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AN ANALYSIS OF THE STABILITY OF EMBANKMENTS ON SOFT GROUNDS

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ABSTRACT

The stability of embankments on soft grounds is examined with the basic argument of yield-design ; from the knowledge of the strength criteria defining the allowable stress states in the constitutive soils a yield-factor is defined with a plain significance : if the yield-factor is less than unity, the embankment is certainly unstable. Upper bounds for the yield-factor can be determined either by static or by kinematic methods which are more convenient. This leads to the yield-factor method for stability analysis, based on the use of rigid-block mechanism. It proves to be efficient as regards for instance computation time and can be extended to deal with many practical cases of embankment stability as well as with earth-dam stability. The comparison of the so-obtained results with results given by a method of slices reveals some interesting discrepancies, proving the yield-factor method to be at least a valuable check for the well established classical methods for stability analysis in soil mechanics. By the yield-factor method a weak line is also determined in the considered embankment : it defines the block whose overall equilibrium satisfying the strength-criteria is the "weakest" ; this may give some information as regards the initial collapse mechanism for an unstable embankment.

CONTENTS

AN ANALYSIS OF THE STABILITY OF EMBANKMENTS ON SOFT GROUNDS

J. SALENÇON O. COUSSY

1. *INTRODUCTION*

Alike the study of the stability of earth slopes, the analysis of the stability of embankments on soft grounds is a theoretical problem far from being completely solved despite its great importance from the practical point of view. Numerous analytical methods have been devised for this kind of study and their very variety is a proof that none of them is thoroughly satisfying. During the recent years this diversity did increase due to the expansion of the finite element method. A good bibliography on the subject can be found in RAULIN, ROUQUES and TOUBOL (1974).

Stability analysis in soil mechanics, for instance when dealing with an embankment, or an earth-slope, aims at solving the following problem : given the embankment by the parameters defining its geometry, given the constitutive soils with the knowledge of the "strength-criteria" defining the allowable stress states, determine whether such a structure is stable or unstable under a loading consisting in the self weight of the system and given additional loads. This problem is the fundamental problem of what we call "yield-design" (an english translation for the French "Calcul à la Rupture").

In soil mechanics the first one to have stated the problem of stability in such a way is certainly C.A. COULOMB in 1773 ; but this kind of problem is also to be dealt with in many other branches of the engineering practice. Recently a theory of yield-design has been built up (SALENÇON, 1976-1978) which covers all these various aspects. It includes some of DRUCKER's (1954) and RADENKOVIC's (1961,1962) works and is, roughly speaking, parallel to what may be found in the book by W.F. CHEN (1975).

The outstanding result of the theory of yield-design is to show that, from the only knowledge of the "stress criterion" defining the allowable stress states for the constitutive material, it is possible and only possible to decide whether a given structure under a given loading (for instance the above-mentioned embankment) is *certainly unstable* or, on the contrary, *potentially stable*. Therefore only a "yield-factor" can be defined for the structure.

The theory makes it possible to point out the origin of the difficulties encountered with some classical methods used for stability analysis in soil mechanics (COUSSY and SALENÇON, 1978). From the practical point of view it shows that upper-bounds for the yield-factor can be determined by means of static considerations or, which is easier, of kinematic ones, through the only use of the principle of virtual works without any intervention of a flow-rule or of anything similar.

The paper will present an analysis of the stability of embankments on soft grounds within the frame of yield-design. The theory of yield-design being only exposed in what concerns the studied problem and on this particular case, readers looking for a more exhaustive view should report to the above-mentioned references. Attention will be focused on the use of rigid block mechanisms to obtain upper bounds for the yield-factor ; this will lead to the "yield-factor method" for stability analysis. Finally some other applications of the method will be evoked, for instance dealing with the stability of earth-dams.

2. YIELD-FACTOR FOR AN EMBANKMENT RESTING ON A SOFT GROUND LAYER

2.1. *The studied problem*

The problem is to analyse the stability of an embankment made of a homogeneous soil with both cohesion and friction, resting on a purely cohesive ground layer which lies on what may be considered as a rigid bed rock.

The geometrical parameters are defined on figure 1.

The "strength-criterion" for the soil constituting the embankment is represented by Coulomb's criterion with a cohesion C and a friction angle ϕ ; it may be written :

$$|\tau| \leqslant C + \sigma \tan \phi \ , \qquad \forall \ \underline{n} \ , \tag{2.1}$$

where σ and τ stand for the normal and tangential components of the stress acting on any typical facet with a normal $\underline{n}$ (compressive stresses are counted positive, that is $\underline{n}$ is the inward normal to the facet).

For the soil in the layer, the strength-criterion reduces to :

$$|\tau| \leqslant C' \tag{2.2}$$

The strength-criteria at the contact between the embankment and the layer, and between the layer and the bed-rock, must also be evoked here ; a discussion on the influence of these criteria on the stability of the structure can be found in Coussy (1978). In order to have matters fixed we shall assume that perfect adhesion takes place at both interfaces, but this assumption will have no influence on the significance of the final results obtained.

The loading for the problem is defined by γ and γ' , the specific weights of the soils in the embankment and in the layer respectively.

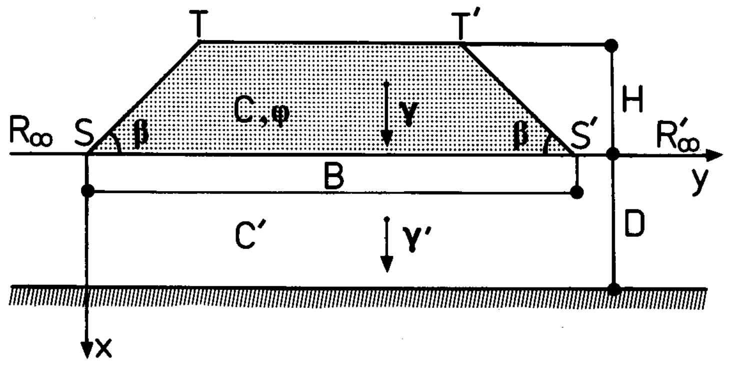

Figure 1 :
Stability analysis of an embankment on a soft ground layer

The problem is then plainly formulated by the question : *given the embankment with its geometry, its strength-criteria, and the loading, is it possible that this structure be stable ?*

The answer to this question derives from a very simple reasoning ; it can immediately be stated that :

(2.3) *In order for the embankment to be stable there must exist at least one stress field* $\underline{\underline{\sigma}}$, *satisfying all the equations of static equilibrium, under the restraint that (2.1) or (2.2), according to the soil, must be fulfilled everywhere.*

As this obvious statement is only a necessary condition, an embankment for which such a stress field *does* exist will be called "*potentially stable*". Conversely an embankment for which such a stress field *doesn't* exist is *certainly unstable*.

2.2 Definition of the yield-factor

The parameters defining the problem can be sorted in three groups :

B, H, D, β, *defining the geometry,*
C, C', ϕ, *defining the strength,*
γ, γ', *defining the loading* (γ, $\gamma'>0$)

Then, taking the geometry and the strength as constant and considering γ and γ' to be variable we get a family of problems of stability analysis for the same embankment, depending on the loading.

As a consequence of the convexity of the strength-criteria (*2.1*) and (*2.2*), the following result can be stated : in the plane of the loading parameters (γ, γ'), the loading-points corresponding to potentially stable embankments form a convex domain including point 0.

Figure 2 shows this domain in the shape sketched by COUSSY (1978) after a priori considerations, but this shape is not involved in the continuation of the demonstration.

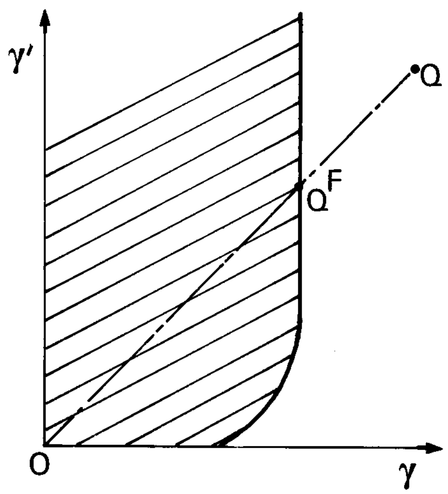

Figure 2 :
Potentially stable embankments with fixed geometry and strength- criteria.

The yield-factor is then defined as follows : considering the embankment under the loading case (γ,γ') represented by point Q in the plane of the loading parameters, the yield-factor is :

(*2.4*) $$F=\overline{OQ^F}/\overline{OQ},$$

where Q^F is the point at the intersection of the vector-radius $\underline{OQ}$ with the boundary of the convex domain of potentially stable embankments.

It is clear form this definition that the scalar factor F exhibits the following property :

(2.5) $$\begin{cases} F<1 \Longrightarrow & \textit{unstable embankment,} \\ F>1 \Longrightarrow & \textit{potentially stable embankment.} \end{cases}$$

F is a function of all the parameters defining the problem ; it results from a dimensional analysis performed on this function that F must assume the form :

(2.6) $$F=F\left(\frac{C'}{\gamma H}\ ,\ \frac{C}{C'}\ ,\ \frac{\gamma'}{\gamma}\ ,\ \frac{B}{H}\ ,\ \frac{D}{H}\ ,\ \beta\ ,\phi\ \right)\ .$$

Besides, from definition (2.4), it is evident that multiplying both γ and γ' by the same positive constant results in multiplying F by the same constant. It follows that F will necessarily be written :

(2.7) $$F=\frac{C'}{\gamma H}\ K^{+}\left(\frac{C}{C'}\ ,\ \frac{\gamma'}{\gamma}\ ,\ \frac{B}{H}\ ,\ \frac{D}{H}\ ,\ \beta,\ \phi\right)$$

where K^+ is a scalar function of the indicated arguments.

2.3. *Significance of the yield-factor*

Equation (2.7) shows that the yield-factor F can be given 3 different meanings :

1) *a factor with respect to cohesions,*
if the embankment with parameters B, H, D, β, C, C', ϕ, γ, γ', is potentially stable, its yield-factor being equal to F>1, then any embankment with the same parameters B, H, D, β, ϕ, γ, γ', and the cohesions $\overline{C} = C/\lambda$ and $\overline{C}' = C'/\lambda$, where λ is >F, is certainly unstable (for this embankment : $\overline{F}$ <1),

2) *a factor with respect to geometry,*
if the embankment with parameters B, H, D, β, C, C', ϕ, γ, γ', is potentially stable,its yield-factor being equal to $F>1$, then any embankment with the same parameters C, C', ϕ, γ, γ', and the geometrical parameters $\overline{B}=\lambda B$, $\overline{H}=\lambda H$, $\overline{D}=\lambda D$, $\overline{\beta}=\beta$, where λ is $> F$, is certainly unstable ($\overline{F} < 1$).

3) *a factor with respect to the specific weights,*
if the embankment with parameters B, H, D, β, C, C', ϕ, γ, γ', is potentially stable, its yield factor being equal to $F>1$, then any embankment with the same parameters B, H, D, β, C, C', ϕ, and the specific weights $\overline{\gamma}'=\lambda\gamma'$ and $\overline{\gamma}=\lambda\gamma$, where λ is $> F$, is certainly unstable (such a case happens when reduced scale models are tested in a centrifuge).

As will appear in the following sections, the yield-factor F defined by eq. (2.4), is very convenient for practical use, especially from the kinematic point of view. The first interpretation given above shows that F appears as a factor with respect to cohesions ; when dealing with classical methods for stability analysis, such as the method of slices, it is usually preferred to refer to a factor with respect to the total strength : that means that the factor instead of concerning the cohesion of each soil, will concern the maximum feasible shear-strengths for a given value of σ , namely $|\tau|=C+\sigma\tan\phi$ and $|\tau|=C'$, therefore it will actually concern both C' and C on the one hand and $\tan\phi$ on the other.

There would be no difficulty in defining a yield-factor $\mathcal{F}$ with respect to the total strength. $\mathcal{F}$ would be the solution of the implicit equation :

$$(2.8) \qquad \mathcal{F} = \frac{C'}{\gamma H}\, K^{+}\left(\frac{C}{C'}, \frac{\gamma'}{\gamma}, \frac{B}{H}, \frac{D}{H}, \beta, \tan^{-1}(\tan\phi/\mathcal{F}) \right) ,$$

and therefore need the use of iterative procedures for its determination. It is clear that the so-defined $\mathcal{F}$ exhibits the same property (2.4) as F and that, for a given embankment, the values of F and $\mathcal{F}$ are on the same side of the crucial value 1 :

$$(2.9) \qquad F\left(\frac{C'}{\gamma H},\frac{C}{C'},\frac{\gamma'}{\gamma},\frac{B}{H},\frac{D}{H},\beta,\phi\right) \gtreqless 1 \Longleftrightarrow \mathcal{F}\left(\frac{C'}{\gamma H},\frac{C}{C'},\frac{\gamma'}{\gamma},\frac{B}{H},\frac{D}{H},\beta,\phi\right) \gtreqless 1 \quad .$$

3. UPPER BOUND FOR F OBTAINED THROUGH STATIC METHODS

The introduction of the yield-factor F given above, as well as all the arguments used are founded on considerations of static equilibrium.

For given values of parameters $\frac{C}{C^T}$, $\frac{\gamma'}{\gamma}$, $\frac{B}{H}$, $\frac{D}{H}$, β, ϕ, the value of the function K^+ in formula (2.7) appears as the highest value of the parameter $\frac{\gamma H}{C^T}$ for which there exists a stress-field $\underline{\underline{\sigma}}$ satisfying the necessary condition (2.3).

Conversely, any value of $\frac{\gamma H}{C^T}$ for which such a stress-field doesn't exist is an upper-bound for K^+. This means that any value of $\frac{\gamma H}{C^T}$ for which the equilibrium equations and the strength-criteria are incompatible, is an upper bound to K^+.

Knowing such an upper bound K_M to K^+ for a given set of parameters ($\frac{C}{C^T}$, $\frac{\gamma'}{\gamma}$, $\frac{B}{H}$, $\frac{D}{H}$, β, ϕ) we get an upper bound F_M of the yield-factor F for any embankment with the same set of parameters ($\frac{C}{C^T}$, $\frac{\gamma'}{\gamma}$, $\frac{B}{H}$, $\frac{D}{H}$, β, ϕ) whatever the value of $\frac{\gamma H}{C^T}$ by the use of the formula :

(3.1) $$F_M = \frac{C'}{\gamma H} K_M \quad .$$

From this point of view derive the methods for stability analysis which are founded on the equilibrium of a block. The idea is simple :

Consider, as represented on figure 3, a curve such as RBA drawn across the embankment itself and the layer.

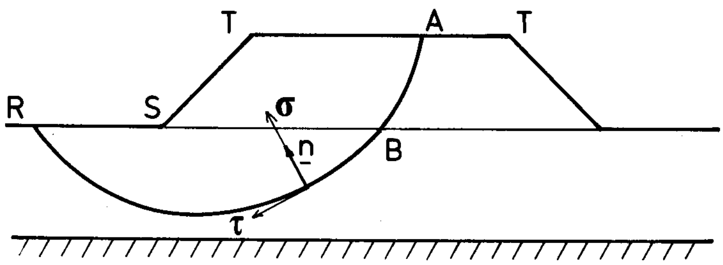

Figure 3 :
Stability analysis by studying the equilibrium of a block.

It results from condition (2.3) that, in order for the embankment to be stable, the overall equilibrium of the block RBATSR must be assured under the conditions (2.1) and (2.2) written along RB and BA respectively for the facets tangent to this curve. This condition is evidently weaker than (2.3).

Thus, any value of $\frac{\gamma H}{C'}$ for which the overall equilibrium of block RBATSR is incompatible with the strength-criteria (2.1) and (2.2) along RBA is an upper bound to K^+.

The difficulty of such an analysis is to prove this incompatibility since for a given curve RBA one doesn't a priori know the optimal distribution of the stress components σ, τ, that is the distribution which would ensure compatibility for the highest value of $\frac{\gamma H}{C'}$. This is the very origin of the differences between classical methods based upon this static approach and which introduce different hypotheses for the distribution $(\sigma,\tau)_{RBA}$: therefore the maximum value so-obtained for $\frac{\gamma H}{C'}$ may lose the character of an upper bound of K^+, which makes it impossible to interpret the results rigorously.

As in the case of earth-slope stability analysis it can be shown that the use of particular forms for the curve RBA makes complementary hypotheses unnecessary ; the problem is then sometimes called "statically determined". The maximum value obtained for $\frac{\gamma H}{C'}$ is a real upper-bound for K^+.

We don't think it useful to insist on this aspect of the question, since the form of curve RBA will be dealt with again in the following sections from the kinematic point of view, which seems to us clearer and more efficient.

4. UPPER BOUNDS FOR F OBTAINED THROUGH KINEMATIC METHODS

The principle of virtual work makes it possible to transform condition (2.3) into the following statement :

(4.1) In order for the embankment to be stable there must exist at least one stress-field $\underline{\underline{\sigma}}$, satisfying the strength criteria (2.1) and (2.2) everywhere ([1]) and such that, in any kinematically admissible velocity-field $\underline{v}$, the power of the stress-field $\underline{\underline{\sigma}}$ balances the power of the gravity forces (γ, γ') :

(4.2) {Power of $\underline{\underline{\sigma}}$ in $\underline{v}$} = {Power of (γ, γ') in $\underline{v}$}

Thus, to prove that F is less than unity it is sufficient to show that, for the considered embankment, in at least one velocity-field $\underline{v}$, conditions (4.2) and (2.1, 2.2) everywhere ([1]) are incompatible.

Said otherwise, for a given set of parmeters $(\frac{C}{C'}; \frac{\gamma}{\gamma'}, \frac{B}{H}, \frac{D}{H}, \beta, \phi)$, to prove that K_M is an upper-bound for K^+, it is sufficient to show that for this value of $\frac{\gamma H}{C'}$, in at least one velocity-field $\underline{v}$, conditions (4.2) and (2.1, 2.2) everywhere ([1]) are incompatible.

At this stage, such a dualized form of the approach given in sections 2 and 3 doesn't appear more convenient than the primal one.

The interest of this kinematic formulation comes from the fundamental property stated below, which is demonstrated in the theory of yield-design and will be proved directly in a particular case in section 5 :

whatever the stress-field $\underline{\underline{\sigma}}$ satisfying (2.1, 2.2) everywhere ([1]), and for any given velocity-field $\underline{v}$, the first member of (4.2) admits an upper bound $P(\underline{v})$ which is a univoquial function of $\underline{v}$.

The form of $P(\underline{v})$ depends on the parameters C, C', ϕ defining the conditions (2.1, 2.2) ; $P(\underline{v})$ is positively linear with respect to $\underline{v}$, non negative, and takes finite values for properly chosen velocity-fields $\underline{v}$.

It follows that : chosen $\underline{v}$ such that $P(\underline{v})$ is finite, any embankment for which the inequality (4.3) holds, is certainly unstable (F is less than unity) :

(4.3) $\quad P(\underline{v}) < \{$ Power of (γ, γ') in $\underline{v}$ },

---

([1]) i.e. (2.1) in the embankment itself and (2.2) in the layer.

since the power of (γ, γ') in $\underline{\upsilon}$, denoted $P(\gamma, \gamma', \underline{\upsilon})$ is evidently bilinear with respect to (γ, γ') and $\underline{\upsilon}$, inequality *(4.3)* separates the plane (γ, γ') of figure 2 into two half-planes as shown on figure 4 : any potentially stable embankment must lie in the half-plane $P(\underline{\upsilon}) > P(\gamma, \gamma', \underline{\upsilon})$.

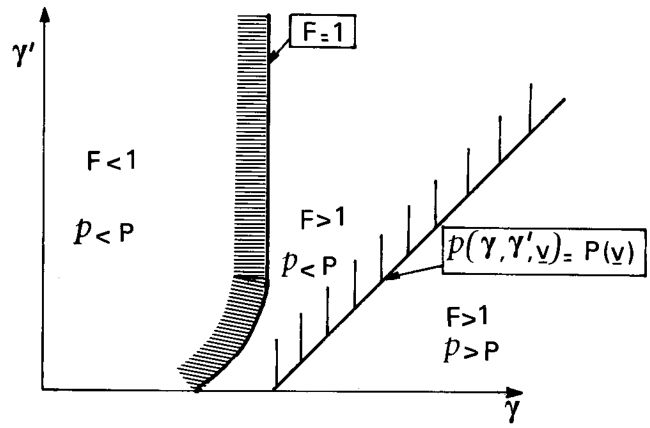

Figure 4 :
Upper-bound for F through the use of kinematic methods.

It is clear from figure 4 that for a considered embankment, any kinematically admissible velocity-field such that the power of the external (i.e. gravity) forces is positive will provide an upper-bound of the yield-factor, namely :

(4.4) $$F_{\upsilon} = P(\underline{\upsilon}) \;/\; P(\gamma, \gamma', \underline{\upsilon}) \geqslant F$$

A better upper-bound will be obtained by minimizing the ratio in *(4.4)* over a range of velocity-fields, i.e. a range of deformation modes of the embankment. An upper-bound so obtained will be denoted F_M. It is plain that for any upper-bound of F, and in particular for F_M, the following implication is valid :

(4.5) $F_M < 1 \Longrightarrow$ unstable embankment.

As it is to be expected from equation *(2.7)*, F_M will be obtained in the form :

(4.6) $$F_M = \frac{C'}{\gamma H} K_M \quad ,$$

where K_M is a function of the same arguments as K^+ ; K_M is an upper bound for K^+.

We shall now examine the use of rigid block deformation modes to obtain an upper bound F_M.

It is to be emphasized that the kinematic approach described here makes no use of any flow rule or anything similar : $P(\underline{v})$ is defined and can be computed from the only knowledge of the strength criteria.

5. *THE YIELD-FACTOR METHOD*

5.1 *Rigid block deformation modes*

In order to apply the kinematic method described in section 4, we consider as a deformation mode of the embankment a rigid block mechanism. Such a mecanism is defined by a center of rotation O and an angular velocity ω.

RBA being a curve drawn across the embankment itself and the layer, it is assumed that the block RBATSR moves as a rigid body round point O with the angular velocity ω, while the rest of the structure remains motionless. Curve RBA is a velocity discontinuity line.

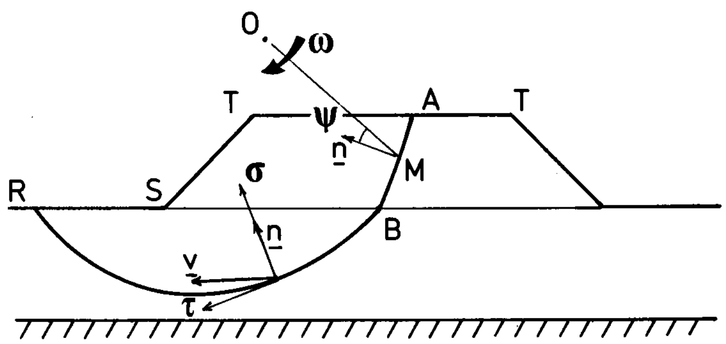

Figure 5 :
Rigid block mechanism.

Looking back at equation (*4.2*), we can see that for this velocity field the first member reduces to :

(*5.1*) $$\{\text{Power of } \underline{\underline{\sigma}} \text{ in } \underline{v}\} = -\int_{RB} (\sigma v_n + \tau v_t)\,ds - \int_{BA} (\sigma v_n + \tau v_t)\,ds,$$

where v_n and v_t are the normal and tangential components of the velocity $\underline{v}$.

Under the strength-criterion (*2.2*) the first integral in equation (*5.1*) is evidently bounded by :

(*5.2*) $$-\int_{RB} (\sigma v_n + \tau v_t)\,ds \leqslant \int_{RB} \pi(\underline{v})\,ds$$

with :

(*5.3*) $$\begin{cases} \pi(v) = +\infty & \text{if } v_n \neq 0 \\ \pi(v) = C'\,|v_t| & \text{if } v_n = 0 \end{cases}$$

As for the second integral, under the strength criterion (*2.1*), it is bounded by :

(*5.4*) $$-\int_{BA} (\sigma v_n + \tau v_t)\,ds \leqslant \int_{BA} \pi(\underline{v})\,ds$$

with (figure 6) :

(*5.5*) $$\begin{cases} \pi(v) = +\infty & \text{if } v_n < |v_t| \tan\phi\ , \\ \pi(v) = C \cot\phi\ v_n & \text{if } v_n \geqslant |v_t| \tan\phi. \end{cases}$$

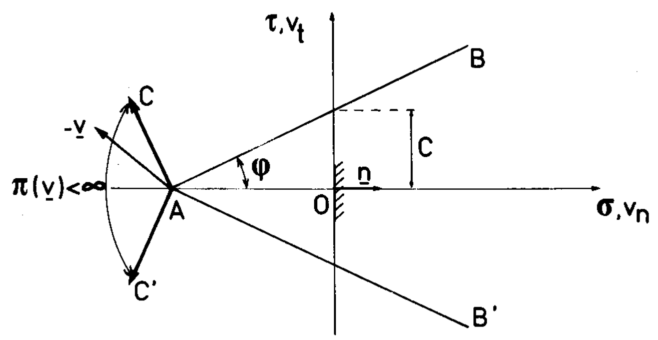

Figure 6 :
Soil with cohesion and friction : intrinsic curve and function $\pi(\underline{v})$.

Therefore $P(\underline{v})$ will be defined as :

$$(5.6) \qquad P(\underline{v}) = \int_{RB} \pi(\underline{v})ds + \int_{BA} \pi(\underline{v})ds$$

The mechanism must be chosen so to obtain a non-trivial upper-bound F_M i.e. $P(\underline{v})$ must be finite. For the first integral in (5.6) to be finite, since $\underline{v}$ is the velocity in a rigid body motion round point O, RB must be *an arc of a circle with a centre O*. For the second integral to be finite it derives from equation (5.5) that in any point M of BA the normal $\underline{n}$ must make an angle ψ, satisfying $\phi \leqslant \psi \leqslant \pi-\phi$, with the vector radius $\underline{OM}$; the angle ψ may not be constant along BA. It must be noticed that the curve BA is never an arc of a circle. As an example BA may be chosen an arc of a log-spiral with a centre O and $\psi \equiv \phi$ (RENDULIC, 1935 ; DRUCKER and PRAGER, 1952).

On the other hand the power of the gravity forces splits into two terms :

1° for the block RBSR, where it reduces to nought since RB is an arc of a circle with a centre O and RSB is horizontal ;
2° for the block BATSB.

It can be proved then (COUSSY and SALENÇON, 1978) that for any given centre O and any given point B, the minimum of ratio F_v is reached when BA is an arc of a log-spiral with a centre O and $\psi \equiv \phi$ (v.z. W.F. CHEN, 1975, for an analogous result).

As a consequence the minimization process for obtaining F_M will be performed on the rigid block mechanisms for which curve RBA is made of an arc of a circle with a centre O in the layer and of an arc of a log-spiral with a centre O and $\psi \equiv \phi$ in the embankment itself (figure 7).

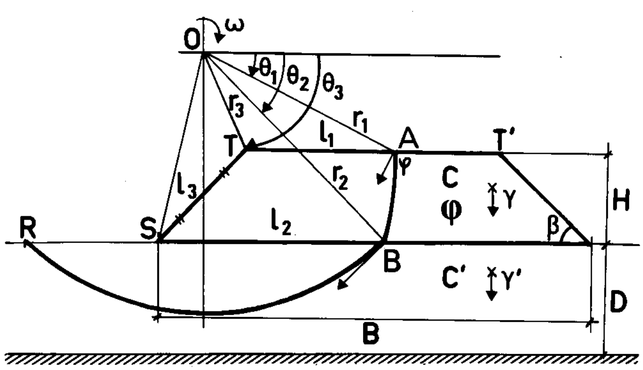

Figure 7 :
Definition of the rigid-block mechanisms used in the "yield-factor method".

5.2. *Determination of* F_M

As shown on figure 7 such a rigid body mechanism may be conveniently described using 3 angular parameters Θ_1, Θ_2, Θ_3 (counted positive clockwise). Detailed calculations show that F_υ takes the form :

$$(5.7) \qquad F_\upsilon = \frac{C'}{\gamma H} K_\upsilon \left(\Theta_1, \Theta_2, \Theta_3, \frac{C}{C'}, \beta, \phi\right),$$

where K_υ is a scalar function of its arguments. F_M will be given by :

$$(5.8) \qquad F_M = \underset{\Theta_1,\Theta_2,\Theta_3}{\text{Min}} \{F_\upsilon\} = \frac{C'}{\gamma H} \underset{\Theta_1,\Theta_2,\Theta_3}{\text{Min}} \{K_\upsilon\}$$

with geometrical restraints on $\Theta_1, \Theta_2, \Theta_3$ involving B/H and D/H. Analytical minimization can be performed with respect to Θ_3 and yields the optimal value of Θ_3 as an explicit function of Θ_1 and Θ_2. It then remains to minimize K_υ with respect to Θ_1 and Θ_2 numerically under the above mentioned restraints. K_υ being an explicit function of Θ_1 and Θ_2 this numerical procedure doesn't need much computer time.

5.3. *Some results obtained through this method. Possibility of extension*

Figure 8 presents some charts of the results obtained by the kinematic approach using rigid-block mechanisms. On account of equation (4.6) and

(5.8) it has been prefered to plot K_M versus D/H for diffent values of the other parameters.

The minimization procedure also leads to a minimizing curve RBA which we propose to call the "weak line" for the given embankment.

This method may be extended to make stability analysis possible in case of an embankment with side-benches, or resting on a multilayer of different purely cohesive soils ; it may happen also that the embankment having been built up gradually a higher value must be assumed for the cohesion of the soil in the layer under the embankment : this case can also be dealt with. So can also the cases of an embankment with a distributed load on TT' or with punctual loads, etc.
For instance Figure 9 presents some charts for K_M in the case of an embankment with side-benches.

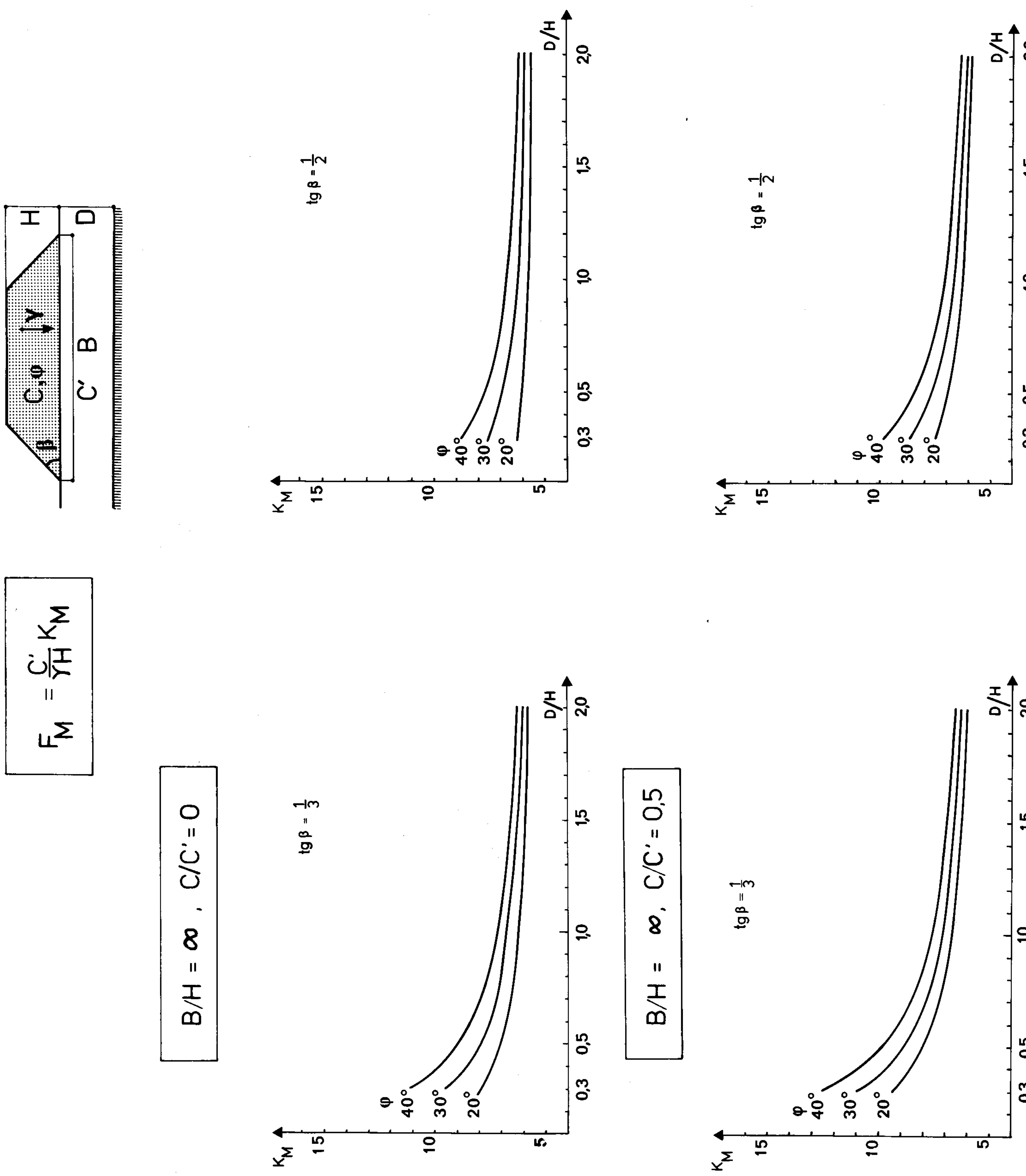

Figure 8 :
Stability analysis of an embankment on a soft ground layer :
charts for K_M.

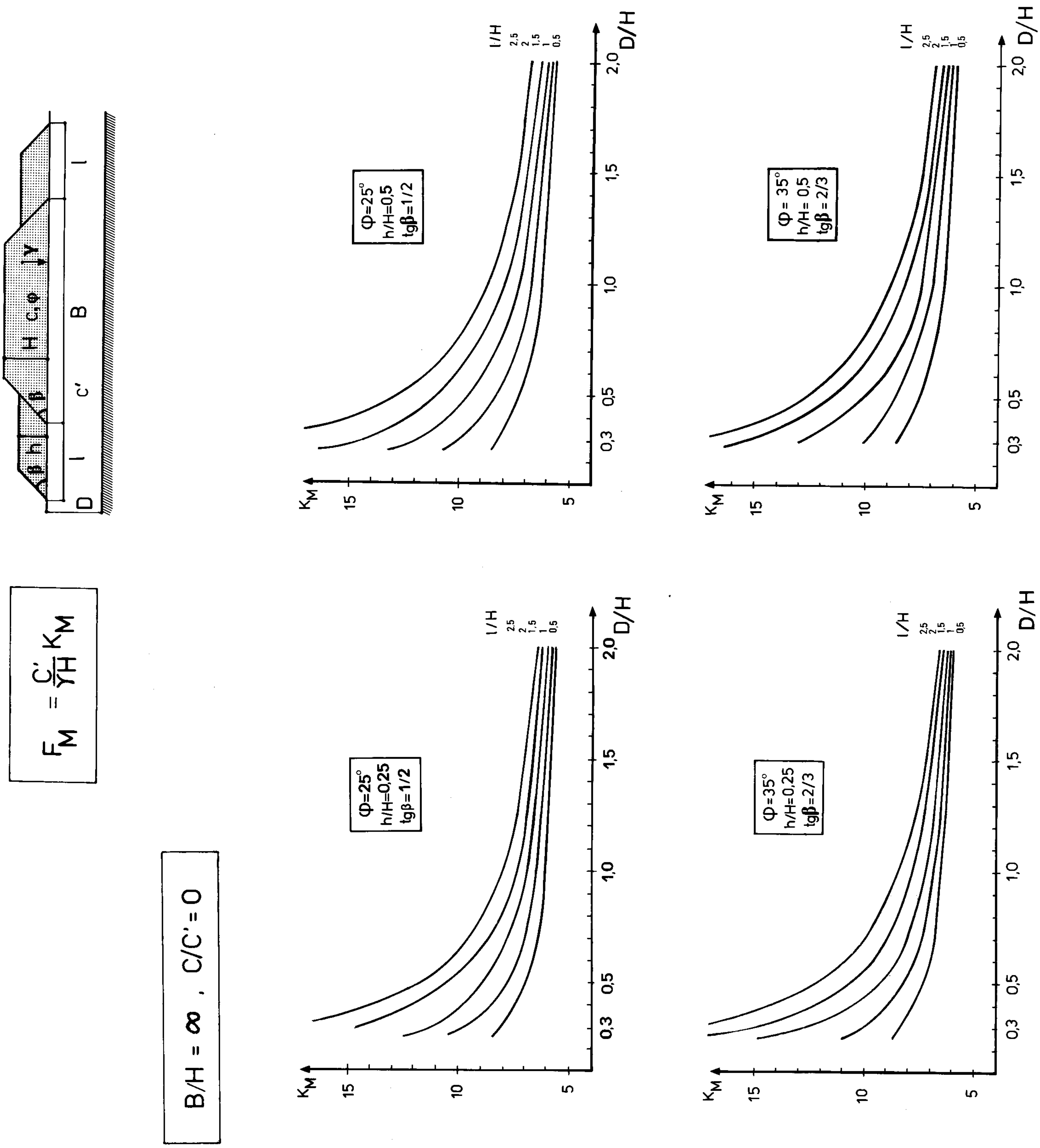

Figure 9 :

Stability analysis of an embankment with side-benches :

Charts for K_M.

5.4. *Critical analysis of the obtained results*

In the following we shall call "yield-factor method" the analysis described hereover which determines an upper bound F_M of F, using rigid block mechanisms.

It is interesting to compare the results obtained by the yield-factor method with those given by a classical method. As an example, a comparison has been carried out with the safety-factor given in the charts obtained by Pilot and Moreau (1973) using a method of slices.

Figure 10 a) shows the ratio "safety factor" / F_M, which is independent of $C'/\gamma H$, as a function of D/H. It can be observed that this ratio decreases, starting from the value 1.30 for D/H = 0.2, down to 0.91 for D/H = 2. It is noticeable that for $D/H < 0.74$, the safety-factor obtained through the method of slices is more optimistic than F_M, this property being plainly marked for $D/H < 0.4$. Its significance is still reinforced by F_M being only an upper bound of the real yield factor F, and important consequences appear for the small values of parameter $C'/\gamma H$. For instance, figure 10b) presents the values of F_M and of the safety factor as functions of D/H, when $C'/\gamma H = 0.1$: it can be seen that for $0.28 < D/H < 0.4$, the safety factor is greater than 1 whereas F_M, an upper bound for the yield factor, is smaller than 1. This means that the instability of the embankment is certain from (*4.5*) despite what might be infered from the value of the "safety"-factor.

We think that such a result must be related to the practical rules which often trust the safety-factor differently according to the constitutive soils. But one must remind that setting a level of confidence greater than 1 to the safety-factor for the embankment to be built, aims not only at preventing instability but chiefly at keeping far enough from instability in order to avoid any preliminary disorder ; the afore presented example shows that this essential purpose may be missed.

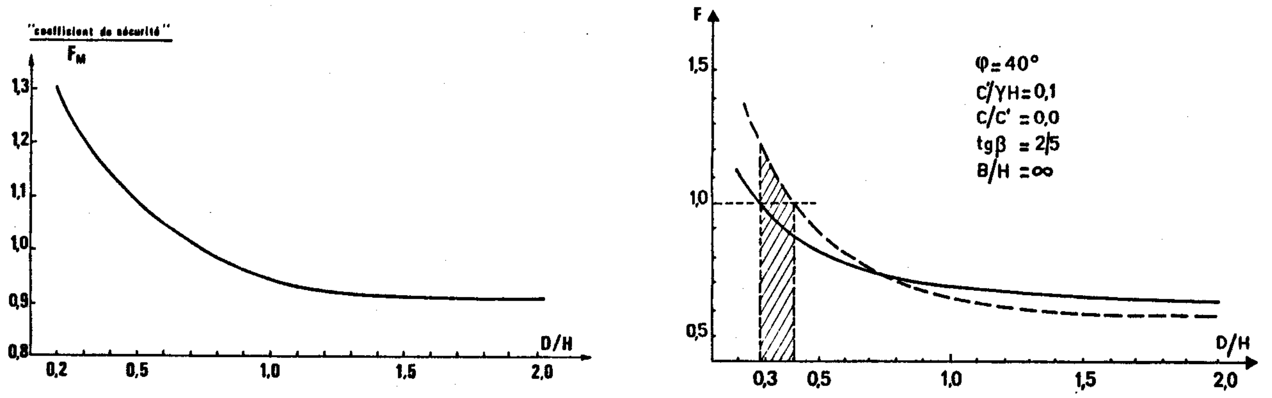

Figure 10 :

A comparison between the yield-factor method and a method of slices.

As for the weak line ? It has been emphasized before, that the method doesn't require any knowledge of a flowrule or anything similar ; this means that the minimizing mechanism cannot in any way pretend to have the significance of the failure mechanism which would take place in case of an unstable embankment. Moreover it must be kept in mind that only the particular range of rigid block mechanisms has been explored to minimize F_v. As a matter of fact the yield-factor method is strictly equivalent to the static analysis presented in section 3 considering the equilibrium of a block : the weak line corresponds, so to speak , to the "weakest" equilibrium.

One must be very cautious when thinking of a comparison with an observed failure mechanism : since in case of unstability great displacements do occur, it is very difficult -not to say risky- to try to imagine the collapse mechanism at its very beginning in the initial geometry ! The case of an embankment on the French river Dives has been used for the comparison presented on figure 11 : the quite good agreement between the calculated weak line and a "remake" of the "real" initial collapse mechanism is worth noticing.

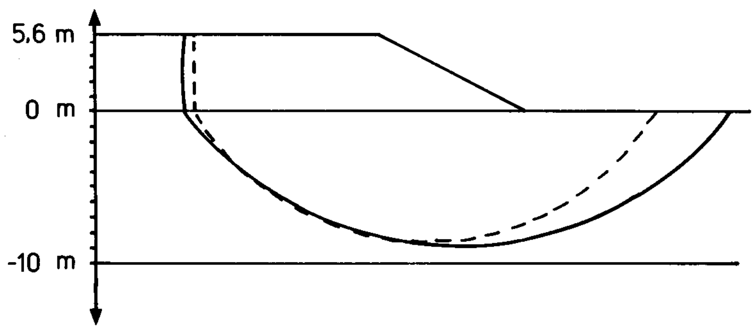

Figure 11 :
Embankment on the river Dives :
calculated weak line (—) and assumed initial collapse mechanism (--)

Finally let us mention that the yield-factor method adequately modified, can be used to analyse earth-dam stability. Effective stresses are considered and the external forces consist in gravity forces (with the different specific weights) and seepage forces. Here again a comparison between the results obtained by the yield-factor method and by a method of slices shows the same kind of discrepancy as before which occurs for non pathological cases.

6. CONCLUSION

The yield-factor method is founded on the only knowledge of the strength-criteria defining the allowable stress-states for the constitutive soils. Therefore the so-obtained upper bound of the yield-factor has a plain significance, ensuring instability when it is less than unity.

Not pretending to be the so-long-waited-for method which will definitely solve any problem of stability-analysis in soil mechanics, it appears to be quite easy to use : it is at least a valuable method for checking the well established classical methods ; examples have been given that this may prove really useful.

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