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BEARING CAPACITY OF SURFACE FOUNDATIONS

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1 - INTRODUCTION - BEARING CAPACITY OF A FOUNDATION

The calculation of surface foundations is usually governed by two conditions, one referring to settlement and the other to foundation "stability". For this second condition, the *bearing capacity* of the foundation is derived from the knowledge of the strength-criterion defining the admissible stress-states for the soil, then the admissible load for the foundation is restricted to a fraction of this bearing capacity, by means of a safety factor. This paper deals with the bearing capacity problem. It aims at improving the corresponding state of knowledge both in classical and non-classical (but non-pathological) cases, leading to a better approach of the safety-factor and making the calculation of foundations possible in circumstances for which no sufficient information was available before.

The following problem stated in the case of a strip footing with an axial load, is typically the problem to be solved for the determination of the bearing capacity of a surface foundation : given a rigid footing with a breadth B, loaded by an axial force Q and acting on a soil defined by its strength-criterion, it is to be determined whether equilibrium is *possible* with this load Q the strength criterion of the soil being satisfied everywhere. This is the fundamental problem of *yield design*. The solution follows from the theory of yield-design (Salençon, 1976-1978) : a value Q^+ can be determined such that equilibrium and strength-criterion are compatible if $Q < Q^+$ and incompatible if $Q > Q^+$. This result being the strongest that can be produced for the proposed problem, it leads to defining the bearing capacity as Q^+ or, more precisely, as the average "pressure" $q_u = Q^+/B$.

Q^+ (and q_u) can be determined, or at least lower bounds be obtained for them using the static approach which derives evidently from their very definition given above. Besides it is shown that Q^+ and q_u can also be determined, or at least upper bounds be easily obtained, through the kinematic approach gained by *dualizing mathematically* the equilibrium equations using the principle of virtual work and introducing the *support function* derived from the sole strength-criterion. These are known as the approaches from inside and outside in the theory of yield design ; it is to be emphasized that though analogous to the theorems of limit analysis, they are not related to the theory of Plasticity.

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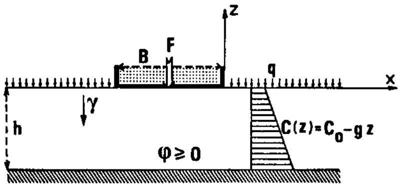
2 - THE STUDIED PROBLEM



The following problem will be studied (figure 1) : bearing capacity of a rigid strip-footing, axially loaded, on a soil layer with a cohesion linearly increasing with depth, resting on a rigid base ; perfect friction is assumed to occur at both interfaces. Though such soils are encountered in current practise (e.g. sea-soils), no common definite rule seems to be available for the calculation of bearing-capacities in that case. It will be shown that the problem can be tackled within the frame of the superposition method, yielding a non-trivial linear lower bound for the bearing capacity which makes use only of previously published results. Then through a new study and the corresponding computations, allowing for all effects simultaneously, the validity of the linear formula will be examined in general, be the soil homogeneous or non-homogeneous and the layer with limited or unlimited thickness. Finally a practical method for the calculation of the bearing capacity will be given based on the use of simple charts.

3 - USE OF THE SUPERPOSITION METHOD

For a soil with $\varphi \neq 0$ the bearing capacity q_u is related by the formula $q_u = q_u^1 + q$, to the bearing capacity q_u^1 corresponding to the problem, geometrically identical to that of § 2, and defined as follows : *homogeneous* soil, with unit weight $\gamma^1 = \gamma + g \cot \varphi$, cohesion $C^1 = C_o + q \tan \varphi$, same friction angle φ and no surface load.

The static approach can be applied to the latter problem without any difficulty, using the bearing capacity factors given by Mandel and Salençon (1972) for a strip-footing on a homogeneous layer of limited thickness. A linear lower bound for q_u is thus obtained, denoted $(q_u)_{superp.}$:

$$q_u \geqslant (q_u)_{superp.} = (q_u^1)_{superp.} + q = \frac{1}{2}(\gamma + g \cot \varphi)\, BN'_\gamma(\varphi\ , B/h) + (C_o + q \tan \varphi)\, N'_c(\varphi\ , B/h) + q. \quad (1)$$

The case of the $\varphi = 0$ soil must be investigated separately :

1° for $C_o = 0$ and $q = 0$, the bearing capacity is $q_u = gB/4$, whatever B/h.

2° in the general case, adding the bearing capacity corresponding to the constant cohesion C_o and the surface load q, and that corresponding to the cohesion gradient g, yields a lower bound $(q_u)_{superp.}$ for the bearing capacity q_u corresponding to the global problem of fig. 1 ($\varphi = 0$) :

$$q_u \geqslant (q_u)_{superp.} = gB/4 + C_o\, N'_c(0,\ B/h) + q. \quad (2)$$

(continuity between eq. (1) and (2) as $\varphi \to 0$ corresponds to the property $N'_\gamma(\varphi,\ B/h) \sim \varphi/2$ as $\varphi \to 0$, whatever B/h).

4 - GENERAL COMPUTATION OF THE BEARING CAPACITY

The exact value of the bearing capacity for the problem of figure 1 is obtained through a global analysis taking simultaneously into account the effects of gravity, cohesion, cohesion-gradient, and surface load. It is first noticed that the bearing capacity q_u is related by : $q_u = q_u^2 + q$, to the bearing capacity q_u^2 corresponding to the problem, geometrically identical to that of § 2, and defined as follows for $\varphi \geqslant 0$: *weightless* soil, with the same friction angle φ and no surface load, and with a $C(z) = (C_o + q \tan\varphi) - (g + \gamma \tan\varphi)z$. The general form of q_u is then obtained through dimensionnal analysis :

$$(3) \qquad q_u = q + (C_o + q \tan\varphi)\, F_c\left[(g + \gamma \tan\varphi)\, B/(C_o + q \tan\varphi), B/h, \varphi\right]$$

and it remains to determine function F_c. This is done by constructing the solution of the latter problem by means of the theory of plane limit equilibriums for non-homogeneous soil (v.z. Olszak, Rychlewski and Urbanowski (1962) for the basic equations).

The best way of presenting the obtained results proves to be by introducing a coefficient μ_c (analogous to that of Lundgren and Mortensen, 1953) in order to compare the values given by eq. (3) and the linear formulae (1) and (2):

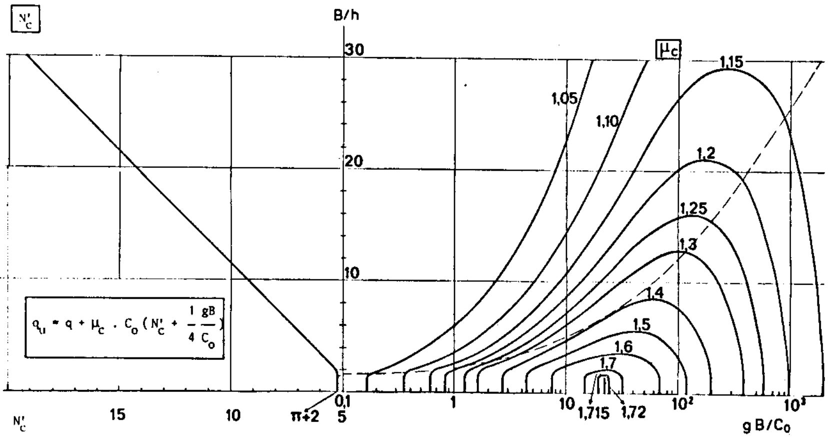
$$(4) \qquad \mu_c\left[(g + \gamma \tan\varphi)\, B/(C_o + q \tan\varphi), B/h, \varphi\right] = \frac{q_u - q}{(q_u)_{superp.} - q}$$

μ_c is obviously always $\geqslant 1$. It is equal to unity at both ends (0 and ∞) of the interval assigned to its first argument and passes by a maximum. Highest values of μ_c are reached when φ is small ($\neq 0°$) and B/h small : the maximum value is reached for $\varphi = 0°$, $B/h < \sqrt{2}$, and $gB/C_o = 23$, and is equal to 1.72 . Thus it is pointed out that the underestimation of the bearing capacity due to the use of the superposition method may be quite important for small values of φ (sea-soils).

5 - PRACTICAL DETERMINATION OF THE BEARING CAPACITY

Simple charts have been proposed by Matar and Salençon (1977), Matar (1978), based on the definition of μ_c (eq. 4) and on the linear formulae (1, 2). As an example the charts for $\varphi = 0°$ are given in fig. 2 . The determination of q_u only requires the values of N'_c and $N'_\gamma \cot\varphi$ and μ_c to be read or interpolated since from (1) and (4), it follows that :

$$q_u = q + \mu_c\left[(C_o + q \tan\varphi)\, N'_c + \frac{1}{2}(g + \gamma \tan\varphi)\, B N'_\gamma \cot\varphi\right].$$



6 - CONCLUSIONS

The interest of these results is attested by various examples of calculations of bearing capacities. For instance let us mention that, for a soil with a cohesion gradient, methods such that "taking only the surface cohesion into account"or "use of an average value of the cohesion" may prove to be either too much conservative, up to the point of making construction impossible, or on the contrary very risky and even dangerous. As the determination of the exact value of the bearing capacity is easy now, the designer is able, with full knowledge of facts to introduce the right safety factor required by the specific problem he is dealing with.

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