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LETTER TO THE EDITOR

Comments on 'Determination of the Tensile Strength of Rock by a Compression Test of an Irregular Test Piece'*

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IN A REPORT to be published at the *Second European Symposium on Comminution* [1] the problem of the stress distribution in a spherical sample submitted to two diametrically opposed concentrated forces has been dealt with. The authors rely upon an article by STERNBERG and ROSENTHAL [2] who have made a very thorough study of the problem. The numerical results which appear in the article by HIRAMATSU and OKA [3] and in the article [1] differ for

$$\sigma_\theta \begin{cases} \theta_0 = 0 \\ \theta \ = 0 \end{cases} \quad (\nu = 0{\cdot}2).$$

The solution which was used in [1] and which was obtained by the Boussinesq potential method is of the form $[\bar{S}] = [\bar{S}_2] + [R_2]$, composed of a part $[\bar{S}_2]$ which is singular at the poles, i.e. the sum of three classical potentials, and of a rest $[R_2]$ which is a series of simple potentials. The numerical calculations are greatly facilitated by the fact that the series corresponding to $[R_2]$ for the stresses and the displacements are uniformly convergent in the whole sphere, including the boundary and the poles, and that their convergence is relatively good. The term which defines the behaviour of the stresses at the poles is $[\bar{S}_2]$ and the Boussinesq potential relative to the concentrated load is the item which preponderates in $[\bar{S}_2]$ and supplies for

$$\sigma_\theta \begin{cases} \theta_0 = 0 \\ \theta \ = 0 \end{cases}$$

an infinite *tension*.

The solution used in article [3] also appears in [2] (we have checked the identity of the formulae given in both). It can be shown by algebraic transformations [2] that it is identical with $[\bar{S}]$. The numerical calculations are, however, as indicated by Sternberg and Rosenthal, difficult to do because the corresponding series have a very slight convergence inside the sphere. Theses series are evidently divergent at the poles. Based on the equation (12b) of [3] we can write, for example:

$$\sigma_\theta \begin{cases} \theta \ = 0 \\ \theta_0 = 0 \end{cases} = \sum_{n \geq 0} M_{2n} \left(\frac{r}{a}\right)^{2n}$$

and we can see when doing the calculations that for a sufficiently large n, M_{2n} is equivalent to $-(1-2\nu)$ $F/\pi a^2 n$.

This suffices to show that for $r/a = 1$ the series should diverge towards $-\infty$ (tension) and not towards $+\infty$ as shown in Fig. 5 of [3].

It seems that an error has been made in the numerical calculation, perhaps as a result of using too small a number of terms in the series which would appear to confirm the fairly good agreement of the result of [3] and of [1] for $r/a < 0{\cdot}5$.

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REFERENCES

1. HABIB P., RADENKOVIC D. and SALENÇON J. Rupture des roches ecrasées entre les plateaux d'une presse, *Proceedings of the Second European Symposium on Comminution*, Amsterdam 20–23 September (1966).

**Int. J. Rock Mech. Min. Sci.* 3, 89–99 (1966).

2. STERNBERG E. and ROSENTHAL F. The elastic sphere under concentrated loads, *J. appl. Mech.* (*ASME*) **74**, 413–421 (1952).
3. HIRAMATSU Y. and OKA Y. Determination of the tensile strength of rock by a compression test of an irregular test piece, *Int. J. Rock Mech. Min. Sci.* **3**, 89–99 (1966).

APPENDIX

Calculation of M_{2n}

It will be remembered that the formula (12b) in [3] was

$$\sigma_\theta = \sum_{n=0}^{\infty} \left\{ P_{2n}(\cos\theta) \times \left(\frac{(2n+3)\lambda - (2n-2)\mu}{4n+3} r^{2n} + \frac{4n^2(2n+2)\lambda + 2n(4n^2+4n-1)\mu}{(2n-1)(2n+1)(4n+3)} a^2 r^{2n-2} \right) \right.$$
$$\left. + \frac{\partial^2 P_{2n}(\cos\theta)}{\partial\theta^2} \times \left(-\frac{(2n+3)\lambda + (2n+5)\mu}{(2n+1)(4n+3)} r^{2n} + \frac{2n(2n+2)\lambda + (4n^2+4n-1)\mu}{(2n-1)(2n+1)(4n+3)} a^2 r^{2n-2} \right) \right\}$$
$$\times \frac{-(4n+3)(4n+1)\{\cos\theta_0 P_{2n}(\cos\theta_0) - P_{2n-1}(\cos\theta_0)\}}{[(8n^2+8n+3)\lambda + (8n^2+4n+2)\mu]\, a^{2n}} p.$$

Taking into consideration that

$$\lim_{\theta_0\to 0} \frac{F}{\pi a^2} \frac{\cos\theta_0 P_{2n}(\cos\theta_0) - P_{2n-1}(\cos\theta_0)}{2(1-\cos\theta_0)} = -(2n+1)\frac{F}{2\pi a^2}$$

$$P_{2n}(\cos\theta) = 1 \text{ for } \theta = 0$$

$$\frac{d^2 P_{2n}(\cos\theta)}{d\theta^2} = -n(2n+1) \text{ for } \theta = 0$$

and by introducing the notations

$$\nu = \frac{\lambda}{2(\lambda+\mu)},\ m = \frac{r}{a},\ \frac{F}{\pi a^2} = \sigma_0,$$

this formula produces for $\theta = 0$ and $\theta_0 \to 0$:

$$\sigma_\theta \Big|_{\substack{\theta=0\\ \theta_0=0}} = \sum_{n=0}^{\infty} (A_{2n} m^{2n} + B_{2n-2} m^{2n-2})$$

with

$$A_{2n} = \frac{\sigma_0}{4} \times \frac{(4n+1)(2n+1)[(4n+2)\nu + 2n^2 + 3n + 2]}{[(4n+1)\nu + 4n^2 + 2n + 1]}$$

$$B_{2n-2} = -\frac{\sigma_0}{4} \times \frac{n(4n+1)[4n^2+4n-1+2\nu]}{[(4n+1\nu) + 4n^2 + 2n + 1]}.$$

We can also write

$$\sigma_\theta \Big|_{\substack{\theta=0\\ \theta_0=0}} = \sum_{n=0}^{\infty} M_{2n} m^{2n}$$

where

$$M_{2n} = A_{2n} + B_{2n}.$$

We can also write down

$$M_{2n} = \frac{\sigma_0}{4}\left[\frac{(4n+1)(2n+1)[(4n+2)\nu + 2n^2 + 3n + 2]}{[(4n+1)\nu + 4n^2 + 2n + 1]} - \frac{(n+1)(4n+5)[4n^2 + 12n + 7 + 2\nu]}{[(4n+5)\nu + 4n^2 + 10n + 7]}\right].$$

Hence we have for a sufficiently large n:

$$M_{2n} = \frac{\sigma_0}{4}(8n\nu - 4n) = -n(1-2\nu)\sigma_0.$$