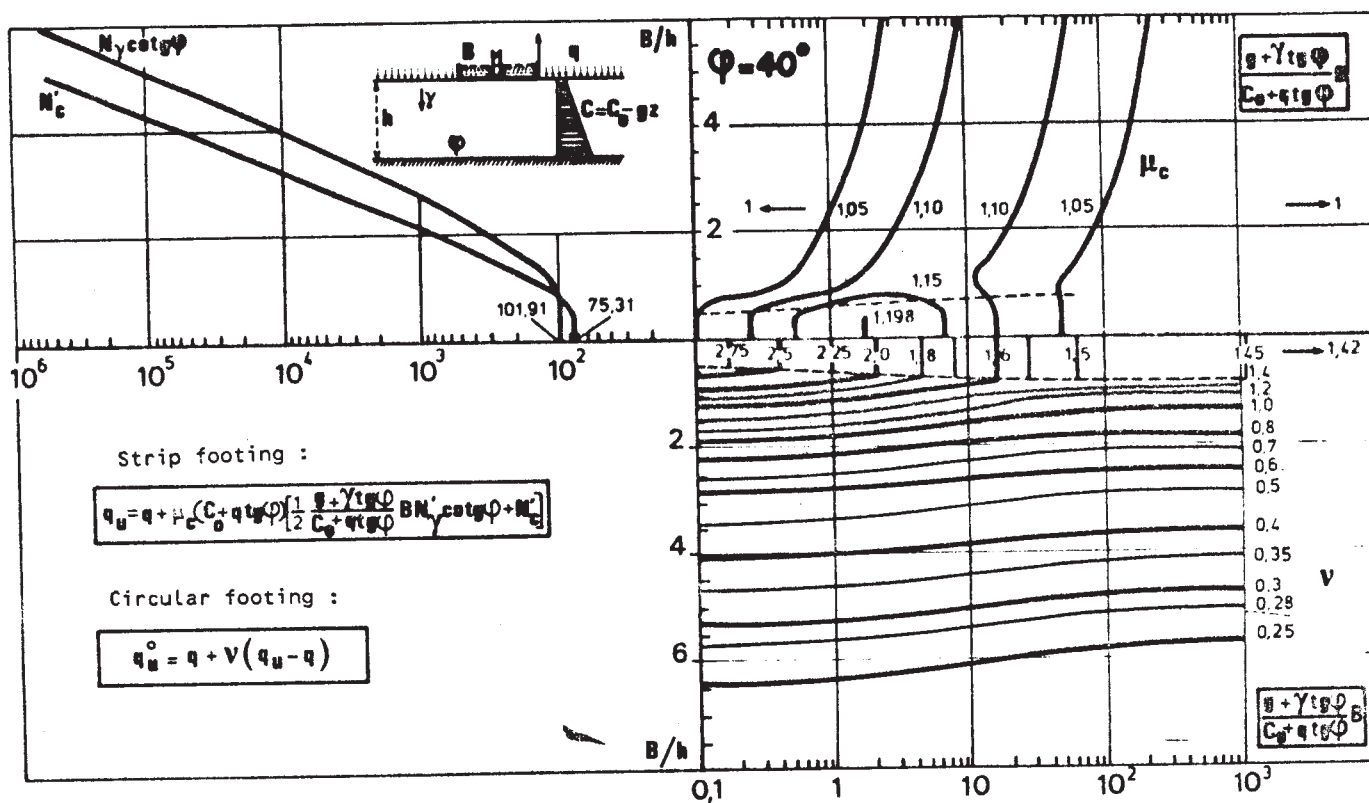$\varphi = 25°$
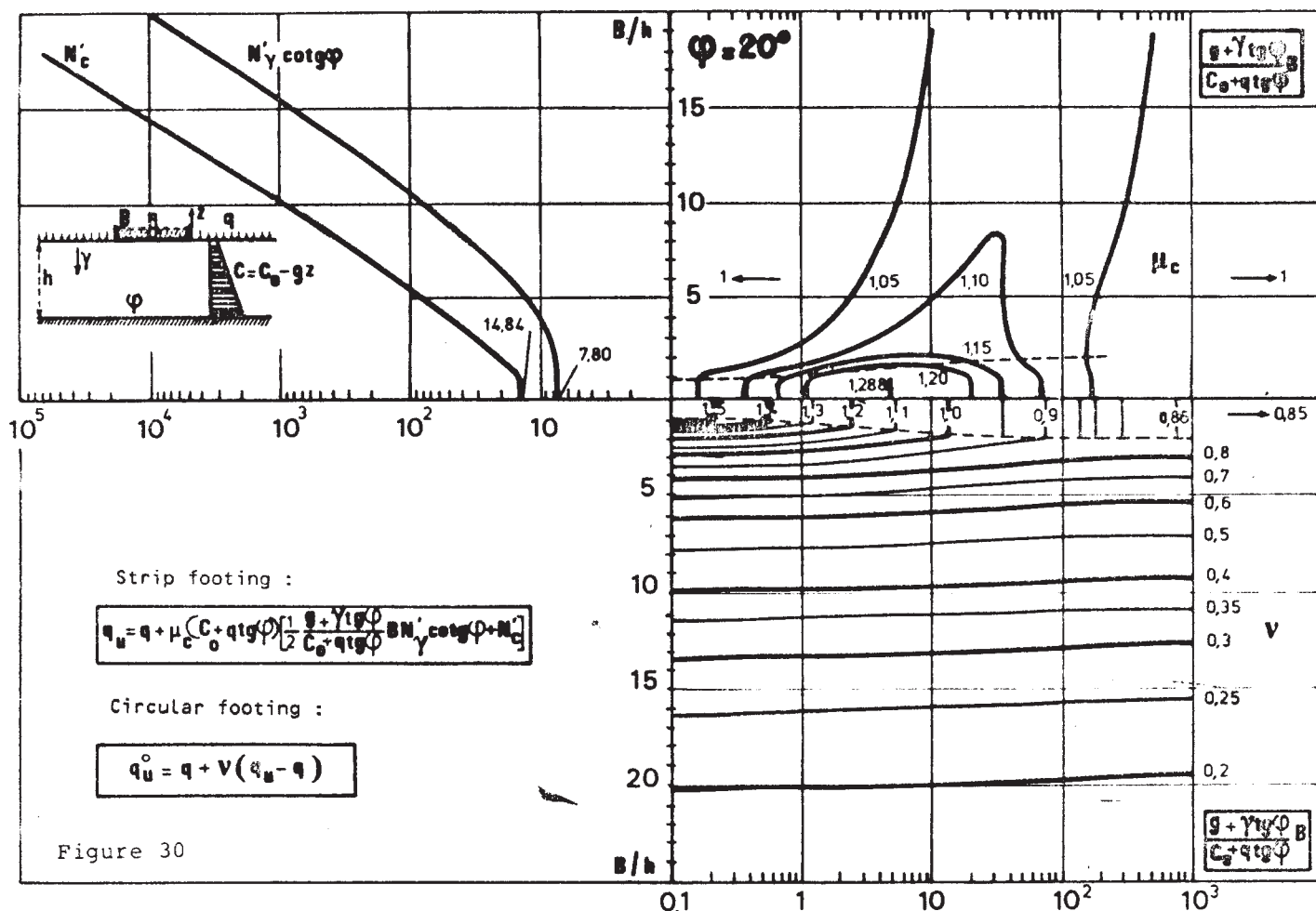
Figure 32
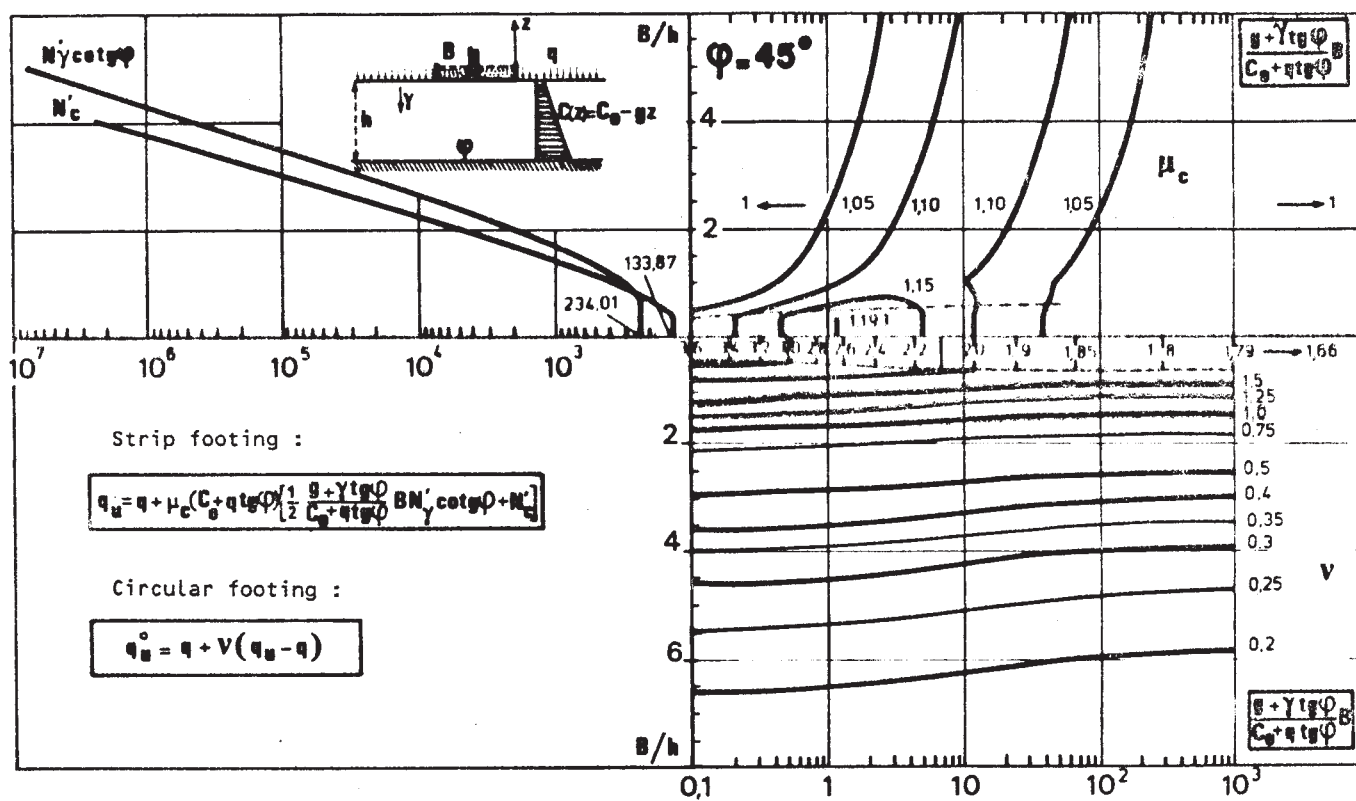

Figure 33

Figure 34

Figure 35

6. EXAMPLES OF THE USE OF THE "$iso\text{-}\mu_c$" MULTI-ENTRY CHARTS

6.1. Presentation

MATAR (1978) gives well detailed examples concerning the use of the superposition method and of the global calculation as well as of various practical "rules" used for the case of a strip-footing resting on a non-homogeneous soil of limited thickness. So as not to encumber our discussion we will only give the most significative ones.

6.2. Example 1 : site A

Friction angle of soil   : ϕ = 0°, then 4°,10°

Foundation width   : B = 40m, then 4m

Lateral surface load   : q = 0

Layer thickness   : h = 10 m

Surface cohesion   : C_o = 10^3 Pa

Descending vertical
cohesion gradient   : g = $2.5 \times 10^3 N/m^3$

Unit weight   : γ = $1.6 \times 10^4 N/m^3$

6.2.1. B = 40 m

From the multi-entry chart of figure 27 (ϕ = 0°) we read :

$$\mu_c = 1.48 \qquad N'_c = 6.25$$

whence $(q_u)_{superp.}$ = 3.12×10^4 Pa

and q_u = 4.62×10^4 Pa

This renders evident the magnitude of the bearing capacity underestimation by the superposition method.

Since, the case a non-homogeneous soil layer of limited thickness has been discussed by MANDEL and SALENÇON (1969 and 1972), in practice, one can think of handling the above problem as if it were a homogeneous soil layer by such rules as :

. only taking the surface cohesion into account, leading to an underestimation of the bearing capacity :

$$(q_u)_o = 0.625 \times 10^4 \text{ Pa}$$

This value yield a superabondance of dimensionings.

. by using an "average" cohesion, for instance :

$$C_m = C_o + g \frac{h'}{2}$$

where h' is the thickness of the disturbed zone corresponding to a homogeneous soil :

$$h' = \min (h, \frac{B\sqrt{2}}{2})$$

Thereby :

$$C_m = 13.5 \times 10^3 \text{ Pa}$$

and the estimated bearing capacity is :

$$\overline{q_u} = 8.44 \times 10^4 \text{ Pa}$$

Besides the difficulty incurred in using this method, it leads to an overestimation of the bearing capacity, which is obviously unacceptable.

A comparison between the bearing capacity values obtained by the different methods is illustrated by figure 36.

An examination of the influence that the friction angle, even when very small, could have on the bearing capacity is not without benefit. For the above example, assuming ϕ = 0°, 4°, 10°, we have :

when ϕ = 0° $\overline{q_u}$ = 4.62 x 10^4 Pa
when ϕ = 4° q_u =10.24 x 10^4 Pa
when ϕ =10° q_u =29.3 x 10^4 Pa

this clearly establishes (fig. 37) the very strong stabilizing effect of the friction angle, from the bearing capacity viewpoint.

6.2.2. B = 4 m

It will be seen that the remarks made above are also valid for the case of a narrower strip-footing.
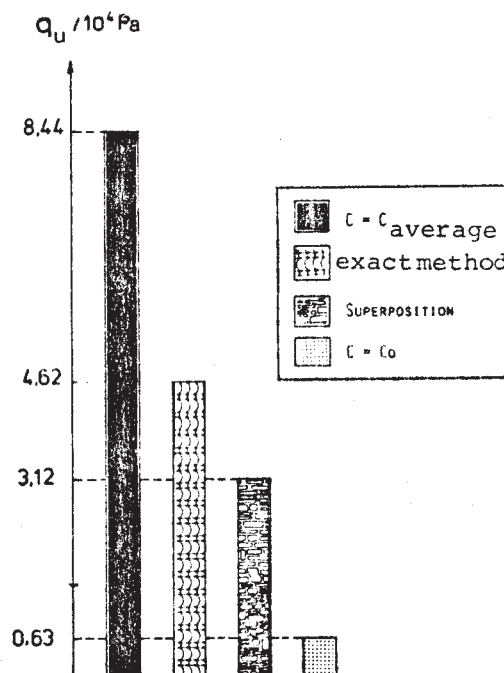
$q_u / 10^4 Pa$

Figure 36

From figure 27, we read :

$$\mu_C = 1.65$$

whence :

$$q_u = 1.26 \times 10^4 \text{ Pa}$$

Empirical methods, when used for the above mentioned non-homogeneous case, lead to :

when taking : $C = C_o$ $(q_u)_o = 0.51 \times 10^4$ Pa

          $C = C_m$ $\overline{q_u} = 2.33 \times 10^4$ Pa

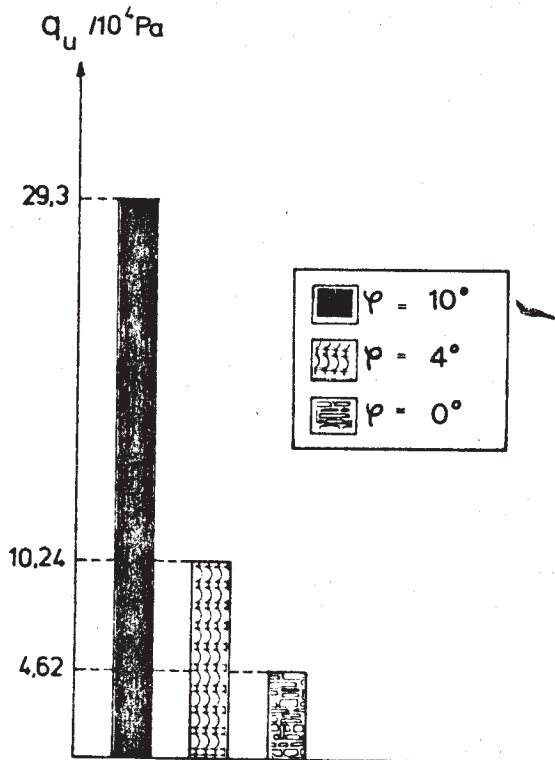Figure 38 shows the comparison between the values obtained from the different methods.

Figure 37

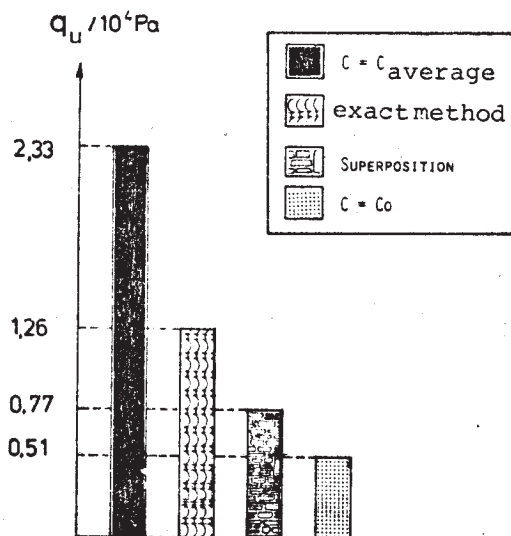Figure 38

The pronounced stabilizing effect of the friction angle can once again be evidenced here.

6.3. Example 2 : site B [1]

Friction angle of soil	: $\phi = 0°$, then $4°, 10°$
Foundation width	: $B = 40$ m
Lateral surface load	: $q = 0$
Layer thickness	: $h = \infty$
Surface cohesion	: $C_o = 0$
Descending vertical cohesion gradient	: $g = 0.6 \times 10^3 \text{N/m3}$
Unit weigt	: $\gamma = 1.6 \times 10^4 \text{N/m3}$

For the purely cohesive soil ($\phi = 0$), the various methods yield :

$$q_u = (q_u)_{superp.} = 0.6 \times 10^4 \text{ Pa}$$

$(q_u)_o = 0$ construction will be impossible

$\overline{q_u} = 4.36 \times 10^4$ Pa unacceptable overestimation

The stabilizing effect of the friction angle is again quite pronounced :

$\phi = 0$ $q_u = 0.6 \times 10^4$ Pa

$\phi = 4°$ $q_u = 3.85 \times 10^4$ Pa

$\phi = 10°$ $q_u = 16.83 \times 10^4$ Pa

6.4. Example 3 : site C

Soil frictional	: $\phi = 30°$
Foundation width	: $B = 4$ m
Unit weight	: $\gamma = 1.8 \times 10^4 \text{N/m3}$
Lateral surface load (footing depth : 1 m)	: $q = 1.8 \times 10^4 \text{N/m3}$
Layer thickness	: $h = \infty$
Surface cohesion	: $C_o = 1.6 \times 10^4$ Pa
Descending vertical cohesion gradient (homogeneous medium)	: $g = 0$

Referring to the multi-entry chart of figure 32, we read :

_____

[1] Clay in Mediterranean (LE TIRANT 1976)

$\mu_c = 1.20$

whence : $q_u = 1.20\ (q_u)_{superp.} - 0.20\ q$

The bearing capacity underestimation resulting from the superposition method is, in this case, around 20 %.

7. COMPARISON WITH THE MODELS'TEST RESULTS

7.1. Presentation

The purpose of this comparison of the theorical study results, given in the preceding chapters and materialized in the form of multi-entry charts for the computation of the strip-footing capacities, with the test results from models has been explained in chapters 1 and 2 (§ 2.6). It is concerned with evaluating the practical import, vis-à-vis the modelized problem, of the notion of bearing capacity as was defined.

Figure 39 schematically represents a test unit within which one must attempt to satisfy all the conditions in order to reproduce the modelized problem, especially from the viewpoint of the strip-footing, for example :

. The foundation is rigid and axially loaded with a constant lineic density.

. Adhesion between foundation and soil is total.

. Test trough is rigid.

. Adhesion between test trough bottom and soil is total.

. The lateral faces of the test trough, perpendicular to the foundation's longitudinal axis, are smooth and in direct contact with the foundation.
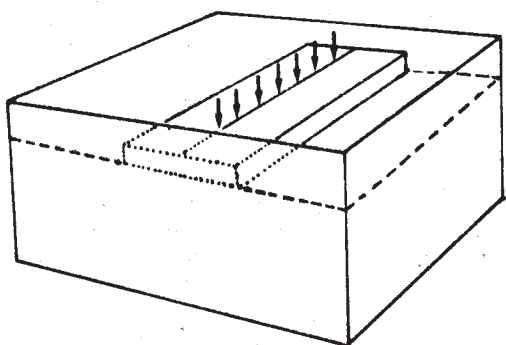

Fig.39 Test trough

7.2. Results and remarks

This comparison is made with the tests carried out by TOURNIER (1) in a study of the bearing capacity of a strip-footing on a homogeneous soil layer of limited thickness. They were performed on a layer of dry sand, of thickness h, average unit weight $\gamma \simeq 1.64 \times 10^4$ N/m3 and with a friction angle of $\phi = 38°$ (determined from triaxial tests). Several values of B/h were considered for footing widths, B = 0.20m 0.35m and 0.50m.

The modelized problem is thus one of the basic problems for the study of the bearing capacity of a footing on a non-homogeneous soil layer. It is in fact problem "s" of § 3.1. corresponding to the theoretical bearing capacity given by question 3.1a :

$$q_u = 1/2\ \gamma BN'_\gamma (B/h,\ \phi)$$

Table II gives the experimentally determined values of the ratio $2q_u/\gamma B$, i. e. the "experimental N'$_\gamma$".

B/h \\ B	0,20 m	0,35 m	0,50 m
0,15	150		
0,33	150	128	90
0,5	150	128	101
0,67			114
0,8	184		
1	231	124	134
1,16		194	
1,41		244	
2	508	192	

Table II

"Experimental N'$_\gamma$" according to TOURNIER and MILOVIC

The results clearly bring out the experimental difficulties (e. g. divergence of the values obtained for a single value of B/h). Various reasons may be suggested for this, of which the difficulty in evaluating foundation failure and thus to select the load value to be used, is not the least important.

(1) TOURNIER (1972), TOURNIER and MILOVIC (1977)

They are represented on figure 40 where it will be noted that the values found are all greater than those on the theoretical curve corresponding to $\phi = 38°$. Various explanations may be proposed.

. Foundation failure does not occur in the initial geometry but follows upon a certain settlement which, for the theoretical bearing capacity to be used, no longer leads to equation 3.1a but rather to the global equation 5.1 so as to account for the surface load.

. Work-hardening of the soil under the foundation can also be evoked.

A dotted curve has been plotted on figure 40. With the exclusion of one deviating point, this dotted curve would represent rather closely the experimental results. It is seen that this curve would be in close agreement with the curve representing $N'_\gamma(B/h, \phi)$, provided that the value used for ϕ be 42°.
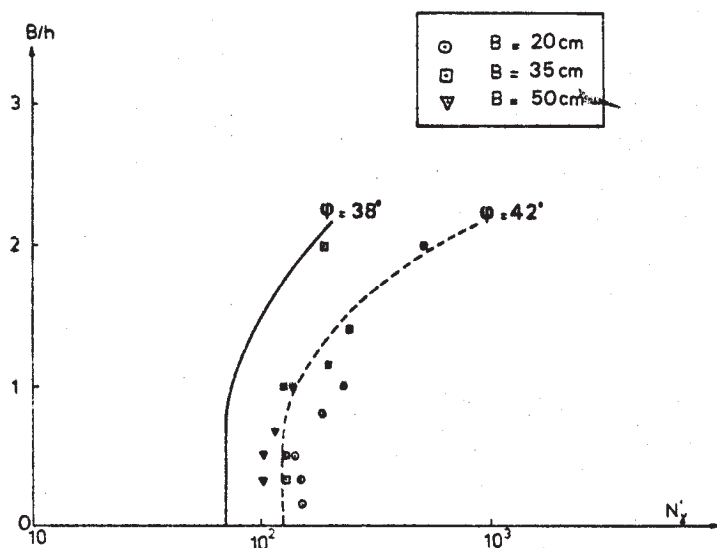

Fig.40

This can lead to questioning the value of the friction angle that is to be used in the calculations concerning strip-footing, i. e. to in fact adopt for each soil a tridimensional failure criterion other than the "intrinsic-curve" of COULOMB, SALENÇON and HALPHEN (1979) discuss this point which is related to the "true triaxial" experiments performed by numerous teams (e. g. LADE and DUNCAN, 1973) and to the considerations of HABIB (1958 and 1961).

The main conclusion to be drawn from this comparison seems to us to be that the exact theoretical value of the bearing capacity derived from the definition of 12.2 and calculated by the theoretical equation 3.1a with the given coefficients using the friction angle read at the triaxial is less than the load which experimentally appears as corresponding to foundation failure. This result tends to-

ward a greater safety margin. It would be worthwhile to confirme this by testing under conditions where the coupling effect would appear, q_u then being calculated by equation 5.1 or 5.2, notably for the case where μ_c reaches its highest values.

8. CONCLUSION

The theoretical aspects, linked to yield design, considered in this paper will not be brought up again ; we will simply concern ourselves here with bringing out the pertinent contributions as related to the computation viewpoint.

An equation was given that, for a soil layer of limited thickness and having cohesion linearly increasing with depth, enables one to take the cohesion gradient into account in order to evaluate, by means of the superposition method (linear equation), the bearing capacity of a strip-footing. This result is based on the "unit weight – vertical cohesion gradient" equivalence.

The formulae and multi-entry charts, conveniently enabling the exact calculation of this bearing capacity by taking all the coupling effects into account, are given.

The use of these "iso-μ_c" multi-entry charts brings into view a favourable correction, possibly large, to the results obtained by the superposition method traditionally employed to calculate the bearing capacity of strip-footings on homogeneous soil.

It offers a precise reply to the commonly encountered problems, handled at present, in an empirical manner, by two types of methods. The dangerous aspect of one of these methods has been shown. The other method is assuredly conservative, but may render the construction expensive and sometimes impossible.

It also enables one to see the very pronounced stabilizing effect, from the viewpoint of bearing capacity, of the friction angle even for soils having little friction contact.

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